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Source: *The Economic Journal*, Vol. 98, No. 392 (Sep., 1988), pp. 701-719

Published by: [Blackwell Publishing](#) for the [Royal Economic Society](#)

Stable URL: <http://www.jstor.org/stable/2233909>

Accessed: 04/10/2010 12:02

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FOOD SUBSIDIES AND POVERTY ALLEVIATION*

Timothy Besley and Ravi Kanbur

The question of food subsidies seems to be at the forefront of nearly all discussion of macroeconomic adjustment in developing countries. The discussions are highly charged, with issues of political expediency being tied to issues of economic efficiency and equity. The annual round of negotiations with the IMF, which for many countries have been centred on conditionality of IMF loans, means that the food subsidy question appears regularly on the agenda, and media coverage of ‘IMF riots’ – disturbances in the wake of the IMF negotiation decreases in food, fuel and transportation subsidies – have brought an awareness of the question to people in developed countries as well.

When faced with the charge that the food (and fuel) subsidies present excessive budgetary exposure, and are responsible for a large part of the fiscal deficit, many LDC governments reply that a reduction of subsidies would seriously affect their poverty alleviation objectives. The counter to this is usually that the pattern of food subsidies, as it stands, is not appropriate to this objective. With better targeting, it is argued, the poverty alleviation objective could be attained at far lower costs. Minimisation of the ‘leakages’ involved in the existing pattern of subsidies thus becomes the key task.

What is the optimal pattern of food subsidies, under budgetary constraints, when the objective is poverty minimisation? This is the question to which this paper is addressed. We wish to derive rules to guide the design of food subsidies for poverty alleviation, in the framework of the modern public finance literature. In the space available we cannot hope to cover all of the issues, but we do hope to make a start in making precise the main trade-offs involved.

A key assumption of this paper is that it is not possible to identify, costlessly, households below the poverty line. While such identification is attempted in many countries, and whatever the initial success, it is unlikely to be administratively feasible in the medium to long run. In principle it requires the means testing of hundreds of thousands, if not millions, of households every year, and it is difficult to believe that accuracy in identification can be purchased cheaply. It is for this reason that generalised subsidies – on commodities that both rich and poor consume – are an attractive option. The disadvantage is of course that from the point of view of the poverty alleviation objective, the leakage to the rich may be large. This is the trade-off we face.

In the macroeconomic adjustment literature the focus has been on the budgetary consequences of food subsidies because it is assumed that producer prices and consumer prices are independent. One way of eliminating the fiscal

* The authors are grateful to two anonymous referees and seminar participants in Oxford, Princeton, Yale, Boston, the World Bank and the I.M.F. for comments and suggestions.

deficit from these operations is to bring producer and consumer prices closer together. But lowering producer prices will lower rural incomes and hence exacerbate the problem of poverty there. Thus added to the question of the pattern of consumer subsidies is the fact that the interests of net sellers and net purchasers of food are diametrically opposed. Unless alternative compensation schemes are followed, a change in the price of food will create gainers and losers (see World Bank, 1986), and the task of analysis is to present a framework in which these gains and losses can be weighed against each other. Given the focus of this paper, the efficacy of policies will be evaluated with respect to their effect on *poverty* at the national level.

In analysing the impact of food subsidy, it matters a great deal whether the subsidy is on the marginal unit of consumption or on infra-marginal units, and the analysis of this paper keeps these two issues separate. After a discussion of the measurement of poverty with distorted prices, in Section I, we consider infra-marginal subsidies in Section II. The most common form of these in developing countries are ration shops, where a given quantity (ration) of a food is provided at below market prices. After characterising the effect of such schemes on incomes and poverty, we ask the question – where should ration shops be located, and what information should guide the targeting of the subsidies to different regions? Section III considers subsidies at the margin, e.g. an import subsidy which lowers the free market price of food. Which commodities should attract the highest subsidy, given budgetary constraints? We give characterisations in terms of indicators that can be estimated from readily available data. Section IV compares the schemes set out in Section III and IV and asks what their relative benefits are in terms of the objective of poverty alleviation. Throughout Sections II–IV, the focus is on consumers. The effect of lower food prices on net sellers of food is ignored. Section V remedies this by taking explicit account of producer poverty. A framework is presented for weighing up the conflicting effects on consumer and producer poverty, and simple rules are suggested for when a lower price of food may indeed lower national poverty, despite the adverse effect it has on producers. Section VI concludes the paper.

I. ON MEASURING POVERTY

Prior to analysing the role of food subsidies in alleviating poverty, we have first of all to specify what we mean by poverty. We need a poverty line and then a poverty index which aggregates information on units below the poverty line. Poverty, like the standard of living, is a multidimensional concept. There are many ways of reducing these many dimensions to a single one, and then specifying a cutoff point such that units below that point are regarded as being in poverty. A detailed discussion of the issues involved would be beyond the scope of this paper (see Kanbur, 1987*a*). In what follows we suggest an approach which is operational and convenient, as well as appropriate, for the questions we are asking.

Consider a consumer facing prices \mathbf{q} and with lump sum income y . His indirect utility function is denoted

$$V(\mathbf{q}, y). \quad (1)$$

Consider also a reference price vector \mathbf{p} which for most purposes we will assume to be the vector of world prices. Following King (1983) the consumer's equivalent income y_E is defined implicitly from:

$$V(\mathbf{p}, y_E) = V(\mathbf{q}, y). \quad (2)$$

Inverting this we get the equivalent income function:

$$y_E = y_E(\mathbf{p}, \mathbf{q}, y), \quad (3)$$

which tells us the lump sum income necessary to keep the consumer as well off at reference prices under the new price structure. As is usual, we interpret y_E as a monetary measure of the consumer's well being.

Having defined a unidimensional measure of the consumer's standard of living we next have to choose a poverty line which cuts off the poor from the non-poor, following Sen's (1981) 'focus' axiom. There are many ways of arriving at such a line, nutrition-based and otherwise. But for our purposes it suffices to have an arbitrary cutoff at z_E , given exogenously, so that those with $y_E \leq z_E$ are classified as being in poverty. Since y_E is monotonically increasing in y , there will be a cutoff in y space corresponding to z_E which we call z :

$$z_E = y_E(\mathbf{p}, \mathbf{q}, z) \Leftrightarrow z = z(\mathbf{p}, \mathbf{q}, z_E), \quad (4)$$

and it is immaterial which space we choose to specify the cutoff (there is also a cutoff in utility space).

Given the cutoff, we need now to specify an index of poverty. This has led to a large literature, discussed recently by Atkinson (1987) and by Pyatt (1987). The axiomatic basis of many of the poverty indices in common use has been considered in this literature, but a consensus seems to have emerged around the properties that poverty measures should have. For example, Foster and Shorrocks (1987) show that the requirement of sub-group consistency, which says that if poverty of any sub-group increases then so does aggregate poverty, and a homogeneity restriction, essentially imply that the poverty measure will be an additively homogeneous function of the normalised poverty gap:

$$\frac{z_E - y_E}{z_E}.$$

Such a measure was in fact proposed by Foster *et al.* (1984). Writing the density function of lump sum income as $f(y)$, the measure is thus:

$$P_\alpha = \int_0^z \left(\frac{z_E - y_E}{z_E} \right)^\alpha f(y) dy \quad \alpha \geq 0. \quad (5)$$

This family of measures contains several well known and commonly used poverty indices as special cases. For instance, when $\alpha = 0$,

$$P_0 = \int_0^z f(y) dy = H, \quad (6)$$

the well known head count ratio or incidence of poverty – the fraction of units below the poverty line. When $\alpha = 1$,

$$\begin{aligned} P_1 &= \int_0^z \left(\frac{z_E - y_E}{z_E} \right) f(y) dy \\ &= HI, \end{aligned} \quad (7)$$

where

$$I = \frac{z_E - \bar{y}_E^p}{z_E} \quad (8)$$

is the poverty gap ratio for the mean poor income, \bar{y}_E^p . Apart from a normalisation, P_1 tells us the total poverty gap of the poor – the amount of money necessary to eradicate poverty assuming perfect targeting and no other costs.

Now, the measure $H (= P_0)$ was criticised by Sen (1976) for being insensitive to the depth of poverty. While P_1 allows for this sensitivity, it is not sensitive to the distribution of income *within* the poor. This extra degree of sensitivity can be introduced in different ways. In the context of the Foster *et al.* (1984) measure the straightforward way of doing this is to choose values of α greater than 1. Thus the P_α class of measures carries within it as members some commonly used indices, and is capable of representing increased sensitivity to the poorest of the poor, as reflected in the choice of α . It also has other convenient properties such as subgroup decomposability, which we will refer to in greater detail in Section II. Of course other poverty measures, such as that of Sen (1976) could be used, but these do not satisfy subgroup decomposability. Unless there is a major objection to the subgroup consistency axiom, which seems to be a reasonable requirement, we feel that the normative and analytical properties of the P_α family of measures are adequate for our purposes. For these reasons we choose to conduct our analysis of food subsidies and poverty alleviation using the P_α class of poverty measures.

II. SUBSIDISED RATIONS: WHERE SHOULD RATION SHOPS BE LOCATED?

In many developing countries, food subsidies are given in the form of generalised rations though ‘fair price’ shops. These rations are made available at a price which is below the market price. Until the reforms of the late 1970s, for example, all Sri Lankans were entitled to a certain quantity of rice, at below free market prices, sometimes completely free (Rasputra, 1986). How should we model the effect of this type of scheme on customers’ well being? If the ration can be resold at the higher free market price then it is clear that the

subsidy is infra-marginal and equivalent to a lump sum income transfer equal to the ration quantity times the difference between the market price and the ration price. We believe that preventing the resale of food rations is extremely difficult in developing countries and we will use the model of 'rations with resale' income transfer in what follows. It should then be clear that since all that is happening is the transfer of lump sum income to consumers, the *pattern* of subsidies across commodities is irrelevant, as is the question of whether a large ration is given with a small subsidy or a small ration is given with a large subsidy. What is important is the net transfer of lump sum income to consumers.

Since the consumption trade-offs at the margin are given by market prices (which we assume to be world prices for convenience), then the equivalent income of a consumer who receives a transfer m is given by:

$$y_E = y_E(\mathbf{p}, \mathbf{p}, y + m) = y + m, \quad (9)$$

where $m = (\mathbf{p} - \mathbf{q})' \bar{\mathbf{x}}$, the value of transfer implied by ration $\bar{\mathbf{x}}$ at the infra-marginal subsidy price \mathbf{q} . Notice that \mathbf{q} does *not* enter (9) since \mathbf{p} reflects the relevant marginal trade-offs in consumption even when the food subsidy scheme is in place.

What is the effect on poverty of cutting back on such rationed food subsidies? The poverty index is:

$$P_\alpha = \int_0^z \left[\frac{z_E - y_E(\mathbf{p}, \mathbf{p}, y + m)}{z_E} \right]^\alpha f(y) dy. \quad (10)$$

We can work directly in terms of the effect on P_α of a change in m :

$$\frac{\partial P_\alpha}{\partial m} = \int_0^z \left(\frac{\alpha}{z_E} \right) \left(\frac{z_E - y_E}{z_E} \right)^{\alpha-1} \left(-\frac{\partial y_E}{\partial m} \right) f(y) dy. \quad (11)$$

From (9), it follows that $\partial y_E / \partial m = 1$. Hence:

$$\begin{aligned} \frac{\partial P_\alpha}{\partial m} &= \left(-\frac{\alpha}{z_E} \right) \int_0^z \left(\frac{z_E - y_E}{z_E} \right)^{\alpha-1} f(y) dy \\ &= \left(-\frac{\alpha}{z_E} \right) P_{\alpha-1}. \end{aligned} \quad (12)$$

Expression (12), which is closely related to the results in Kanbur (1987*b*), tells us that the marginal impact on P_α of a small cut in the food subsidy budget is proportional to $P_{\alpha-1}$. Thus if $\alpha = 1$ then the impact is proportional to $P_0 = H$, i.e. the head count ratio. The intuition behind this should be clear. A marginal cut in the food subsidy budget reduces everybody's income in the population by the same small amount. The poverty gap goes up by the amount of the cut per person times the number of the poor. Allowing for the normalisation $H (= P_0)$ is the indicator of the severity of the effect on P_1 . The elasticity of poverty with respect to food subsidy cuts is given by:

$$\epsilon_\alpha = -\frac{m}{P_\alpha} \frac{\partial P_\alpha}{\partial m} = \alpha \left(\frac{m}{z_E} \right) \left(\frac{P_{\alpha-1}}{P_\alpha} \right). \quad (13)$$

When $\alpha = 1$ this collapses to:

$$\epsilon_1 = \frac{m P_0}{z_E P_1} = \frac{m}{z_E - \bar{y}_E^p}. \quad (14)$$

Thus the poverty elasticity in this special case is seen to depend on how much the food subsidy transfer 'fills' the poverty gap of the average poor person.

We have so far assumed that *all* consumers have access to the public food distribution system through ration shops. While this may be a good description for small countries, in larger countries where the population is more geographically dispersed the location of ration shops turns out to be an important component of *targeting* income transfers to the poor. The idea is that only people within a certain radius of the ration shop will find it worthwhile to come to the ration shop, and the poverty characteristics of different geographical regions could be used to decide whether or not to locate a ration shop there. Such a procedure is reported in World Bank (1986):

A coupon program that distributed food every two weeks through government-run supermarkets used income to determine who could participate in Recife, Brazil. The program revealed several problems... Building on lessons from the evaluations, the Brazilian program was modified, with apparent success, to reach very low-income neighborhoods without coupons or down payments. Common basic foods now are subsidized for all customers of many registered small neighborhood stores in selected poverty areas. Any leakages of benefits to people not in need is much less expensive than administering the cumbersome coupon program.

The above commentary on an operational scheme raised two points of interest. First, it confirms the basic tenet of this paper as regards the costliness of means testing and targeting individual households. Second, it raises the question of where to locate the ration shops or, rather, to which neighbourhoods the subsidies should be targeted towards 'poverty areas'. Our object is to make the latter claim precise.

Suppose that we can divide the population into two mutually exclusive groups by region (the analysis extends easily to many regions). We have survey information about the distribution of income (and hence about poverty) in the two regions. How should this information be used to target increases or cuts in food subsidy expenditure? Let the regions be indexed by 1 and 2, and let the poverty in the two regions taken separately by $P_{1,\alpha}$ and $P_{2,\alpha}$. If the proportion of national population in the two regions is λ_1 and λ_2 ($\lambda_1 + \lambda_2 = 1$), then Foster *et al.* (1984) show that national poverty can be written:

$$P_\alpha = \lambda_1 P_{1,\alpha} + \lambda_2 P_{2,\alpha}. \quad (15)$$

Now, increasing the budgetary subsidy in region 1 by an amount b_1 will increase each person's income there by an amount b_1/λ_1 . Using this in conjunction with the structure of argument in (12) gives us:

$$\frac{\partial P_\alpha}{\partial b_1} = \lambda_1 \frac{\partial P_{1,\alpha}}{\partial b_1} = \left(-\frac{\alpha}{z_E} \right) P_{1,\alpha-1}. \quad (16)$$

Similarly, directing a subsidy b_2 to region 2 will increase each person's income thereby by an amount b_2/λ_2 , giving us

$$\frac{\partial P_\alpha}{\partial b_2} = \lambda_2 \frac{\partial P_{2,\alpha}}{\partial b_2} = \left(-\frac{\alpha}{z_E} \right) P_{2,\alpha-1}. \quad (17)$$

What (16) and (17) tell us is that if the objective is to minimise P_α at the national level, then the region with the higher $P_{\alpha-1}$ (*not* the region with the higher P_α) should be favoured at the margin. At least for marginal changes, then, what is relevant is *not* the value of poverty index P_α in the two regions, but the value of the *indicator* associated with P_α , namely $P_{\alpha-1}$, in the two regions. Thus when $\alpha = 1$ the policy conclusion is to target towards regions with high head count ratios even though the objective is to minimise P_1 . Ration shops should thus be located in neighbourhoods with high incidences of poverty – this is the strategy that will be most cost effective in reducing the national poverty gap.

III. SUBSIDIES AT THE MARGIN: WHICH FOODS SHOULD CARRY A HIGH SUBSIDY?

Alongside the subsidised ration scheme described and analysed in the previous section, many developing countries have food subsidy schemes which subsidise the consumption of food at the margin. In these cases trade in the commodity is in the hands of the government which, for example, purchases on the world market and sells to consumers at a lower price but without a ration – consumers can buy as much as they wish. In most countries both infra-marginal and marginal subsidy schemes are in operation (see for example the description of arrangements in Egypt by Scobie, 1983). Since subsidies at the margin affect consumer budgeting, this has to be taken into account in our analysis of poverty.

Consider the case where the government alters the subsidy rates on two commodities while maintaining its budget balanced. We write:

$$q_i = p_i + t_i; \quad i = 1, 2, \dots, n,$$

for the post tax prices of the n commodities, and note from the definition of P_α that:

$$\frac{\partial P_\alpha}{\partial t_i} = \frac{\alpha}{z_E} \int_0^z \left[\frac{z_E - y_E(\mathbf{p}, \mathbf{q}, y)}{z_E} \right]^{\alpha-1} \left(-\frac{\partial y_E}{\partial q_i} \right) f(y) dy. \quad (18)$$

Letting consumer demands be $x_i(\mathbf{q}, y)$ the government's budget constraint is:

$$\int_0^\infty [\sum_k t_k x_k(\mathbf{q}, y)] f(y) dy = B. \quad (19)$$

If the commodity taxes to be altered are t_1 and t_2 then it follows from differentiating (19) that for budget balance:

$$\frac{dt_1}{dt_2} = - \frac{\int_0^\infty \left(\sum_k t_k \frac{\partial x_k}{\partial t_2} + x_2 \right) f(y) dy}{\int_0^\infty \left(\sum_k t_k \frac{\partial x_k}{\partial t_1} + x_1 \right) f(y) dy}. \quad (20)$$

Using (18) and (20) yields

$$\frac{dP_\alpha}{dt_1} = \frac{\alpha}{z_E} \int_0^z \left(\frac{z_E - y_E}{z_E} \right)^{\alpha-1} \left[\left(-\frac{\partial y_E}{\partial q_1} \right) - \left(-\frac{\partial y_E}{\partial q_2} \right) \frac{\int_0^\infty \left(\sum_k t_k \frac{\partial x_k}{\partial t_1} + x_1 \right) dF}{\int_0^\infty \left(\sum_k t_k \frac{\partial x_k}{\partial t_2} + x_2 \right) dF} \right] dF(y), \quad (21)$$

where we replace $f dy$ with dF for notational convenience.

The most straightforward case that we can consider is that in which we are initially at a no-tax position so that $t_k = 0$ for all k . Let us use the following notation

$$\bar{x}_i = \int_0^\infty x_i dF \quad (22)$$

$$\bar{x}_i^P = \left(\int_0^z x_i dF \right) / \int_0^z dF \quad (23)$$

for the mean consumption of commodity i by the population as a whole and by the poor, respectively. From the definition of y_E we have that:

$$\frac{\partial y_E}{\partial q_i} = \frac{\partial V(\mathbf{q}, y)}{\partial q_i} / \frac{\partial V(\mathbf{p}, y_E)}{\partial y_E}.$$

Using Roy's identity this becomes

$$\frac{\partial y_E}{\partial q_i} = \left[-\frac{\partial V(\mathbf{q}, y)}{\partial y} / \frac{\partial V(\mathbf{p}, y_E)}{\partial y_E} \right] x_i(\mathbf{q}, y). \quad (24)$$

If $\mathbf{p} = \mathbf{q}$, then $y_E = y$, so that:

$$\frac{\partial y_E}{\partial q_i} \Big|_{\mathbf{p}=\mathbf{q}} = -x_i(\mathbf{q}, y). \quad (25)$$

Thus (21) now becomes

$$\frac{dP_\alpha}{dt_1} \Big|_{\mathbf{p}=\mathbf{q}} = \frac{\alpha}{z} \bar{x}_1 \int_0^z \left(\frac{z_E - y_E}{z_E} \right)^{\alpha-1} \left(\frac{x_1}{\bar{x}_1} - \frac{x_2}{\bar{x}_2} \right) dF. \quad (26)$$

Expression (26) gives us the effect on poverty of a reallocation of food subsidies at the margin. It shows that the net effect depends on how large total consumption of a commodity is relative to its consumption by the poor, weighted by $[(z_E - y_E)/z_E]^{\alpha-1}$. It should be clear that the result in (26) does not

depend upon the specific form of the Foster *et al.* poverty measure. Any additively separable poverty measure will yield a similar result, with appropriate replacement of the weighting function. For special cases, consider $\alpha = 1$, so that the objective is to minimise the poverty gap at the national level. Then (26) collapses to:

$$\frac{dP_1}{dt_1} = \frac{\bar{x}_1}{z} H\left(\frac{\bar{x}_1^P}{\bar{x}_1} - \frac{\bar{x}_2^P}{\bar{x}_2}\right), \quad (27)$$

which says that the subsidy should be targeted towards that commodity for which \bar{x}_i^P/\bar{x}_i is highest, if the objective is to minimise P_1 . The above argument makes precise a claim that is often made, and repeated in World Bank (1986).

The main determinant of a food's suitability for subsidy is the share of it that goes to the target population. If a food is consumed exclusively by the target group, the subsidy will be very efficient; a dollar's worth of subsidy will provide almost a dollar of added income to the target group. But if the target population consumes only 30 percent of a subsidized food, the subsidy is much less efficient. This efficiency varies according to the food chosen. In Brazil, for example, a dollar spent on subsidizing bread transfers about 18 cents to the low-income population and a dollar spent on subsidizing legumes, about 39 cents... Food subsidies for consumers can be even more efficient if further selectivity is introduced by, say, subsidizing inferior grades consumed by the poor.

Our formal analysis shows the microfoundations of the above argument, and the conditions under which it holds true. If either the axioms underlying the measure are not satisfied or $\alpha \neq 1$, then the simplicity of the rule in (27) disappears. However, suppose that the indirect utility function has the quasi homothetic form, so that:

$$y_E = a(\mathbf{p}, \mathbf{q}) + b(\mathbf{p}, \mathbf{q}) y, \quad (28)$$

and

$$x_i(\mathbf{q}, y) = \delta_{i0}(\mathbf{q}) + \delta_{i1}(\mathbf{q}) y, \quad (29)$$

(i.e. Engel curves are linear), then with a general $\alpha (> 1)$, (26) becomes:

$$\left. \frac{dP_\alpha}{dt_1} \right|_{\mathbf{p}=\mathbf{q}} = \frac{\alpha \bar{x}_1}{z} P_{\alpha-1} \left(\frac{\delta_{10}}{\bar{x}_1} - \frac{\delta_{20}}{\bar{x}_2} \right) + \alpha \bar{x}_1 (P_{\alpha-1} - P_\alpha) \left(\frac{\delta_{11}}{\bar{x}_1} - \frac{\delta_{21}}{\bar{x}_2} \right). \quad (30)$$

Hence the effect on poverty depends on current values of P_α and $P_{\alpha-1}$ as well as the demand parameters δ_{i0} and δ_{i1} . A simple statement such as 'target towards the commodity which has the highest ratio of consumption by poor total consumption' is no longer valid, although it can be shown that (26) can also be written as:

$$\left. \frac{dP_\alpha}{dt_1} \right|_{\mathbf{p}=\mathbf{q}} = \frac{\alpha \bar{x}_1}{z} P_{\alpha-1} \left(\frac{\hat{x}_1}{\bar{x}_1} - \frac{\hat{x}_2}{\bar{x}_2} \right), \quad (31)$$

where:

$$\hat{x}_1 = \delta_{i0} + \delta_{i1} \hat{y}$$

is the consumption of the 'representative' poor person defined by income level \hat{y} :

$$\hat{y} = \frac{\int_0^z \left(\frac{z_E - y_E}{z_E} \right)^{\alpha-1} y dF}{\int_0^z \left(\frac{z_E - y_E}{z_E} \right)^{\alpha-1} dF} = z \left(1 - \frac{P_\alpha}{P_{\alpha-1}} \right),$$

recalling that when $\mathbf{p} = \mathbf{q}$, $y_E = y$. In this case of linear Engel curves the simple rule in (27) is seen to be a special case of (31), which has a more general weighting scheme to define the representative poor person – in the special case of $\alpha = 1$ the representative poor person is in fact the average poor person.

While they give us neat results, the empirical relevance of linear Engel curves can be questioned. Econometric analysis (e.g. Deaton, 1981) seems to show non-linearity in observed Engel curves. If we let the Engel curve be a quadratic

$$x_i(\mathbf{q}, y) = \delta_{i0}(\mathbf{q}) + \delta_{i1}(\mathbf{q})y + \delta_{i2}(\mathbf{q})y^2,$$

then it can be shown (see Appendix A 1) that:

$$\left. \frac{dP_\alpha}{dt_1} \right|_{\mathbf{p}=\mathbf{q}} = \frac{\alpha \bar{x}_1}{z} \left\{ P_{\alpha-1} \left(\frac{\delta_{10}}{\bar{x}_1} - \frac{\delta_{20}}{\bar{x}_2} \right) + (P_{\alpha-1} - P_\alpha) \left(\frac{\delta_{11}}{\bar{x}_1} - \frac{\delta_{21}}{\bar{x}_2} \right) z \right. \\ \left. + [(P_{\alpha+1} - P_\alpha) + (P_{\alpha-1} - P_\alpha)] \left(\frac{\delta_{12}}{\bar{x}_1} - \frac{\delta_{22}}{\bar{x}_2} \right) z^2 \right\}. \quad (32)$$

As can be seen, in this case it is the values of $P_{\alpha+1}$, P_α and $P_{\alpha-1}$ which matter. These can be calculated from household income and expenditure surveys, and if we have the Engel curve coefficients δ_{ij} , then the efficacy of reallocating food subsidies can be assessed using the above analysis. If it were felt necessary to use a higher order polynomial to represent the Engel curve, then the generalisation derived in the Appendix A 1 can be used.

Consider now the case where we do not start from all taxes being zero, i.e. we allow $\mathbf{p} \neq \mathbf{q}$. Then (26) no longer holds, and we have to use the more general form in (21). However, if:

$$\frac{\partial V(\mathbf{q}, y)}{\partial y} \bigg/ \frac{\partial V(\mathbf{p}, y_E)}{\partial y_E} = \sigma(\mathbf{p}, \mathbf{q}, y, y_E) \quad (33)$$

is independent of y then the analysis is simplified. This independence obtains if we use the pre-reform price vector (even if it is tax distorted) as the reference price vector, as suggested by King (1983). In this case $\mathbf{p} = \mathbf{q}$ and $\sigma = 1$. However, if this is felt to be inappropriate then we could restrict preferences such that even with $\mathbf{p} \neq \mathbf{q}$, σ is independent of y . It is shown in Appendix A 2 that this happens for the linear expenditure system. This is true more generally for linear Engel curves.

With (33) satisfied, (21) now becomes:

$$\frac{dP_\alpha}{dt_1} = \frac{\alpha \sigma}{z_E} \int_0^z \left(\frac{z_E - y_E}{z_E} \right)^{\alpha-1} \left[x_1 - x_2 \frac{(1+d_1)\bar{x}_1}{(1+d_2)\bar{x}_2} \right] dF, \quad (34)$$

where

$$d_i = \int_0^\infty \sum_k t_k \frac{\partial x_k}{\partial t_i} dF / \int_0^z x_i dF \quad (35)$$

is related to the Mirrlees (1976) 'index of discouragement' for taxes (notice, however, that while Mirrlees uses compensated demands, we have used uncompensated demands). Using this we get a simple rearrangement of equation (21) as,

$$\frac{dP_\alpha}{dt_1} = \sigma (1 + d_1) \bar{x}_1 \frac{\alpha}{z_E} P_{\alpha-1} \left[\frac{\hat{x}_1}{\bar{x}_1 (1 + d_1)} - \frac{\hat{x}_2}{\bar{x}_2 (1 + d_2)} \right], \quad (36)$$

where

$$\hat{x}_i = \frac{\int_0^z \left(\frac{z_E - y_E}{z_E} \right)^{\alpha-1} x_i(\mathbf{q}, y) dF}{\int_0^z \left(\frac{z_E - y_E}{z_E} \right)^{\alpha-1} dF} \quad (37)$$

is the representative consumption of the poor, consumption being weighted by the social weights $[(z_E - y_E)/z_E]^{\alpha-1}$. If x_i is linear in y then we get back to the analogues of (31) and (32).

Our earlier arguments are now modified in the following way. When $\alpha = 1$, we look now for food commodities which have not only a high ratio of poor consumption to total consumption (assuming $1 + d_i > 0$), but also for those which have a low index of discouragement. When $\alpha > 1$ the appropriate generalisation applies. The above results can be seen as a special case of the general analysis of Diamond (1975) who characterised optimality of tax rates in terms of compensated d_i s and the 'distributional characteristic' of a commodity θ_i , defined as the normalised covariance between consumption of good i and an appropriately defined distributional weight (the net social marginal utility of income). With poverty as the objective the distributional characteristic has a particular form given by the structure of the poverty index. Not surprisingly, it depends upon the consumption of the poor relative to total consumption. For special cases, straightforward operational rules in reallocating food subsidies are thus seen to have firm foundations in micro theory, and our analysis presents conditions under which these rules are exactly optimal, and indicates when they are only approximately optimal.

IV. THE CHOICE BETWEEN MARGINAL AND INFRA-MARGINAL SUBSIDIES

In Section III we looked at the principles by which the pattern of subsidies at the margin may be altered to achieve better the objective of poverty alleviation at fixed budgetary costs, while in Section II a similar exercise was undertaken for the distribution of infra-marginal subsidies for a commodity. Intuitively, infra-marginal subsidies transfer purchasing power independently of current income while subsidies at the margin do so in proportion to current consumption of the commodity in question, and hence (to the first order) in

proportion to income. For a given budget, therefore, infra-marginal subsidies are better at alleviating poverty. However, there may be administrative difficulties in achieving full coverage of the population, and there may be take-up problems associated with individuals not going to ration shops because they thereby declare themselves to be poor. By contrast, a price subsidy, through an import subsidy, for example, is comprehensive and reaches the whole population. It is this trade-off that we wish to make precise in this section.

Denoting the subsidy at the margin by s_i , the budgetary cost of this subsidy is:

$$B = \int_0^z s_i x_i(\mathbf{q}, y) dF. \quad (38)$$

Thus

$$\frac{dB}{ds_i} = \int_0^\infty \left(x_i + s_i \frac{\partial x_i}{\partial q_i} \right) dF. \quad (39)$$

We are focusing on commodity i by assuming no other taxes on other commodities, and we sharpen the focus further by evaluating all changes at $s_i = 0$. Thus:

$$\frac{dB}{ds_i} = \int_0^\infty x_i dF = \bar{x}_i. \quad (40)$$

The effect on poverty of a small increment in the budgetary deficit, transmitted to the population via an increment of the subsidy s_i , is given by:

$$\left(\frac{dP_\alpha}{dB} \right)_M = \frac{dP_\alpha}{ds_i} \frac{ds_i}{dB} = -\frac{\alpha}{z\bar{x}_i} \int_0^z \left(\frac{z_E - y_E}{z_E} \right)^{\alpha-1} x_i(\mathbf{p}, y) dF, \quad (41)$$

where 'M' stands for 'marginal' and it is understood that all derivatives are evaluated at $s_i = B = 0$.

Now, if the same increment in the deficit were to be transferred to the population via an infra-marginal (IM) subsidy which gave m to each person through some combination of a ration plus a subsidy on the world price for that ration, then $B = m$ and:

$$\left(\frac{dP_\alpha}{dB} \right)_{IM} = -\frac{\alpha}{z} \int_0^z \left(\frac{z_E - y_E}{z_E} \right)^{\alpha-1} dF. \quad (42)$$

For $\alpha = 1$, (41) and (42) become:

$$\left(\frac{dP_1}{dB} \right)_M = -\frac{1}{z} H \frac{\bar{x}_i^P}{\bar{x}_i} \quad (43)$$

$$\left(\frac{dP_1}{dB} \right)_{IM} = -\frac{1}{z} H. \quad (44)$$

Thus, so long as $\bar{x}_i^P < \bar{x}_i$ i.e. the mean consumption of the poor is less than the mean consumption of the population as a whole, then for $\alpha = 1$ it is always better to use the infra-marginal subsidy. More generally, it is shown in

Appendix A 3 that this result holds true for $\alpha > 1$ so long as x_i is an increasing function of y . It then follows that under these conditions the absolute value of (41) is always less than the absolute value of (42); in other words the reduction of poverty is always greater using the infra-marginal method rather than the method of subsidies at the margin.

We can quantify this difference by asking the following question. Suppose only a fraction g of the intended transfer m reached the population when the infra-marginal method was used; what rate of 'take-up' g would make one indifferent between the two methods? In this case (42) would clearly become:

$$\left(\frac{dP_\alpha}{dB}\right)_{IM,g} = -\frac{\alpha g}{z} \int_0^z \left(\frac{z_E - y_E}{z_E}\right)^{\alpha-1} dF. \quad (45)$$

When $\alpha = 1$,

$$\begin{aligned} \left(\frac{dP_1}{dB}\right)_M &= \left(\frac{dP_1}{dB}\right)_{IM,g} \\ &\Rightarrow g = \frac{\bar{x}_i^P}{\bar{x}_i}. \end{aligned} \quad (46)$$

Hence, if the take-up rate is the same as the ratio of mean consumption of the poor to overall mean consumption then we would be indifferent between additive and multiplicative transfers. If Engel curves were proportional to income then \bar{x}_1^P/\bar{x}_1 is simply \bar{y}^P/\bar{y} i.e. the ratio of mean poor income to mean national income. As an approximation, the latter ratio thus gives us an indication of the relative benefits of using infra-marginal subsidies.

This suggests that food subsidy programmes should be tailored to the structures of particular countries. Where the ratio \bar{y}^P/\bar{y} is low the poor benefit less from subsidies at the margin, and the same budgetary transfer would yield better results if effected via infra-marginal subsidies. But if the 'depth' of poverty is not so great then the administrative benefits of price subsidies at the margin will dominate. Alternatively, the superiority of infra-marginal schemes may rest rather heavily upon individuals being able to benefit from ration shops, and being able to turn their ration shop food into cash. Empirical evidence on such behaviour would seem to be essential if we are to understand its efficacy as a means of alleviating poverty.

V. PRODUCERS VERSUS CONSUMERS

So far we have assumed that producer prices and consumer prices for food are kept independent of each other. However, as noted in World Bank (1986), in many countries government policy operates through allowing in extra imports of food, thereby lowering prices for consumers *and* for producers. The effects of this on poverty are unambiguous: 'In countries in which many of the people facing chronic food security are rural landless or urban poor who must buy their goods, lower food prices will improve food security. In countries in which

many of the poor produce more food than they consume, however, lower food prices will worsen security' (World Bank, 1986).

Of course the government may allow in more imports and then insulate the producers from the effects of lower prices by keeping producer prices high and bearing the cost in terms of its fiscal deficit. But if it does not or cannot do so it faces an inevitable trade-off between producer poverty and consumer poverty, which it is the object of this section to formalise and make precise. Once again, we are looking to establish the microfoundations of simple rules of thumb such as – if producer poverty exceeds consumer poverty, lower food prices increase poverty overall.

To start the analysis, notice that for producers we interpret lump sum income y as being generated by a profit function:

$$y = \Pi(\mathbf{q}, \mathbf{k}),$$

where the k 's are non-traded household specific endowments such as land. Then it is easy to see that for producers:

$$\left. \frac{\partial y_E}{\partial q_i} \right|_{\mathbf{p}=\mathbf{q}} = -[x_i(\mathbf{q}, y) - r_i(\mathbf{q}, \mathbf{k})], \quad (47)$$

where r_i is the production of commodity i and $x_i - r_i$ is the net demand for i (we suppose $r_i > x_i$ for 'producers').

Considering only urban consumers and rural producers (ignoring the rural landless who depend on the rural producers for their income), if λ_1 is the proportion of the former while λ_2 is the proportion of the latter, then:

$$\begin{aligned} P_\alpha &= \lambda_1 P_{1,\alpha} + \lambda_2 P_{2,\alpha} \\ &= \lambda_1 \int_0^z \left(\frac{z_E - y_E}{z_E} \right)^\alpha dF_1 + \lambda_2 \int_0^z \left(\frac{z_E - y_E}{z_E} \right)^\alpha dF_2, \end{aligned} \quad (48)$$

where F_1 and F_2 are the distributions of lump sum income among consumers and producers, that among the latter being generated by the underlying distribution of non-traded factors such as land. Consider now a subsidy on the world price from which producers are *not* insulated. It follows then that:

$$\left. \frac{dP_\alpha}{dt} \right|_{\mathbf{p}=\mathbf{q}} = \lambda_1 \frac{\alpha}{z} \int_0^z \left(\frac{z_E - y_E}{z_E} \right)^{\alpha-1} x dF_1 + \lambda_2 \frac{\alpha}{z} \int_0^z \left(\frac{z_E - y_E}{z_E} \right)^{\alpha-1} (-n) dF_2, \quad (49)$$

where we drop the commodity subscript since we are dealing with only one commodity, and n is the net supply of producers.

The two components of (49) precisely capture the different effects on poverty via the effects on the poverty of consumers and the effects on the poverty of producers. If $\alpha = 1$, then:

$$\left. \frac{dP_1}{dt} \right|_{\mathbf{p}=\mathbf{q}} = \frac{\alpha}{z} (\lambda_1 H_1 \bar{x}_1^P - \lambda_2 H_2 \bar{n}_2^P), \quad (50)$$

where \bar{n}_2^P is the mean net supply of poor producers while \bar{x}_1^P is the mean demand of poor as before. It is *not* enough, then, simply to know the incidence of poverty

among consumers and producers taken separately. In order to know the effects on poverty we also have to know the 'exposure' of the poor to price changes – this is given naturally by \bar{x}_1^P for the consumer poor and by \bar{n}_2^P for the producer poor. If the latter is large then, as the World Bank (1986) suggests, food subsidies may end up increasing poverty, and (49) and (50) give us the precise conditions when that will happen for $\alpha = 1$. The components of (50) are estimable from available data on household income and expenditure patterns for the rural and the urban sector.

The above analysis applies equally well, of course, to the case where there is no government intervention but the country is a small open economy and the price of food falls in the international market because of shifts in global food balances (for further discussion of these scenarios see Kanbur, 1986). Consider now the following question, which is designed to bring out the producer/consumer conflict at its sharpest. Starting from a position of no intervention in a small open economy the government is considering using a given increase in fiscal deficit to either (i) increase the producer price above the world level or (ii) decrease the consumer price below the world level. Which should it do, if its object is to alleviate as much poverty as it can with the resources it has? What indicators about the characteristics of poverty among producers and consumers are relevant in guiding this choice?

Let the producer subsidy be s_p , and the consumer subsidy s_c . Then the equal budgetary outlay constraint is:

$$\lambda_1 \int_0^\infty s_c x(\mathbf{q}, y) dF_1 = \lambda_2 \int_0^\infty s_p n(\mathbf{q}, y) dF_2 = B, \quad (51)$$

where n is net supply. Evaluating derivatives as $s_c = s_p = B = 0$, we get the result that:

$$\frac{ds_c}{dB} = \frac{1}{\lambda_1 \bar{x}_1} \quad (52)$$

$$\frac{ds_p}{dB} = \frac{1}{\lambda_2 \bar{n}_2}. \quad (53)$$

The effect on poverty of the two strategies is given by:

$$\left(\frac{dP_\alpha}{dB} \right)_{CONS} = \frac{dP_\alpha ds_c}{ds_c dB} = -\frac{\alpha}{z \bar{x}_1} \int_0^z \left(\frac{z_E - y_E}{z_E} \right)^{\alpha-1} x dF_1 \quad (54)$$

$$\left(\frac{dP_\alpha}{dB} \right)_{PROD} = \frac{dP_\alpha ds_p}{ds_p dB} = -\frac{\alpha}{z \bar{n}_2} \int_0^z \left(\frac{z_E - y_E}{z_E} \right)^{\alpha-1} n dF_2. \quad (55)$$

If $\alpha = 1$, then (54) and (55) become:

$$\left(\frac{dP_1}{dB} \right)_{CONS} = -\frac{\alpha \bar{x}_1^P}{z \bar{x}_1} H_1 \quad (56)$$

$$\left(\frac{dP_2}{dB} \right)_{PROD} = -\frac{\alpha \bar{n}_2^P}{z \bar{n}_2} H_2. \quad (57)$$

Thus the critical indicators are the ratio of consumption by the poor to total consumption, and the ratio of net supply by the poor to the total net supply. For the special case of $\alpha = 1$ we get the indicators we would expect at the intuitive level, and these indicators are estimable from available data. For the general case of $\alpha > 1$ the intuitive rules fail, but their generalisations are given in (54) and (55). If we are willing to make special assumptions with regard to preferences and production technology, we can simplify these even for $\alpha > 1$. For example, if

$$x(q, y) = \frac{\theta y}{q},$$

i.e. we have a proportional expenditure system, then (54) becomes:

$$\left(\frac{dP_\alpha}{dB}\right)_{CONS} = -\frac{\alpha\theta}{z\bar{x}_1 q} \int_0^z \left(\frac{z_E - y_E}{z_E}\right)^{\alpha-1} y dF_1 = -\frac{\alpha\theta}{z\bar{x}_1 q} (P_{1,\alpha-1} - P_{1,\alpha}). \quad (58)$$

On the other hand, if the commodity that we are considering is the only one produced and the profit function is of the form:

$$\Pi(q, z) = kq^\gamma,$$

which happens, for example, if the production function is Cobb–Douglas, then:

$$r = \frac{\partial \pi}{\partial q} = \gamma k q^{\gamma-1} = \frac{\gamma \pi}{q}.$$

If producer preferences also lead to a proportional expenditure system, then:

$$x(q, y) = \frac{\theta y}{q}$$

and

$$n(q, y) = r - x = \frac{y}{q} (\gamma - \theta).$$

Hence

$$\left(\frac{dP_\alpha}{dB}\right)_{PROD} = -\frac{\alpha(\gamma - \theta)}{z\bar{n}_2 q} (P_{2,\alpha-1} - P_{2,\alpha}). \quad (59)$$

Thus with these specialisations the appropriate indicators to use include P_α and $P_{\alpha-1}$ among producers and among consumers. This example reveals that it is *not* simply a matter of checking on the incidences of poverty in the two sectors, although they are an important part of the whole story. This will hold *a fortiori* in more complex cases.

VI. CONCLUSION

The issue of targeting food subsidies to achieve maximum impact on poverty is one of great practical and policy significance. Our task has been to formulate the problem of the optimal pattern of food subsidies within the framework of the modern public finance literature. Taking explicit account of the budget constraint, and of preferences and technology, we have derived rules for

reallocating food subsidies in a manner which would reduce poverty. It should be clear that the specific rules derived in this paper depend on the choice of poverty measure. We have argued that the P_z class of measures has appealing normative properties as well as analytical convenience. Some of our rules can be generalised to alternative poverty measures – for example, some of those within the additively separable class. Others cannot, in particular, measures which do not satisfy subgroup consistency (Foster and Shorrocks (1987)) will not have the decomposability properties that we use in our analysis.

We believe that our results are of interest from the policy perspective. At the operational level, there exist many heuristic and intuitively justified rules of thumb which dominate the policy discussion. World Bank (1986) gives many examples of these, some of which are quoted in the main text of the paper. Our analysis has shown the conditions under which these operational rules of thumb have justification. Moreover, we have indicated some of the generalisations which are appropriate when these are not met.

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Date of receipt of final typescript: October 1987

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APPENDIX

A 1

What is needed in general is to evaluate an expression of the type:

$$\int_0^z \left(\frac{z-y}{z} \right)^{\alpha-1} x_i(\mathbf{q}, y) dF(y). \quad (\text{A } 1)$$

Consider a polynomial representation of the Engel Curve:

$$x_i(\mathbf{q}, y) = \sum_{j=0}^n \delta_{ij}(\mathbf{q}) y^j \tag{A 2}$$

and note that we can get an arbitrarily good approximation for any smooth function using this representation. Using (A 2) and (A 1), we get:

$$\int_0^z \left(\frac{z-y}{z}\right)^{\alpha-1} x_i dF = \int_0^z \left(\frac{z-y}{z}\right)^{\alpha-1} \left(\sum_{j=0}^n \delta_{ij} y^j\right) dF. \tag{A 3}$$

Now consider the term:

$$\delta_{ij} z^j \int_0^z \left(\frac{z-y}{z}\right)^{\alpha-1} \frac{y^j}{z^j} dF. \tag{A 4}$$

Define:

$$(1-\mu)^j = \left(\frac{y}{z}\right)^j \tag{A 5}$$

and note that

$$\mu = \frac{z-y}{z}. \tag{A 6}$$

From the binomial expansion,

$$(1-\mu)^j = \sum_{k=0}^j \binom{j}{k} (-1)^k \mu^k. \tag{A 7}$$

Thus (A 4) now becomes:

$$\delta_{ij} z^j \int_0^z \left(\frac{z-y}{z}\right)^{\alpha-1} \left[\sum_{k=0}^j \binom{j}{k} (-1)^k \left(\frac{z-y}{z}\right)^k \right] dF = \delta_{ij} z^j \sum_{k=0}^j \binom{j}{k} (-1)^k P_{\alpha-1+k}. \tag{A 8}$$

Using this in (A 3) yields:

$$\int_0^z \left(\frac{z-y}{z}\right)^{\alpha-1} x_i(\mathbf{q}, y) dF(y) = \sum_{j=0}^n \delta_{ij} z^j \sum_{k=0}^j \binom{j}{k} (-1)^k P_{\alpha-1+k}. \tag{A 9}$$

The general form of the expression in (A 9) can now be inserted into (26) of the text to give us the general expression:

$$\left. \frac{dP_\alpha}{dt_1} \right|_{\mathbf{p}-\mathbf{q}} = \frac{\alpha}{z \bar{x}_1} \left[\sum_{j=0}^n \left(\frac{\delta_{1j}}{\bar{x}_1} - \frac{\delta_{2j}}{\bar{x}_2} \right) z^j \sum_{k=0}^j \binom{j}{k} (-1)^k P_{\alpha-1+k} \right]. \tag{A 10}$$

Expressions (30) and (32) can then be derived as special cases of (A 10), with $n = 1$ and $n = 2$, respectively.

A 2

If the direct utility function is:

$$U(x) = \sum_{i=1}^n \theta_i \log(x_i - \underline{x}_i),$$

then the indirect utility function is:

$$V(\mathbf{q}, y) = \sum_{i=1}^n \theta_i \log \theta_i - \sum_{j=1}^n \theta_j \log q_j + \log(y - \sum_{j=1}^n q_j x_j).$$

Thus

$$\frac{\partial V(\mathbf{q}, y)}{\partial y} = \frac{1}{y - \sum_{j=1}^n q_j x_j}$$

and

$$\frac{\partial V(\mathbf{p}, y_E)}{\partial y_E} = \frac{1}{y_E - \sum_{j=1}^n p_j x_j}$$

But from the definition of y_E ,

$$\begin{aligned} V(\mathbf{q}, y) &= V(\mathbf{p}, y_E) \\ &\Rightarrow - \sum_{j=1}^n \theta_j \log q_j + \log \left(y - \sum_{j=1}^n q_j x_j \right) \\ &= - \sum_{j=1}^n \theta_j \log p_j + \log \left(y_E - \sum_{j=1}^n q_j x_j \right) \\ &\Rightarrow \log \left(\frac{y - \sum q_j x_j}{y_E - \sum q_j x_j} \right) = \sum \theta_j \log q_j - \sum \theta_j \log p_j. \end{aligned}$$

The right-hand side of the above expression is independent of y , but the left hand side is a monotonic transform of the ratio of $\partial V(\mathbf{q}, y)/\partial y$ and $\partial V(\mathbf{p}, y_E)/\partial y_E$.

A 3

Consider

$$\begin{aligned} \frac{1}{\bar{x}_i} \int_0^z \left(\frac{z_E - y_E}{z_E} \right)^{\alpha-1} x_i(\mathbf{q}, y) dF &= \frac{H}{\bar{x}_i} \int_0^z \left(\frac{z_E - y_E}{z_E} \right)^{\alpha-1} x_i(\mathbf{q}, y) \frac{dF}{H} \\ &= \frac{H}{\bar{x}_i} \mathcal{E} \left[\left(\frac{z_E - y_E}{z_E} \right)^{\alpha-1} x_i \right], \end{aligned}$$

where \mathcal{E} is the expectation operator defined over the distribution of poor incomes.

$$< \frac{H}{\bar{x}_i} \mathcal{E} \left[\left(\frac{z_E - y_E}{z_E} \right)^{\alpha-1} \right] \mathcal{E}(x_i)$$

since x_i and $[(z_E - y_E)/z_E]^{\alpha-1}$ are negatively correlated (for $\alpha > 1$)

$$\begin{aligned} &= H \frac{\bar{x}_i^P}{\bar{x}_i} \int_0^z \left(\frac{z_E - y_E}{z_E} \right)^{\alpha-1} \frac{dF}{H} \\ &= \frac{\bar{x}_i^P}{\bar{x}_i} \int_0^z \left(\frac{z_E - y_E}{z_E} \right)^{\alpha-1} dF \\ &< \int_0^z \left(\frac{z_E - y_E}{z_E} \right)^{\alpha-1} dF \end{aligned}$$

since $\bar{x}_i^P < \bar{x}_i$ if x_i is an increasing function of y .