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DECOMPOSING MULTIPLIERS FOR ECONOMIC SYSTEMS INVOLVING REGIONAL AND WORLD TRADE*

Jeffery I. Round

This paper is concerned with the analysis of linkages between economic systems and the spillover and possible feedback effects one system may have on another. It is not a new topic, the initial stimulus in the literature coming from Machlup (1943) and the seminal papers by Metzler (1950) and Chipman (1950). Almost precisely the same questions raised by these authors have been readdressed subsequently by many writers and in a variety of contexts. Most recently, Goodwin (1983) has developed a world matrix multiplier with much the same overall objectives as Machlup and Metzler in mind.

Metzler's proposition was to understand 'the mechanism by which an expansion or contraction of income in one region or country is transmitted to other regions or countries'.¹ In addition, he assumed that 'the conclusions apply without modification to the regions within a single country, or, indeed to any regional classification of the world economy, such as the economies comprising Eastern Europe, Western Europe, Latin America, and similar regions'.² In fact the question has also long occupied the attention of many regional economists working in the context of sub-national regions. The question of interdependence between sub-national regional economies generally assumes a special importance simply because such regions are relatively open economies and are subject to fewer natural and artificial barriers to commodity trade and factor movements than may exist between countries.³ Even so, interest has focused almost exclusively on regional interdependence through commodity trade and within the framework of multiregional input-output systems.⁴ However, there are some exceptions; Klein and Glickman (1977) have developed a multiregional model of a fairly aggregative nature which incorporates other regional interdependencies. A review of this and similar models has been carried out by Glickman (1977).

Paralleling this interest in inter-regional interdependence at the sub-national level, there has been a revived interest in questions of linkage at the international level, with the advent of the project LINK model (pioneered by Klein and Hickman),⁵ the United Nations model of the world economy (Leontief *et al.*

* This paper sets out my own interpretation of, and solution to, a problem which I have worked on jointly with Graham Pyatt. Sections I and II in particular are the result of our joint work. Sir Richard Stone also showed me an unpublished research note of his which greatly clarified the research issues involved. Therefore, I wish to record more than the usual debt of gratitude to them both, although of course I must accept responsibility for the outcome.

¹ Metzler (1950, p. 329).

² Metzler (1950, p. 330).

³ Ohlin (1933).

⁴ There is a vast number of examples in such studies. The field is summarised in Richardson (1972), and a most notable recent study is Polenske (1980), which is a regional model for the United States.

⁵ See Hickman (1974) and Waelbroek (1976).

1977), and especially the OECD international linkage model (OECD, 1979). The OECD model is explicit in its concern to capture 'the transmission of economic impulses from one OECD country to another' and to 'focus on the international linkage process'.¹ The same is perhaps true of the several other models of world trade, of which the ten-region model by Thorbecke and Field (1974) is a most notable example. Goodwin (1983) has adapted their estimates in order to examine the balance of payments effects, internationally, of demand expansions originating in individual country blocs. He also considers the international transmission of inflation, again using a linear multiplier model based on five country blocs which together comprise the world economy.

The present paper should be viewed against this background and is an attempt to analyse further the nature of linkages between systems. It arises from three particular motivations. First, it is a consequence of some work on a social accounting matrix (SAM) framework for Malaysia which explicitly incorporates a regional dimension.² This framework describes the magnitude and nature of the various transactions between and within East Malaysia and peninsular Malaysia. However, in conjunction with this is a second motivation, which is to seek a regional extension of a multiplier decomposition methodology previously developed to determine some particular relationships between output, factor demands and income (Pyatt and Round, 1979). The regional extension is, of course, to attempt to identify specifically regional components of a multiplier matrix, a matrix which in general depicts the effects of exogenous outlays on endogenous incomes within the system as a whole. All of this rests on an ability to recognise income and expenditure loops both between and within the regions of the system.

Previous work especially relevant to this paper is contributions by Miller (1966; 1969), Yamada I. (1961) and Yamada H. and Ihara (1968). Although concentrating exclusively on input-output multipliers and commodity trade, these authors established the kernel of a more general enquiry into the nature of inter-regional and international linkages. More recently, and still within the context of regional input-output analysis, in a previous paper (Round, 1979) I have shown the potential importance of feedback effects within interdependent systems as well as the importance of paying particular regard to the accounting constraints of the system as a whole.

The rest of this paper is organised into five sections. The next section sets out an aggregate SAM framework for Malaysia, and is included to illustrate the nature of a regional accounting framework which embraces a broader set of accounts and transactions than would be captured by input-output. Although principally provided for illustrative purposes, it does have the added advantage that it is a two-region system, and multiplier decomposition is particularly simple when only two regions are involved. Section II develops the analysis for this case. At the heart of some of the particular complexities of decomposing regional multipliers is the existence of two-way trade in the 'same' commodity, or more generally, two-way transfers between regions which are classified within the same functional account. Section III considers this question as it

¹ OECD (1979, p. 73).

² See Pyatt and Round (1984); and Chander *et al.* (1980).

relates to two- and three-region systems and, with this as a basis, section IV extends the regional multiplier decomposition to three regions. In this section we also note some of the difficulties in extending this analysis to higher-order regional systems.

I. A REGIONAL SAM FOR MALAYSIA

The development of regional accounting systems in general, and an accounting framework appropriate to the geographical relationships between East and West Malaysia in particular, are more fully discussed elsewhere (Pyatt and Round, 1984). For present purposes it will suffice to show only the resulting framework, together with a brief explanation of some of the statistical entries. This should assist in understanding the accounting structure on which the analytical work in later sections will be based.

The social accounting matrix (SAM) for Malaysia has eleven principal accounts, including two accounts relating to current and capital transactions with the rest of the world. For each region, considered as part of an integrated regional system, each of the nine own-region 'domestic' accounts records incomes and outlays wholly within the region as well as the origin of receipts and the destination of payments externally. The external transactions are recorded in some detail between the region and other regions, and the rest of the world. Essentially embracing concepts enunciated by Stone (1961), a distinction has been drawn between functional flows (transactions) and geographical (inter-regional) flows. Thus, all transactions between different (functional) accounts are represented as taking place within each region. Inter-regional flows in the accounting system therefore represent transfers which simply augment the receipts of an account in one region and simultaneously deplete the same (functional) account in the other region.

Table 1 shows an aggregate version of the regional SAM for Malaysia. It can be seen that the inter-regional blocks are diagonal submatrices, and although the recorded entries are small relative to those elsewhere in the matrix, in this instance they happen to be the estimated inter-regional transfers which fulfil the augmenting role described earlier. In summary, therefore, this way of representing regional accounts allows the functional and geographical elements of transactions to be clearly distinguished.

In more disaggregated versions of this matrix, such as commodity disaggregations of trade flows, the diagonality of the inter-regional matrices is maintained, although their overall dimensions are increased.¹ Partly as a result of a relatively small amount of regional interdependence, and partly due to assumptions made in unscrambling rather meagre data sources, some of the inter-regional transfers are recorded zeros.² These are different from definitional zeros, which are shown as blank diagonal entries in the inter-regional submatrices in Table 1.

¹ There are some complications in the treatment of trade, such as accounting for the distinction between c.i.f. valuation of imports and f.o.b. valuation of exports. See Pyatt and Round (1984).

² Table 1 is adapted from Pyatt and Round (1984), table 8.1. Interregional transfers for the factors, households and companies accounts were originally shown as being zero. However, the entries in the social accounting matrix shown in Table 1 have been arbitrarily adjusted to show small inter-regional transfers for these accounts, for illustrative purposes only.

Table 1. *Aggregate regional accounts for*

Incomes	Expenditure: East Malaysia									
	Institutions' current					Consolidated capital (6)	Commodities (9)	Activities (10)	Indirect taxes (11)	
	Wants (1)	Factors (2)	Households (3)	Companies (4)	Government (5)					
East Malaysia										
Wants (1)	—	—	1,179·0	—	—	—	—	—	—	—
Factors (2)	—	—	—	—	—	—	—	1,563·0	—	—
Institutions										
Households (3)	—	1,376·0	60·0	10·9	25·0	—	—	—	—	—
Companies (4)	—	149·0	—	—	7·8	—	—	—	—	—
Government (5)	—	—	36·1	32·5	—	—	—	—	—	253·0
Consolidated capital (6)	—	—	196·8	106·6	36·8	—	—	—	—	—
Commodities (9)	1,179·0	—	—	—	243·0	349·0	—	1,421·9	—	—
Activities (10)	—	—	—	—	—	—	3,108·7	—	—	—
Indirect taxes (11)	—	—	—	—	—	—	129·2	123·8	—	—
West Malaysia										
Wants (1)	—	—	—	—	—	—	—	—	—	—
Factors (2)	—	10·0	—	—	—	—	—	—	—	—
Institutions										
Households (3)	—	—	14·2	—	—	—	—	—	—	—
Companies (4)	—	—	—	4·0	—	—	—	—	—	—
Government (5)	—	—	—	—	235·0	—	—	—	—	—
Consolidated capital (6)	—	—	—	—	—	0	—	—	—	—
Commodities (9)	—	—	—	—	—	—	218·2	—	—	—
Activities (10)	—	—	—	—	—	—	—	—	—	—
Indirect taxes (11)	—	—	—	—	—	—	—	—	—	—
Rest of World										
Current (7*)	—	40·8	—	6·0	2·8	—	1,046·7	—	—	—
Capital (8*)	—	—	—	—	—	85·0	—	—	—	—
Total										
	1,179·0	1,575·8	1,486·1	160·0	550·4	434·0	4,502·8	3,108·7	253·0	

Source. Adapted from Pyatt

II. MULTIPLIERS AND THE TWO-REGION SYSTEM

The multiplier decompositions, established in Pyatt and Round (1979), were designed to show some of the separable effects and linkages which may take place between designated endogenous accounts of an economic system. In that study the chosen endogenous accounts were factors of production, households and the production accounts, while the consolidated capital, external and non-household institution (such as government) current accounts were designated as exogenous accounts. By endogenising households as well as factors and production, the impact multiplier \mathbf{M} of exogenous outlays \mathbf{x} on endogenous incomes \mathbf{y} , represented as

$$\mathbf{y} = \mathbf{M}\mathbf{x} \quad (1)$$

captures the reciprocal relationship between income distribution and the structure of production. But it can further be shown that \mathbf{M} can be multiplicatively decomposed to give

$$\mathbf{y} = \mathbf{M}_3 \mathbf{M}_2 \mathbf{M}_1 \mathbf{x}, \quad (2)$$

East and West Malaysia (M \$ million)

Expenditure: West Malaysia												Total
Wants (1)	Factors (2)	Institutions' current			Consolidated capital (6)	Commodities (9)	Activities (10)	Indirect taxes (11)	Rest of World			
		House- holds (3)	Com- panies (4)	Government (5)					Current (7*)	Capital (8*)		
—	—	—	—	—	—	—	—	—	—	1,179.0	(1)	
—	2.0	—	—	—	—	—	—	10.8	—	1,575.8	(2)	
—	—	10.0	—	—	—	—	—	4.2	—	1,486.1	(3)	
—	—	—	1.0	—	—	—	—	2.2	—	160.0	(4)	
—	—	—	—	226.6	—	—	—	2.2	—	550.4	(5)	
—	—	—	—	—	93.8	—	—	—	—	434.0	(6)	
—	—	—	—	—	—	98.1	—	1,211.8	—	4,502.8	(9)	
—	—	—	—	—	—	—	—	—	—	3,108.7	(10)	
—	—	—	—	—	—	—	—	—	—	253.0	(11)	
—	—	6,349.2	—	—	—	—	—	—	—	6,349.2	(1)	
—	—	—	—	—	—	9,038.1	—	209.4	—	9,257.5	(2)	
—	7,319.7	391.7	98.0	171.2	—	—	—	11.8	—	8,006.6	(3)	
—	1,401.9	—	—	125.9	—	—	—	7.9	—	1,539.7	(4)	
—	—	351.9	959.1	—	—	—	1,549.0	12.9	—	3,107.9	(5)	
—	—	903.8	262.5	818.6	—	—	—	—	-85.0	1,899.9	(6)	
6,349.2	—	—	—	1,742.0	2,014.9	—	7,891.3	4,111.8	—	22,327.4	(9)	
—	—	—	—	—	—	17,296.1	—	—	—	17,296.1	(10)	
—	—	—	—	—	—	1,182.3	366.7	—	—	1,549.0	(11)	
—	533.9	—	219.1	23.6	—	3,750.9	—	—	—	5,640.8	(7*)	
—	—	—	—	—	-208.8	—	—	38.8	—	-85.0	(8*)	
6,349.2	9,257.5	8,006.2	1,539.7	3,107.9	1,899.9	22,327.4	17,296.1	1,549.0	5,640.8	-85.0		

and Round (1984), table 8.1.

where the components of \mathbf{M} can be ascribed to the separate effects of multipliers wholly within a group of accounts (\mathbf{M}_1); the effect of an exogenous injection which feeds back upon itself but via other parts of the system (\mathbf{M}_2); or simply the effect an increase in income in one group of accounts has upon another (\mathbf{M}_3). The separation of these effects gives a useful picture of structural independence within the endogenous accounts of the system.

Interpreting \mathbf{M} as simply *ex post* accounting structure, then it can be derived quite simply as follows. Defining \mathbf{T} as the matrix of endogenous account transactions, bordered by the vector \mathbf{x} , a row vector \mathbf{w}' of payments by endogenous accounts to the consolidated exogenous account, and a scalar ϕ representing the consolidated payment between exogenous accounts; then the complete social accounting matrix \mathbf{S} where

$$\mathbf{S} \equiv \begin{bmatrix} \mathbf{T} & \mathbf{x} \\ \mathbf{w}' & \phi \end{bmatrix} \quad (3)$$

has the property that

$$\mathbf{S}\mathbf{i} = \mathbf{S}'\mathbf{i} = \begin{bmatrix} \mathbf{y} \\ \theta \end{bmatrix}, \quad (4)$$

where \mathbf{y} is the vector of endogenous account totals and θ a scalar of total receipts and payments by the exogenous account. Thus, defining

$$\mathbf{T} = \mathbf{A}\hat{\mathbf{y}}, \quad (5)$$

where \mathbf{A} is a matrix of average propensities with respect to income across the broad group of endogenous transactions, it follows that

$$\mathbf{y} = (\mathbf{I} - \mathbf{A})^{-1}\mathbf{x} = \mathbf{M}\mathbf{x}. \quad (6)$$

Alternatively, under the assumption that prices are fixed, and with information about the extent to which expenditure elasticities differ from unity, \mathbf{A} can represent marginal expenditure propensities and consequently \mathbf{M} is an *ex ante* impact multiplier showing the effect of exogenous injections \mathbf{x} on endogenous incomes \mathbf{y} at given prices.

Extending the basic SAM and decomposing multipliers further according to a regional dimension means that attention will be focused on the patterns of inter-regional trade and transfers.

It will be expositionally more convenient to use average propensities for deriving *ex ante* multipliers although there are additional difficulties with this assumption at the regional level beyond those already recognised for an individual economy.¹ The starting point of the analysis is a two-region system, which enables the regional SAM for Malaysia to be used for illustrative purposes. The choice of endogenous and exogenous accounts will be essentially similar to that in Pyatt and Round (1979) referred to earlier. Hence the wants, factors, household (current) and commodities production activities accounts are designated as being endogenous, while all other accounts (government and companies current accounts, consolidated capital accounts and the rest-of-world accounts) are combined into a single exogenous account. Table 2 illustrates this reorganisation and consolidation of the SAM for Malaysia.

Table 2 can be expressed analytically for two regions 1 and 2 as follows:

$$\mathbf{S} \equiv \begin{bmatrix} \mathbf{T}_{11} & \hat{\mathbf{t}}_{12} & \mathbf{x}_1 \\ \hat{\mathbf{t}}_{21} & \mathbf{T}_{22} & \mathbf{x}_2 \\ \mathbf{w}'_1 & \mathbf{w}'_2 & \phi \end{bmatrix}, \quad (7)$$

where the account totals are

$$\mathbf{S}\mathbf{i} = \mathbf{S}'\mathbf{i} = \begin{bmatrix} \mathbf{y}_1 \\ \mathbf{y}_2 \\ \theta \end{bmatrix}. \quad (8)$$

The analytical structure of \mathbf{S} would be maintained even if the endogenous accounts were disaggregated further into say, separate factor, household or

¹ Within the context of a fixed price model this creates no special problem: marginal coefficients can be obtained via expenditure elasticities as indicated above. But the analysis is still limited by the fixed-price assumption, and for regional systems this implies that relative prices between regions are also fixed which, in turn, implies that exchange rates are fixed. The latter assumption may not be too serious for regions within a country or within the same currency area, but for broader geographical regions and different countries this could be especially restrictive.

Table 2
Consolidated regional Malaysian SAM (M \$ million)

Receipts	Expenditures						Total
	Endogenous accounts			Exogenous account			
	East Malaysia		West Malaysia	Household current		Production	
Factor	Household current	Production	Factor	Household current	Production	Exogenous account	
Endogenous accounts							
East Malaysia							
Factor	—	—	1,563.0	2.0	—	—	10.8
Household current	1,376.0	60.0	—	—	10.0	—	40.1
Production	—	1,179.0	1,421.9	—	—	98.1	1,803.8
West Malaysia							
Factor	10.0	—	—	—	—	9,038.1	209.4
Household current	—	14.2	—	7,319.7	391.7	—	281.0
Production	—	—	218.2	—	6,349.2	7,891.3	7,868.7
Exogenous account	189.8	232.9	1,299.7	1,935.8	1,255.7	5,299.9	4,835.9
Total	1,575.8	1,486.1	4,502.8	9,257.5	8,006.6	22,327.4	10,213.8

Source. Aggregation of Table 1.

commodity accounts. Expressing the proportion of inter-regional transfers from region i to j as a proportion of total outlays in region i as $\hat{\mathbf{b}}_{ji}$, where

$$\hat{\mathbf{t}}_{ji} = \hat{\mathbf{b}}_{ji} \mathbf{y}_i, \tag{9}$$

and \mathbf{T}_{ii} as the product of average propensities \mathbf{B}_{ii} and total outlays \mathbf{y}_i ,

$$\mathbf{T}_{ii} = \mathbf{B}_{ii} \hat{\mathbf{y}}_i, \tag{10}$$

gives a final two-equation system in y and x :

$$\mathbf{y}_1 = \mathbf{B}_{11} \mathbf{y}_1 + \hat{\mathbf{b}}_{12} \mathbf{y}_2 + \mathbf{x}_1, \quad \mathbf{y}_2 = \hat{\mathbf{b}}_{21} \mathbf{y}_1 + \mathbf{B}_{22} \mathbf{y}_2 + \mathbf{x}_2. \tag{11}$$

The commodity trade element of $\hat{\mathbf{b}}_{ji}$ shows the proportion of imports to the total supply (domestic plus imports) in region i .

This may be written as

$$\begin{bmatrix} \mathbf{y}_1 \\ \mathbf{y}_2 \end{bmatrix} = \begin{bmatrix} \mathbf{B}_{11} & \hat{\mathbf{b}}_{12} \\ \hat{\mathbf{b}}_{21} & \mathbf{B}_{22} \end{bmatrix} \begin{bmatrix} \mathbf{y}_1 \\ \mathbf{y}_2 \end{bmatrix} + \begin{bmatrix} \mathbf{x}_1 \\ \mathbf{x}_2 \end{bmatrix} \tag{12}$$

and solved as

$$\begin{bmatrix} \mathbf{y}_1 \\ \mathbf{y}_2 \end{bmatrix} = \begin{bmatrix} (\mathbf{I} - \mathbf{B}_{11})^{-1} & \mathbf{o} \\ \mathbf{o} & (\mathbf{I} - \mathbf{B}_{22})^{-1} \end{bmatrix} \left\{ \begin{bmatrix} \hat{\mathbf{b}}_{12} & \mathbf{o} \\ \hat{\mathbf{b}}_{21} & \mathbf{o} \end{bmatrix} \begin{bmatrix} \mathbf{y}_1 \\ \mathbf{y}_2 \end{bmatrix} + \begin{bmatrix} \mathbf{x}_1 \\ \mathbf{x}_2 \end{bmatrix} \right\}. \tag{13}$$

This becomes

$$\begin{bmatrix} \mathbf{y}_1 \\ \mathbf{y}_2 \end{bmatrix} = \begin{bmatrix} \mathbf{o} & \mathbf{D}_{12} \\ \mathbf{D}_{21} & \mathbf{o} \end{bmatrix} \begin{bmatrix} \mathbf{y}_1 \\ \mathbf{y}_2 \end{bmatrix} + \begin{bmatrix} (\mathbf{I} - \mathbf{B}_{11})^{-1} & \mathbf{o} \\ \mathbf{o} & (\mathbf{I} - \mathbf{B}_{22})^{-1} \end{bmatrix} \begin{bmatrix} \mathbf{x}_1 \\ \mathbf{x}_2 \end{bmatrix} \tag{14}$$

where

$$\mathbf{D}_{12} = (\mathbf{I} - \mathbf{B}_{11})^{-1} \hat{\mathbf{b}}_{12} \quad \text{and} \quad \mathbf{D}_{21} = (\mathbf{I} - \mathbf{B}_{22})^{-1} \hat{\mathbf{b}}_{21}$$

so that

$$\begin{bmatrix} \mathbf{y}_1 \\ \mathbf{y}_2 \end{bmatrix} = \begin{bmatrix} \mathbf{I} & -\mathbf{D}_{12} \\ -\mathbf{D}_{21} & \mathbf{I} \end{bmatrix}^{-1} \begin{bmatrix} (\mathbf{I} - \mathbf{B}_{11})^{-1} & \mathbf{o} \\ \mathbf{o} & (\mathbf{I} - \mathbf{B}_{22})^{-1} \end{bmatrix} \begin{bmatrix} \mathbf{x}_1 \\ \mathbf{x}_2 \end{bmatrix} \tag{15}$$

or

$$\mathbf{y} = \mathbf{M}_{rx} \mathbf{M}_{r1} \mathbf{x}, \tag{16}$$

where \mathbf{y} and \mathbf{x} are stacked vectors of endogenous account incomes and exogenous outlays, respectively; and \mathbf{M}_{rx} and \mathbf{M}_{r1} are representations of the pre-multiplying matrices in expression (15). Both \mathbf{M}_{rx} and \mathbf{M}_{r1} are multiplier matrices since $\mathbf{M}_{rx} \geq \mathbf{I}$ and $\mathbf{M}_{r1} \geq \mathbf{I}$. Together, they capture the total repercussions within and between endogenous accounts in the inter-regional system. Individually, they have interesting interpretations.

\mathbf{M}_{r1} is the intra-regional multiplier matrix. It shows the multiplier effects that result from linkages wholly within each of the regions taken separately. Thus, an exogenous outlay of \mathbf{x}_1 in region 1 may create multiplier repercussions within that region via $(\mathbf{I} - \mathbf{B}_{11})^{-1}$, but the zero off-diagonal submatrices indicate that the multiplier matrix as a whole captures no inter-regional repercussion; and similarly for region 2.¹

The second component of the overall multiplier matrix, which premultiplies the intra-regional multiplier matrix in equation (18), is \mathbf{M}_{rx} , and this may be

¹ Note that $(\mathbf{I} - \mathbf{B}_{11})^{-1}$ and $(\mathbf{I} - \mathbf{B}_{22})^{-1}$ can each be decomposed further, according to Pyatt and Round (1979), as indicated in equation (2).

referred to as the 'inter-regional' multiplier matrix. It captures all of the (spatial) repercussions between the accounts of one region and those of the other, but it excludes all of the 'within region' effects since these have already been accounted for by \mathbf{M}_{r1} . Note that the inter-regional multiplier essentially depends upon the linkages represented by $\hat{\mathbf{b}}_{12}$ and $\hat{\mathbf{b}}_{21}$, and the degree of departure of \mathbf{M}_{rx} from the identity matrix depends on the strength of bilateral trade linkages and other endogenous inter-regional transfers.

Some previous attempts to decompose input-output multiplier matrices relating to two- and three-region systems by Miller (1966; 1969) used some results about the inverse of a partitioned matrix to trace the linkages through the regional system term by term. The same results can be of use in multiplicatively decomposing the inter-regional multiplier matrix \mathbf{M}_{rx} further.

$$\begin{aligned} \mathbf{M}_{rx} &= \begin{bmatrix} \mathbf{I} & -\mathbf{D}_{12} \\ -\mathbf{D}_{21} & \mathbf{I} \end{bmatrix}^{-1} \\ &= \begin{bmatrix} (\mathbf{I} - \mathbf{D}_{12}\mathbf{D}_{21})^{-1} & (\mathbf{I} - \mathbf{D}_{12}\mathbf{D}_{21})^{-1}\mathbf{D}_{12} \\ (\mathbf{I} - \mathbf{D}_{21}\mathbf{D}_{12})^{-1}\mathbf{D}_{21} & (\mathbf{I} - \mathbf{D}_{21}\mathbf{D}_{12})^{-1} \end{bmatrix} \\ &= \begin{bmatrix} (\mathbf{I} - \mathbf{D}_{12}\mathbf{D}_{21})^{-1} & \mathbf{0} \\ \mathbf{0} & (\mathbf{I} - \mathbf{D}_{21}\mathbf{D}_{12})^{-1} \end{bmatrix} \begin{bmatrix} \mathbf{I} & \mathbf{D}_{12} \\ \mathbf{D}_{21} & \mathbf{I} \end{bmatrix} \quad (17) \\ &= \mathbf{M}_{r3}\mathbf{M}_{r2}, \end{aligned}$$

where \mathbf{M}_{r2} and \mathbf{M}_{r3} represent the matrices in equation (17). Like \mathbf{M}_{r1} , \mathbf{M}_{r3} is block diagonal and hence shows the extent to which outlays in one region affect the incomes of endogenous accounts in the same region. But \mathbf{M}_{r3} shows the component multipliers accounting for inter-regional feedback effects and may therefore be referred to as an inter-regional 'closed loop' multiplier matrix. The final matrix \mathbf{M}_{r2} is also simple and of interesting structure. It too is a multiplier matrix (since $\mathbf{M}_{r2} \geq \mathbf{I}$) and has identity matrices in its diagonal. Analogous to the decomposition in equation (2), \mathbf{M}_{r2} is an inter-regional 'open loop' multiplier matrix. It captures the effect that one region has upon the other, after accounting for all 'own-region' effects: hence the block diagonal matrices are identity matrices. The total inter-regional multiplier effect for 'own regions' is obtained as the product of corresponding diagonal blocks of \mathbf{M}_{r3} and \mathbf{M}_{r1} ; while the equivalent multiplier effect of one region upon the other is the product of the appropriate inter-regional 'open loop' effect and the total 'own-region' effect for the former region. Hence the total multiplier relationship in the regional system can be expressed as

$$\mathbf{y} = \mathbf{M}_{r3}\mathbf{M}_{r2}\mathbf{M}_{r1}\mathbf{x}. \quad (18)$$

This clarifies the nature of the separate effects involved in the regional system.

Tables 3, 4, 5 and 6 show the results of decomposing multipliers derived from Table 2. They clearly illustrate the strength of the contribution of the \mathbf{M}_{r1} multipliers, and the relative weakness of the inter-regional closed loop multipliers \mathbf{M}_{r3} , in this particular case. The remaining inter-regional open loop effects, which are also relatively small, are captured in \mathbf{M}_{r2} .

Table 3
Multipliers matrix M

Receipts	Expenditures					
	East Malaysia			West Malaysia		
	Factors	Households	Production	Factors	Households	Production
East Malaysia						
Factors	1.5789	0.6629	0.8018	0.0079	0.0095	0.0104
Households	1.4369	1.6454	0.7297	0.0090	0.0110	0.0106
Production	1.6677	1.9097	2.3097	0.0216	0.0269	0.0292
West Malaysia						
Factors	0.1154	0.1198	0.1342	1.7041	0.8905	1.0678
Households	0.1104	0.1161	0.1189	1.4168	1.7919	0.8878
Production	0.2604	0.2856	0.3189	1.7393	2.1996	2.6377

Table 4
Multipliers matrix M_{r1}

Receipts	Expenditures					
	East Malaysia			West Malaysia		
	Factors	Households	Production	Factors	Households	Production
East Malaysia						
Factors	1.5779	0.6618	0.8005	0	0	0
Households	1.4358	1.6443	0.7284	0	0	0
Production	1.6648	1.9065	2.3061	0	0	0
West Malaysia						
Factors	0	0	0	1.7028	0.8889	1.0661
Households	0	0	0	1.4157	1.7904	0.8836
Production	0	0	0	1.7363	2.1959	2.6337

Table 5
Multipliers matrix M_{r2}

Receipts	Expenditures					
	East Malaysia			West Malaysia		
	Factors	Households	Production	Factors	Households	Production
East Malaysia						
Factors	1.0	0	0	0.0003	0.0008	0.0035
Households	0	1.0	0	0.0003	0.0021	0.0032
Production	0	0	1.0	0.0004	0.0024	0.0101
West Malaysia						
Factors	0.0108	0.0085	0.0517	1.0	0	0
Households	0.0090	0.0171	0.0429	0	1.0	0
Production	0.0110	0.0210	0.1276	0	0	1.0

Table 6
Multipliers matrix \mathbf{M}_{r3}

Receipts	Expenditures					
	East Malaysia			West Malaysia		
	Factors	Households	Production	Factors	Households	Production
East Malaysia						
Factors	1·0001	1·0001	0·0005	0	0	0
Households	0·0001	1·0001	0·0005	0	0	0
Production	0·0001	0·0003	1·0014	0	0	0
West Malaysia						
Factors	0	0	0	1·0000	0·0002	0·0006
Households	0	0	0	0·0000	1·0001	0·0005
Production	0	0	0	0·0001	0·0004	1·0014

III. CROSS HAULS AND INTER-REGIONAL CLOSED LOOP MULTIPLIERS

In the two-region multiplier system discussed in the previous section, the second multiplier matrix, referred to as an inter-regional closed loop multiplier matrix, was expressed as

$$\mathbf{M}_{r3} = \begin{bmatrix} (\mathbf{I} - \mathbf{D}_{12}\mathbf{D}_{21})^{-1} & \mathbf{0} \\ \mathbf{0} & (\mathbf{I} - \mathbf{D}_{21}\mathbf{D}_{12})^{-1} \end{bmatrix}. \quad (19)$$

This has some interesting features, especially in so far as it relates to the phenomenon of 'cross-hauling' of commodities between regions, although it is more general and relates to all inter-regional transfers. To simplify exposition in this and subsequent sections it will prove convenient to limit endogeneity to some disaggregation of the production accounts, so that \mathbf{M} is now essentially a matrix of input-output multipliers.

Cross-hauling is defined as the occurrence of trade flows of the 'same' commodity in two geographical directions. It might be argued that cross-hauling is observed only as a result of economic inefficiency. But even in a world of economic efficiency and full information, cross-hauling could still arise from one or more of three kinds of aggregation within the production accounts. These may be termed commodity, spatial and intertemporal aggregation factors.¹ The first reflects the fact that however finely the line is drawn between accounts there may be some qualitative differences between ostensibly similar commodities produced in different plants and locations. Spatial aggregation factors arise because regions are rarely (if ever) point-economies. So for a particular spatial configuration of sources of supply and demand it is possible for it to be more efficient for requirements in region *A* to be supplied by region *B*, and those of *B* by *A*. Finally, transactions are recorded during a particular time period, usually of one year duration. Hence, because of seasonal factors two-way shipments may occur during that time period. Bearing in mind these possibilities, it is of interest to see how bilateral cross-hauling is captured in the multiplier \mathbf{M}_{r3} .

¹ For a discussion see Polenske (1980, p. 107).

$\mathbf{M}_{7,3}$ is block diagonal. It is a multiplier matrix because $\mathbf{M}_{7,3} \geq \mathbf{I}$, with strict inequality between corresponding elements arising if either or both of $\mathbf{D}_{12}\mathbf{D}_{21}$ and $\mathbf{D}_{21}\mathbf{D}_{12}$ exceeds zero. Now because of the structures of \mathbf{D}_{12} and \mathbf{D}_{21} (see equation (14)) both $\mathbf{D}_{12}\mathbf{D}_{21}$ and $\mathbf{D}_{21}\mathbf{D}_{12}$ are likely to exceed zero, and the diagonal elements of $\mathbf{M}_{7,3}$ may exceed unity. These diagonal elements are the own-region own-commodity multiplier effects of exogenous outlays. Part of this multiplier may be due to cross hauls if corresponding elements of $\hat{\mathbf{a}}_{12}$ and $\hat{\mathbf{a}}_{21}$ were non-zero; but the remainder would be due to the nature of system linkages which might give non-zero diagonal elements in $\mathbf{D}_{12}\mathbf{D}_{21}$ or $\mathbf{D}_{21}\mathbf{D}_{12}$, or, even failing that, in higher-order interactions such as $(\mathbf{D}_{12}\mathbf{D}_{21})^2$, $(\mathbf{D}_{12}\mathbf{D}_{21})^3$, ...

For two-region systems, the cross-hauling phenomenon is especially simple (two-way trade between two regions) even though this may account for only part of the total own-commodity inter-regional closed loop multiplier. However, for third and higher-order regional systems, the nature of the cross hauls is more complex. In addition to what may be termed 'bilateral' cross hauls, there can be multilateral cross hauls between three or more regions. To view this more closely, consider a three-region system, whose SAM corresponding to (7) would be

$$\mathbf{S} \equiv \begin{bmatrix} \mathbf{T}_{11} & \hat{t}_{12} & \hat{t}_{13} & \mathbf{x}_1 \\ \hat{t}_{21} & \mathbf{T}_{22} & \hat{t}_{23} & \mathbf{x}_2 \\ \hat{t}_{31} & \hat{t}_{32} & \mathbf{T}_{33} & \mathbf{x}_3 \\ \mathbf{w}'_1 & \mathbf{w}'_2 & \mathbf{w}'_3 & \phi \end{bmatrix}. \quad (20)$$

Now for any one commodity, the structure of trade flows between regions would be

$$\mathbf{X} = \begin{bmatrix} 0 & t_{12} & t_{13} \\ t_{21} & 0 & t_{23} \\ t_{31} & t_{32} & 0 \end{bmatrix} \quad (21)$$

and if bilateral cross-hauls are netted out from the matrix (so that at least one of t_{ij} and t_{ji} is zero), then (20) would have three possible structures. Either

$$\mathbf{X}_1 = \begin{bmatrix} 0 & t_{12} & 0 \\ 0 & 0 & t_{23} \\ t_{31} & 0 & 0 \end{bmatrix} \quad \text{or} \quad \mathbf{X}_2 = \begin{bmatrix} 0 & 0 & t_{13} \\ t_{21} & 0 & 0 \\ 0 & t_{32} & 0 \end{bmatrix} \quad \text{or} \quad \mathbf{X}_3 = \begin{bmatrix} 0 & t_{12} & t_{13} \\ 0 & 0 & t_{23} \\ 0 & 0 & 0 \end{bmatrix}, \quad (22)$$

where, in each case, $t_{ij} \geq 0$. We see that \mathbf{X}_1 and \mathbf{X}_2 are circular permutation matrices, so in this case there are circular shipments of the commodity around the system. It is in this sense that multilateral cross hauls may be said to exist, even though bilateral cross hauls are ruled out. If, on the other hand, \mathbf{X}_3 emerges then there are no bilateral or multilateral cross hauls left in the system.

This digression on the nature of cross hauls serves two purposes. The first is to show that, in the two-region case, the inter-regional closed loop multiplier arises partly as a direct result of bilateral cross hauls and partly as a result of complex feedbacks which take an injection into a particular regional commodity account around the regional system and back to source. The second is that a two-region system is likely to be a special case, and with third- and higher-order region

systems there may be at least two broad groups of inter-regional closed loop effects, stemming from the existence of bilateral and multilateral cross hauls of commodities in particular, and bilateral and multilateral linkages and transfers in general.

IV. MULTIPLIERS AND THE THREE-REGION SYSTEM

Beginning with the three-region SAM represented in (20), the initial stages of the decomposition algebra described in equations (9) to (12) can be reproduced with only minor modification. For three regions, the analogue of equation (12) is

$$\begin{bmatrix} \mathbf{y}_1 \\ \mathbf{y}_2 \\ \mathbf{y}_3 \end{bmatrix} = \begin{bmatrix} \hat{\mathbf{B}}_{11} & \hat{\mathbf{b}}_{12} & \hat{\mathbf{b}}_{13} \\ \hat{\mathbf{b}}_{21} & \mathbf{B}_{22} & \hat{\mathbf{b}}_{23} \\ \hat{\mathbf{b}}_{31} & \hat{\mathbf{b}}_{32} & \mathbf{B}_{33} \end{bmatrix} \begin{bmatrix} \mathbf{y}_1 \\ \mathbf{y}_2 \\ \mathbf{y}_3 \end{bmatrix} + \begin{bmatrix} \mathbf{x}_1 \\ \mathbf{x}_2 \\ \mathbf{x}_3 \end{bmatrix}, \tag{23}$$

which solves in the same way as equation (13) to yield

$$\mathbf{y} = (\mathbf{I} - \mathbf{D})^{-1} (\mathbf{I} - \hat{\mathbf{B}})^{-1} \mathbf{x}, \tag{24}$$

where
$$(\mathbf{I} - \hat{\mathbf{B}})^{-1} = \begin{bmatrix} (\mathbf{I} - \mathbf{B}_{11})^{-1} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & (\mathbf{I} - \mathbf{B}_{22})^{-1} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & (\mathbf{I} - \mathbf{B}_{33})^{-1} \end{bmatrix}$$

and
$$(\mathbf{I} - \mathbf{D})^{-1} = \begin{bmatrix} \mathbf{I} & -\mathbf{D}_{12} & -\mathbf{D}_{13} \\ -\mathbf{D}_{21} & \mathbf{I} & -\mathbf{D}_{23} \\ -\mathbf{D}_{31} & -\mathbf{D}_{32} & \mathbf{I} \end{bmatrix}^{-1}$$

and where, for instance

$$\mathbf{D}_{ij} = (\mathbf{I} - \mathbf{B}_{ii})^{-1} \hat{\mathbf{b}}_{ij}.$$

Note also that the method of arriving at (24) is quite general for an *r*-region system.

As in the two-region case, equation (24) may be viewed as the product of an inter-regional multiplier \mathbf{M}_{rx} and intra-regional multiplier \mathbf{M}_{r1} , where

$$\mathbf{M}_{r1} = (\mathbf{I} - \hat{\mathbf{B}})^{-1} \quad \text{and} \quad \mathbf{M}_{rx} = (\mathbf{I} - \mathbf{D})^{-1}.$$

However, unlike the two-region case, \mathbf{M}_{rx} does not exhibit the same simple structure for immediate further decomposition into inter-regional open and closed loop effects. The previous section offers clues as to why this is so: for two regions cross hauls and linkages are limited to the bilateral variety, \mathbf{D} is an especially simple circular permutation matrix, and decomposition into open and closed multipliers follows immediately. But for three regions both bilateral and trilateral cross hauls are admissible. Hence a slightly modified approach is called for in order to decompose \mathbf{M}_{rx} further.

Attention may be focused on the \mathbf{M}_{rx} multiplier by simply rewriting equation (24) as

$$\mathbf{y} = (\mathbf{I} - \mathbf{D})^{-1} \mathbf{h}, \tag{25}$$

where

$$\mathbf{h} = (\mathbf{I} - \hat{\mathbf{B}})^{-1} \mathbf{x}.$$

Therefore, for three regions, the solution to (25) may be expressed as the solution to three vector equations of the form

$$\mathbf{y}_i - \mathbf{D}_{ij} \mathbf{y}_j - \mathbf{D}_{ik} \mathbf{y}_k = \mathbf{h}_i,$$

where $i, j, k = 1, 2$ and 3 and $i \neq j \neq k$. Using Gauss-Seidel methods for solving simultaneous linear equations, the solution for \mathbf{y}_i may be expressed as

$$\begin{aligned} \mathbf{y}_i &= [\mathbf{I} - \mathbf{D}_{ij}\mathbf{D}_{ji} - (\mathbf{D}_{ik} + \mathbf{D}_{ij}\mathbf{D}_{jk})(\mathbf{I} - \mathbf{D}_{kj}\mathbf{D}_{jk})^{-1}(\mathbf{D}_{ki} + \mathbf{D}_{kj}\mathbf{D}_{ji})]^{-1} \\ &\times \{ \mathbf{h}_i + [\mathbf{D}_{ij} + (\mathbf{D}_{ik} + \mathbf{D}_{ij}\mathbf{D}_{jk})(\mathbf{I} - \mathbf{D}_{kj}\mathbf{D}_{jk})^{-1}\mathbf{D}_{kj}] \mathbf{h}_j \\ &+ (\mathbf{D}_{ik} + \mathbf{D}_{ij}\mathbf{D}_{jk})(\mathbf{I} - \mathbf{D}_{kj}\mathbf{D}_{jk})^{-1} \mathbf{h}_k \}. \end{aligned} \tag{26}$$

It can easily be observed that equation (26) contains a number of similar factors throughout the several terms in the expansion, and there is also a certain degree of symmetry in the result. For instance, the solutions for $\mathbf{y}_1, \mathbf{y}_2$ and \mathbf{y}_3 may be written in matrix form as

$$\begin{bmatrix} \mathbf{y}_1 \\ \mathbf{y}_2 \\ \mathbf{y}_3 \end{bmatrix} = \begin{bmatrix} \mathbf{H}_{11} & \mathbf{o} & \mathbf{o} \\ \mathbf{o} & \mathbf{H}_{22} & \mathbf{o} \\ \mathbf{o} & \mathbf{o} & \mathbf{H}_{33} \end{bmatrix} \begin{bmatrix} \mathbf{I} & \mathbf{L}_{12} & \mathbf{K}_{13} \\ \mathbf{K}_{21} & \mathbf{I} & \mathbf{L}_{23} \\ \mathbf{L}_{31} & \mathbf{K}_{32} & \mathbf{I} \end{bmatrix} \begin{bmatrix} \mathbf{h}_1 \\ \mathbf{h}_2 \\ \mathbf{h}_3 \end{bmatrix} \tag{27}$$

or as $\mathbf{y} = \mathbf{M}_{r3}\mathbf{M}_{r2}\mathbf{h}$, (28)

where

$$\left. \begin{aligned} \mathbf{H}_{ii} &= [\mathbf{I} - \mathbf{D}_{ij}\mathbf{D}_{ji} - (\mathbf{D}_{ik} + \mathbf{D}_{ij}\mathbf{D}_{jk})(\mathbf{I} - \mathbf{D}_{kj}\mathbf{D}_{jk})^{-1}(\mathbf{D}_{ki} + \mathbf{D}_{kj}\mathbf{D}_{ji})]^{-1}, \\ \mathbf{K}_{ik} &= (\mathbf{D}_{ik} + \mathbf{D}_{ij}\mathbf{D}_{jk})(\mathbf{I} - \mathbf{D}_{kj}\mathbf{D}_{jk})^{-1} \\ \text{and } \mathbf{L}_{ij} &= \mathbf{D}_{ij} + \mathbf{K}_{ik}\mathbf{D}_{kj}. \end{aligned} \right\} \tag{29}$$

Already the multiplying matrices in equations (27) and (28) exhibit familiar structure: \mathbf{M}_{r3} shows the ‘circular region’ multipliers, or what has been referred to earlier as the inter-regional closed loop multiplier. \mathbf{M}_{r2} , on the other hand, contains the inter-regional open loop multiplier. Nevertheless, the complex structure of relations in (29) may be simplified further.

Consider the way in which exogenous outlays in one region may leak into other regions and feed back on to itself. There are four routes. For any region, say region 1, there are two kinds of bilateral feedbacks (via region 2 or region 3) and two kinds of trilateral feedbacks (via region 2 first and then region 3, or vice versa). Each of these may contain cross-haul elements as particular components. This may be formalised by considering matrix \mathbf{D} , which relates exclusively to inter-regional linkages. It may be expressed as the sum of two circular permutation matrices, namely:

$$\mathbf{D} = \mathbf{E} + \mathbf{F}$$

where $\mathbf{E} = \begin{bmatrix} \mathbf{o} & \mathbf{D}_{12} & \mathbf{o} \\ \mathbf{o} & \mathbf{o} & \mathbf{D}_{23} \\ \mathbf{D}_{31} & \mathbf{o} & \mathbf{o} \end{bmatrix}$ and $\mathbf{F} = \begin{bmatrix} \mathbf{o} & \mathbf{o} & \mathbf{D}_{13} \\ \mathbf{D}_{21} & \mathbf{o} & \mathbf{o} \\ \mathbf{o} & \mathbf{D}_{32} & \mathbf{o} \end{bmatrix}$.

Clearly, since $\mathbf{F}^2 = \begin{bmatrix} \mathbf{o} & \mathbf{D}_{13}\mathbf{D}_{32} & \mathbf{o} \\ \mathbf{o} & \mathbf{o} & \mathbf{D}_{21}\mathbf{D}_{13} \\ \mathbf{D}_{32}\mathbf{D}_{21} & \mathbf{o} & \mathbf{o} \end{bmatrix}$

(and similarly for \mathbf{E}^2) it means that there are two ways of going from i to j , either directly, or indirectly via k . These are terms included in the expansion of (29). Furthermore, since

$$\mathbf{EF} = \begin{bmatrix} \mathbf{D}_{12}\mathbf{D}_{21} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{D}_{23}\mathbf{D}_{32} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{D}_{31}\mathbf{D}_{13} \end{bmatrix}$$

(and similarly for \mathbf{FE}) as well as

$$\mathbf{E}^3 = \begin{bmatrix} \mathbf{D}_{12}\mathbf{D}_{23}\mathbf{D}_{31} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{D}_{23}\mathbf{D}_{31}\mathbf{D}_{12} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{D}_{31}\mathbf{D}_{12}\mathbf{D}_{23} \end{bmatrix}$$

it means that \mathbf{EF} , \mathbf{FE} , \mathbf{E}^3 and \mathbf{F}^3 capture the four ways of going round the system and back to the source.

With the establishment of these summaries of inter-regional linkages, the multipliers \mathbf{M}_{r3} and \mathbf{M}_{r2} may be written more succinctly, for instance, as

$$\begin{aligned} \mathbf{M}_{r3} &= \begin{bmatrix} \mathbf{H}_{11} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{H}_{22} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{H}_{33} \end{bmatrix} \\ &= [\mathbf{I} - \mathbf{EF} - (\mathbf{F} + \mathbf{E}^2) (\mathbf{I} - \mathbf{FE})^{-1} (\mathbf{E} + \mathbf{F}^2)]^{-1} \end{aligned} \tag{30}$$

and

$$\begin{aligned} \mathbf{M}_{r2} &= \begin{bmatrix} \mathbf{I} & \mathbf{L}_{12} & \mathbf{K}_{13} \\ \mathbf{K}_{21} & \mathbf{I} & \mathbf{L}_{23} \\ \mathbf{L}_{31} & \mathbf{K}_{32} & \mathbf{I} \end{bmatrix} \\ &= [\mathbf{I} + \mathbf{E} + (\mathbf{F} + \mathbf{E}^2) (\mathbf{I} - \mathbf{FE})^{-1} + (\mathbf{F} + \mathbf{E}^2) (\mathbf{I} - \mathbf{FE})^{-1} \mathbf{F}] \end{aligned} \tag{31}$$

Bearing in mind the structures of \mathbf{E} and \mathbf{F} , \mathbf{EF} , \mathbf{E}^3 , etc., it can be seen that expression (30) does indeed capture all the inter-regional closed loop multipliers, based on bilateral and trilateral cross hauls and indirect linkages. Likewise, since each of the three terms in the expansion of \mathbf{M}_{r2} beyond the identity matrix has zero block diagonals, it contains the inter-regional open loop multipliers. One final stage is to note that the direct bilateral multipliers may be extracted by rewriting \mathbf{M}_{r3} as \mathbf{M}_{r3}^* and factoring as follows:

$$\begin{aligned} \mathbf{M}_{r3}^* &= [\mathbf{I} - \mathbf{EF} - (\mathbf{F} + \mathbf{E}^2) (\mathbf{I} - \mathbf{FE})^{-1} (\mathbf{E} + \mathbf{F}^2)]^{-1} \\ &= (\mathbf{I} - \mathbf{EF})^{-1} [\mathbf{I} - (\mathbf{F} + \mathbf{E}^2) (\mathbf{I} - \mathbf{FE})^{-1} (\mathbf{E} + \mathbf{F}^2) (\mathbf{I} - \mathbf{EF})^{-1}]^{-1} \\ &= \mathbf{M}_{r4} \mathbf{M}_{r3} \end{aligned} \tag{32}$$

In summary, therefore, the overall impact multiplier for the three-region system may be expressed, through the combination of (24), (28) and (32), as

$$\mathbf{y} = \mathbf{M}_{r4} \mathbf{M}_{r3} \mathbf{M}_{r2} \mathbf{M}_{r1} \mathbf{x} \tag{33}$$

Thus the impact multiplier may be decomposed into four separate multiplier components, each having a particular interpretation regarding system linkage.

However, there are two problems with the derivation of (33) which might limit its usefulness as a regional augmentation of the $\mathbf{M}_3\mathbf{M}_2\mathbf{M}_1$ decompositions for single economies.

The first problem is that although there is just one way in which a two-region system may be decomposed, for three or more regions the decomposition is not unique. When equation (29) was expressed in terms of \mathbf{E} and \mathbf{F} , a route was implicitly chosen to work the repercussions around the system. Thus, interchanging \mathbf{E} and \mathbf{F} gives an alternative expression for the three-region decomposition. However, if either \mathbf{E} or \mathbf{F} is zero, which would be a special case of the situation when bilateral cross hauls do not exist, then the decomposition is not only unique but obviously simplifies to the well-known structure of $\mathbf{M}_3\mathbf{M}_2\mathbf{M}_1$.¹ The second problem, not unrelated to the first, is that the decomposition becomes hard to generalise for higher-order regional systems. For r regions the number of primary circuits equals $(r-1)!$ and the secondary circuits in the hierarchy begin to assume importance as r increases beyond 4. Hence it becomes difficult to spot the natural extensions of (30) and (31) for higher-order systems.

V. CONCLUSION

This paper has attempted to extend the Pyatt and Round multiplier decompositions for the case where interdependent regional systems are involved. Although for expositional convenience the analysis in Sections III and IV has focused on commodity trade and input-output multipliers, the structure of the SAM presented in Section I and the subsequent computations clearly show that the analysis generally extends to those cases where more than the production accounts are endogenised. Thus the ultimate aim of the analysis could be thought of as one of decomposing the impact multiplier linking exogenous outlays to endogenous incomes into a multiplicative series of functional and geographical components, thereby attempting to identify the relative contributions to the total impacts which can be accounted for by the separate transfer, open and closed loop multipliers in both the geographical and functional domains. For two regions this can be achieved quite nicely; for three regions, as shown in the previous section, there is some limited success; but for higher-order systems the problem has not been successfully dealt with by the approach suggested in this paper. Regrettably from the point of view of implementation most of the empirically based system models, such as the UN world model, the OECD model, and the Thorbecke-Field model, and many of the sub-national regional models, do include relations between more than three geographical areas. Nevertheless, there is a measure of success in that analysis does go some way towards understanding more about the nature of structural interdependence between economic systems involving two or three regions.

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¹ For example, if $\mathbf{E} = \mathbf{O}$, then $\mathbf{M} = \mathbf{M}_{r3}\mathbf{M}_{r2}\mathbf{M}_{r1} = (\mathbf{I} - \mathbf{F}^3)^{-1}(\mathbf{I} + \mathbf{F} + \mathbf{F}^2)(\mathbf{I} - \tilde{\mathbf{C}})^{-1}$.

REFERENCES

- Chander, R., Gnasegarah, S., Pyatt, G. and Round, J. I. (1980). 'Social accounts and the distribution of income: the Malaysian economy in 1970.' *Review of Income and Wealth*, vol. 26, no. 1, pp. 67-85.
- Chipman, J. S. (1950). 'The multi-sector multiplier.' *Econometrica*, vol. 18, pp. 355-74.
- Glickman, N. J. (1977). *Econometric Analysis of Regional Systems*. New York: Academic Press.
- Goodwin, R. M. (1983). 'The world matrix multiplier.' In *Essays in Linear Economic Structure*, pp. 30-56. London: Macmillan.
- Hickman, B. (1974). 'International transmission of economic fluctuations and inflation.' In *International Aspects of Stabilisation Policies*. Federal Reserve Bank of Boston.
- Klein, L. R. and Glickman, N. J. (1977). 'Econometric model-building at the regional level.' *Regional Science and Urban Economics*, vol. 7, pp. 3-23.
- Leontief, W., Carter, A. P. and Petri, P. A. (1977). *The Future of the World Economy*. New York: Oxford.
- Machlup, F. (1943). *International Trade and National Income Multiplier*. Philadelphia: The Blakiston Company.
- Metzler, L. A. (1950). 'A multiple-region theory of income and trade.' *Econometrica*, vol. 18, pp. 329-54.
- Miller, R. E. (1966). 'Interregional feedback effects in input-output models: some preliminary results.' *Papers and Proceedings of the Regional Science Association*, vol. 17, pp. 105-25.
- (1969). 'International feedbacks in input-output models: some empirical results.' *Western Economic Journal*, vol. 7, pp. 41-50.
- OECD (1979). *The OECD International Linkage Model*, Economic Outlook, Occasional Studies, Paris: OECD.
- Ohlin, B. (1933). *Interregional and International Trade*. Cambridge, Mass.: Harvard University Press.
- Polenske, K. R. (1980). *The U.S. Multiregional Input-Output Accounts and Models*. Lexington, Mass.: Lexington Books, D. C. Heath.
- Pyatt, G. and Round, J. I. (1978). 'Regional accounts and regional multipliers in a SAM framework.' Paper presented to the World Bank Conference on Social Accounting Methods in Development Planning, Cambridge, U.K.
- and — (1979). 'Accounting and fixed price multipliers in a social accounting matrix framework.' *ECONOMIC JOURNAL*, vol. 89, pp. 850-73.
- and — (1984). 'Improving the Macroeconomic Data Base: A SAM for Malaysia, 1970.' *World Bank Staff Working Papers* No. 646. Washington, D.C.: World Bank.
- Richardson, H. W. (1972). *Input-Output and Regional Economics*. New York: John Wiley and Sons.
- Round, J. I. (1979). 'Compensating feedback effects in interregional input-output models.' *Journal of Regional Science*, vol. 19, no. 2, pp. 145-55.
- Stone, J. R. N. (1961). 'Social accounts at the regional level: a survey.' In *Regional Economic Planning: Techniques of Analysis for Less Developed Areas* (ed. W. Isard and J. Cumberland). Paris: OECD.
- Thorbecke, E. and Field, A. J. (1974). 'A ten-region model of world trade.' In *International Trade and Finance* (ed. Willy Sellekaerts) (Essays in honour of Jan Tinbergen). London: Macmillan.
- Waelbroeck, J. (ed.) (1976). *The models of Project LINK*. Amsterdam: North-Holland.
- Yamada, H. and Ihara, T. (1968). 'Input-output analysis of interregional repercussion.' *Papers and Proceedings of the Third Far East Conference of the Regional Science Association*, pp. 3-29.
- Yamada, I. (1961). *Theory and Applications of Interindustry Analysis*. Tokyo: Kinokuniya Bookstore Co. Ltd.