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SOCIAL WELFARE UNDERPINNINGS OF URBAN BIAS AND UNEMPLOYMENT

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The reallocation of labour from rural to urban areas, although an historical concomitant of growth in *per capita* incomes, is widely viewed to be a troublesome process. It is a general belief that poor countries are somehow over-urbanised and migration excessive, that urban unemployment or underemployment is a problem, and that the investments to accommodate an expanding urban population are unjustifiably burdensome. Many governments have reacted to migration by attempting to control it via internal passport systems and the forced removal of migrants to rural areas. Mozambique is a recent case in point. These concerns contrast sharply with the conclusions of models of the migration process. For instance, Bhagwati and Srinivasan (1974) argue that input and output tax-subsidy schemes can solve all policy problems raised by migration, without recourse to direct interference with the migration decision.[1]

In this paper we develop a model suggesting greater difficulties in designing policy toward migration and the allocation of labour between rural and urban areas than those identified in the existing literature on this problem. The critical factor responsible for our conclusions is that some goods, notably education, medical care and electricity, cost relatively more in the rural areas than in the urban areas of LDCs. While these cost differentials are not well documented (Linn, 1982), we believe that they are substantial and help to account for the relative dearth of these goods and services in the countryside. For instance, Turvey and Anderson (1977, p. 162) consider that the cost of providing electricity in rural areas is typically 2·5 to 4 times higher than in the cities.

This differential-cost phenomenon has implications for the efficient allocation of resources in LDCs and for the rural–urban distribution of income. It bears on a variety of decisions: unemployment policy, shadow wage pricing of labour, migration policy, taxation of inputs and outputs, and the provision of public services. In effect, agriculturalists are inefficient or high-cost consumers as a result of their location. As a consequence, there is an incentive, from the social welfare viewpoint, to discriminate against rural residents if feasible. This general type of conclusion in a situation of an asymmetric utility possibilities frontier has been noted by other researchers: see Mirrlees (1972) and Arnott and Riley (1977). It has been dubbed 'the unequal (*ex post*) treatment of (*ex ante*) equals' in the first-best optimum. We develop these ideas in the context of the dual economy with differential costs of consumption in Section II.

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[1] Other papers using the Harris–Todaro migration mechanism include: Corden and Findlay (1975), Fields (1975), Gersovitz (1974), Neary (1981), and Stiglitz (1974, 1982). In the Stiglitz (1974) model of labour turnover and migration there is some indication that optimal unemployment may not be zero and that a first-best solution cannot be attained without restrictions on migration. However, the maximising behaviour of job search from the employee's viewpoint is not sufficiently specified to prove this conclusion.

Discrimination of the type embodied in the first-best optimum can be achieved in a competitive equilibrium with an appropriate subsidy scheme if migration is controlled either physically or through a tax on the act of migration. If, however, rural and urban residents must have the same utility, the government cannot improve on the unrestricted competitive equilibrium. We discuss this equal-utilities equilibrium in Section III.

Our central focus is, however, a third type of equilibrium. As discussed in Section IV, it is one in which policy makers, while unable to control migration directly, are able to force a wedge between the expected utility of urban residents and the utility of rural residents by inducing urban unemployment. There is then a trade-off between unemployment (wasted resources) and sectoral inequality (desired given the differential costs of consumption). We discuss when a government can raise social welfare by intervening in a competitive economy to create unemployment. These conditions depend crucially on the proportion of urban jobs that are potentially available to migrants (non-patronage jobs). Consider the extreme case in which there are no non-patronage jobs; rural residents have no incentive to migrate to the city, and with appropriate government inter-vention, the first best can be achieved. At the other extreme, rural residents have the same access to urban jobs as urban residents. In this case, urban unemploy-ment idles labour without generating any socially desirable inequality, and the government must be resigned to the equal-utilities allocation. In intermediate cases it may be worth creating some unemployment to generate some inequality. We term the resulting equilibrium a second-best, unemployment equilibrium. The example beginning in footnote 3 on page 417 makes some of these points in an explicit and consistent manner under rather stark assumptions.

The final section of the paper discusses the shadow wage implications of the model of differential costs. We show that the shadow wage depends on whether individuals hired for government projects are ensured an urban sector job or are from the group facing possible unemployment in their search for urban employment.

I. THE GENERAL MODEL

There is a fixed number of *ex ante* identical individuals in the closed economy. An individual may work either in the manufacturing (M) sector located in the urban area, or in the agricultural (A) sector, or be unemployed (u). Individuals must reside and consume in the sector where they work or are unemployed – a critical indivisibility or non-convexity.

Tastes are described by the strictly concave utility function $U(a, m)$, where a is consumption of the agricultural good, and m consumption of the manufactured good, including public services. It is the m-good that is relatively more expensive to provide in the rural areas. The social valuation of utility is given by the concave function, $S(U)$.

The policy-maker's objective is to maximise social welfare,

$$W = L_M S[U(a_M, m_M)] + L_A S[U(a_A, m_A)] + L_u S[U(a_u, m_u)], \qquad (1)$$

where L_i is the proportion of individuals in activity i, and a_i and m_i are the

quantities they consume. The unemployed are not distinguished by sector, since the model is such that only urban unemployment occurs, as will soon be evident.

The production of both outputs uses only labour. Each worker has a constant marginal product of α in agriculture and 1 in manufacturing. These assumptions are made for analytical convenience because the consideration of factor-income distribution when fixed factors other than labour exist raises thorny questions about the distribution of factor ownership and the taxation of factor rents. We hope to relax this assumption in future research but believe that valuable insight can be gained most easily in this simplified framework.

Agricultural goods are privately consumed and costless to transport, and therefore

$$\alpha L_A - a_A L_A - a_M L_M - a_u L_u = 0. \tag{2}$$

The output of the manufacturing sector is used either in the city or in the countryside:

$$L_M - \theta m_A L_A - m_M L_M - m_u L_u = 0, \tag{3}$$

where $\theta > 1$ is the ratio of the cost of the m-good in the countryside to its cost in the urban area. The parameter θ represents evaporation, as in the loss of electric power as a result of the relatively greater transmission distances in the rural sector.[1] Labour balance (everyone is engaged in some activity) implies that

$$1 - L_A - L_M - L_u = 0. \tag{4}$$

Finally, we develop a migration equilibrium condition which is applicable when there is free migration, but not otherwise. We assume that a rural migrant does not have access to all jobs in the urban area. The parameter, β, denotes the fraction of all manufacturing jobs that a migrant has a chance of obtaining. This parameter has at least two interpretations. First, $(1 - \beta)$ may be conceptualised as the fraction of urban sector jobs that is available through patronage or connections, which incorporates the fact that *urban-born* city dwellers may be more efficient in search than migrants. Alternatively, $(1 - \beta)$ may represent a policy parameter with the government able to ensure that $(1 - \beta)$ of the urban jobs go to city-dwellers, but unable to assign the remainder. The extent of, and rules applicable to, government employment are an important determinant of β, but at this stage we do not attempt to model the micro-foundations of β. We refer to β as the proportion of non-patronage jobs.

[1] There are alternative formulations to this. The additional costs of consuming the m-good in the rural sector may involve an implicit sacrifice of the a-good rather than the m-good. For instance, travel to the city for medical care or education involves time away from agricultural production. Or, the good that costs more in the rural sector may also have to be produced there, often because it is a service. In this case there are two additional costs in the countryside: that incurred because producers of the agricultural good consume at higher cost and that incurred because producers of the good with differential costs are themselves located in the rural sector where they too consume some of the differential-cost good. In this specification there needs to be some other good that the urban sector can exchange for the a-good, if the equilibrium without government intervention is to have any urban sector when the a-good is necessary to survival. Otherwise, these two alternative formulations seem to lead to largely similar qualitative conclusions to the one we investigate in the paper. Finally, it should be noted that some goods may be cheaper to consume in the countryside, offsetting the effects we discuss. If this type of good predominates, which we doubt, our conclusions would be reversed, with the rural sector having the role of the urban and vice versa.

Each individual in the urban sector without a patronage job has an equal chance of obtaining a non-patronage job, an adaptation of the assumption of Harris and Todaro (1970). The probability (π) of employment for a migrant is then

$$\pi = \frac{\beta L_M}{\beta L_M + L_u}. \tag{5}$$

Migration occurs until the expected utility of a migrant equals the opportunity cost of leaving agriculture (U^A),

$$U^A = \pi U^M + (1 - \pi) U^u. \tag{6}$$

Substitution of (5) into (6) yields

$$L_u(U^A - U^u) - \beta L_M(U^M - U^A) = 0, \tag{9}$$

the form of the equilibrium condition we employ.

The planner's problem is to choose the a_j's, m_j's, and L_j's to maximise (1) subject to (2)–(4), and (7) as well if the migration constraint is applicable. The Lagrangean form is

$$\begin{aligned}
\max \mathscr{L} = & \; L_M S[U(a_M, m_M)] + L_A S[U(a_A, m_A)] \\
& + L_u S[U(a_u, m_u)] + \lambda_A[\alpha L_A - a_A L_A - a_M L_M - a_u L_u] \\
& + \lambda_M[L_M - \theta m_A L_A - m_M L_M - m_u L_u] + \phi(1 - L_A - L_M - L_u) \\
& + \eta[L_u(U^A - U^u) - \beta L_M(U^M - U^A)],
\end{aligned} \tag{8}$$

where $\lambda_A, \lambda_M, \theta$ and η are multipliers.

The corresponding first-order conditions are

$$\frac{\partial \mathscr{L}}{\partial a_M} = L_M S'_M U^M_a - \lambda_A L_M - \eta \beta L_M U^M_a = 0, \tag{9a}$$

$$\frac{\partial \mathscr{L}}{\partial a_A} = L_A S'_A U^A_a - \lambda_A L_A + \eta(\beta L_M + L_u) U^A_a = 0, \tag{9b}$$

$$a_u \frac{\partial \mathscr{L}}{\partial a_u} = a_u[L_u S'_u U^u_a - \lambda_A L_u - \eta L_u U^u_a] = 0, \tag{9c}$$

$$\frac{\partial \mathscr{L}}{\partial m_M} = L_M S'_M U^M_m - \lambda_M L_M - \eta \beta L_M U^M_m = 0, \tag{10a}$$

$$\frac{\partial \mathscr{L}}{\partial m_A} = L_A S'_A U^A_m - \lambda_M \theta L_A + \eta(\beta L_M + L_u) U^A_m = 0, \tag{10b}$$

$$m_u \frac{\partial \mathscr{L}}{\partial m_u} = m_u(L_u S'_u U^u_m - \lambda_M L_u - \eta L_u U^u_m) = 0, \tag{10c}$$

$$\frac{\partial \mathscr{L}}{\partial L_M} = S_M - \lambda_A a_M - \phi - \eta \beta(U^M - U^A) + \lambda_M(1 - m_M) = 0, \tag{11a}$$

$$\frac{\partial \mathscr{L}}{\partial L_A} = S_A + \lambda_A(\alpha - a_A) - \phi - \lambda_M \theta m_A = 0, \tag{11b}$$

$$L_u \frac{\partial \mathscr{L}}{\partial L_u} = L_u[S_u - \lambda_A a_u - \phi + \eta(U^A - U^u) - \lambda_M m_u] = 0. \tag{11c}$$

Equations ($9c$), ($10c$) and ($11c$) are written explicitly in Kuhn–Tucker form to reflect the fact that $a_u = 0$, $m_u = 0$ and $L_u = 0$ are possibilities. To simplify the algebra, we adopt the realistic assumption that a_M, a_A, m_M, $m_A > 0$.[1]

In the next sections we examine the properties of the solution of particular forms of this planning problem.

II. THE FIRST-BEST ALLOCATION

In the first-best situation, the planner has control over where individuals locate. The migration constraint (7) is therefore inapplicable, and $\eta = 0$. Propositions 2.1 and 2.2 describe some basic characteristics of the first-best equilibrium:

PROPOSITION 2.1. *Unemployment is zero* ($L_u = 0$).

Proof. Suppose not. Then from ($9a$), ($9c$), ($10a$) ($10c$), and concavity of $U(a, m)$, $a_M = a_u$ and $m_M = m_u$. But then ($11a$) is inconsistent with the term in parentheses in ($11c$) equalling zero, which implies that $L_u = 0$. Contradiction.

PROPOSITION 2.2. (i) $U^M > U^A$ *iff* m *is a normal good*, (ii) $m_M > m_A$ *and* (iii) $a_M \gtreqless a_A$ *as* $S''U_aU_m + S'U_{am} \gtreqless 0$.

Proof: From ($9a$) and ($9b$), $S'_M U_a^M = S'_A U_a^A$, while from ($10a$) and ($10b$), we have $S'_M U_m^M = \theta^{-1} S'_A U_m^A$. Define $\tilde{U} = S(U)$. We may compare the situation of a manufacturing worker to that of an agricultural worker by totally differentiating $\tilde{U}_a = k_0$ and $\tilde{U}_m = k_1\theta$ with respect to θ to prove that $da/d\theta$ has sign $-\tilde{U}_{am}$, $dm/d\theta < 0$ and $d\tilde{U}/d\theta < 0$ iff m is normal.

Despite the *ex ante* equality of individuals in the two sectors, the first-best optimum involves inequality.[2] In the 'normal' case, which we assume in the rest of the paper, urban dwellers are better off than rural dwellers. The fact that individuals must be located *entirely* in one sector or the other for purposes of both production and consumption is the source of the non-convexity in the model leading to this result. Agriculturalists are inefficient consumers from a Benthamite perspective. Urban bias is then the natural outcome of technological conditions that yield a $\theta > 1$.[3]

If policy is restricted to taxes/subsidies on input and output prices, the first-best cannot be attained. With free migration, nothing prevents agriculturalists from migrating until (7) is satisfied, which is inconsistent with the first-best allocation.

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[1] Consumption levels of each good by the three types of individuals must be non-negative or meet some other imposed subsistence thresholds. Without these constraints, the solution to the maximisation can lead to unbounded consumption quantities – for instance, where both the utility and social welfare functions are homogeneous of degree one and the migration constraint does not apply.

[2] So long as some of each good is produced and the manufacturing good has a non-zero income elasticity.

[3] The unequal treatment of equals can be presented even more starkly by the following example. Assume each individual *must* consume a^+ of the agricultural good to survive, but *never* desires to consume more. Further let $\theta = \infty$ so that rural residents cannot consume *any* of the manufactured good ($U_m(a, 0)$ is assumed finite). Then labour allocation in the first best will be $\alpha L_A = (L_M + L_A) a^+ = a^+$. Each agriculturalist will receive $U(a^+, 0)$ while each urban resident will receive $U(a^+, 1)$, i.e. utility from the whole of his product plus a subsidy of a^+. Clearly every agriculturalist would wish to become an urban resident. There is, however, no escape from the inequality in this economy. Even a Rawlsian would have to accept that the lot of the worst-off members cannot be improved. We continue this example in footnotes 1 on page 418 and 1 on page 419.

If individuals can be physically allocated between sectors, so that migration to the urban sector can be stopped, then it is possible to attain the first best. The planner sets L_A and L_M to satisfy the first-order conditions of the planning problem and prohibits labour movement. Of course, a physical restriction on migration is not necessary; a prohibitive tax on migrants, if enforceable, would yield the same result. From (9) and (10), the consumer price of the manufacturing good with the agricultural good as numeraire should be $p \equiv \lambda_M/\lambda_A$ in the city and $\theta\lambda_M/\lambda_A$ in the countryside. Incomes in the urban and rural sectors should satisfy (11). These conditions can be satisfied by imposing an *ad valorem* subsidy of $s = (S_M - \phi)/\lambda_M$ on production of the manufactured good and an *ad valorem* tax of $t = (\phi - S_A)/\lambda_A$ on production of the agricultural good. There are other possible decentralisation mechanisms, but restriction of migration by itself is not enough. It can be shown that the price ratio λ_M/λ_A may exceed or fall below α; thus relative consumer prices are not determined by the labour theory of value despite the linear structure of the technology. This result also derives from the assumption that individuals must consume goods in the location where they produce their output.

III. ANALYSIS OF THE EQUAL-UTILITIES ALLOCATION

When there are no patronage jobs so that $\beta = 1$, we have

PROPOSITION 3.1. (i) $L_u = 0$, (ii) $U^M = U^A$, (iii) $m_M > m_A$ and $a_M < a_A$.

Proof. From $(9a)$, $(9c)$, $(10a)$, $(10c)$, and the properties of the utility function, $a_M = a_u$ and $m_M = m_u$. Therefore, $(11a)$ and $(11c)$ imply $L_u = 0$. Hence $(9a)$, $(9b)$, $(10a)$ and $(10b)$ imply $U^M = U^A$. But this implies that agricultural and manufacturing workers are on different points on the same indifference curves, and hence $m_M > m_A$ and $a_M < a_A$.

The equal-utilities allocation is important both as the outcome of $\beta = 1$ and as a reference point for the discussion of the unemployment allocation.[1]

Neither input nor output taxes are required to decentralise this allocation. The equilibrium relative price in the city will be established at $p = \lambda_M/\lambda_A$. As can be seen from $(9a, b)$ and $(10a, b)$, this price satisfies the consumers' equilibrium conditions. From $(11a, b)$, the value of consumption equals the value of income when $p = \lambda_M/\lambda_A$, and there are no taxes.

Finally, the labour theory of values does not hold in this equilibrium either:

PROPOSITION 3.2. $p = \lambda_M/\lambda_A < \alpha$.

Proof. From Proposition (3.1), and the facts that (a_A, m_A) and (a_M, m_M) are on the same indifference curve and $\alpha = pm_A + a_A$, while $p = pm_M + a_M$.

We now turn to the role that unemployment can play as a partial tax on migration, allowing the planner to achieve an allocation that may be superior to the equal-utilities allocation.

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[1] Continuing the example of footnote 3 on page 417, without a restriction of migration, it would be impossible for urban residents to consume a^+ because they would have nothing to trade agriculturalists (since $\theta = \infty$). Then $L_M = 0$, as everyone returns to the rural sector to obtain agricultural goods and receives $U(a^+, 0)$. Agricultural goods would be in a surplus of $(\alpha - a^+)$ *per capita*. This example makes clear that the difference in social welfare between the first best and the *EUA* can be arbitrarily large.

IV. ANALYSIS OF THE UNEMPLOYMENT ALLOCATION

By lowering the probability that a migrant will obtain an urban job, urban unemployment discourages migration. Unemployment therefore acts as a tax or partial restriction on migration. We have already shown that a prohibitive tax or full restriction on migration can be used in decentralising the first-best allocation. Thus intuition suggests that the deliberate creation of unemployment in an otherwise laissez-faire competitive economy, might be welfare-improving in some circumstances. Such a strategy would not permit attainment of the first-best allocation, however, since there is no unemployment in the first best. We now discuss when an allocation characterised by positive unemployment dominates the *laissez-faire*, zero-unemployment equilibrium of Section III, and when it does what determines the (second-best) optimal amount of unemployment.

As noted earlier, unemployment can be generated in our economy by the creation of manufacturing jobs that provide a higher level of utility than agricultural jobs. If *all* urban jobs are non-patronage jobs, however, all individuals would have the same expected utility. Creating unemployment would have no beneficial effects, since it would not be successful in diverting goods from the rural resident to the *average* urban resident (with utility of U^A when $\beta = 1$). With patronage jobs, however, the creation of unemployment may generate desired inequality. Goods are effectively redistributed from everyone else to those residents in the city with patronage jobs (who are efficient pleasure machines).[1]

We first examine sufficient conditions for an allocation with positive urban unemployment to dominate the (zero-unemployment) equal-utilities allocation (EUA). We then analyse a special case based on a Cobb–Douglas utility function for which we can derive necessary and sufficient conditions as well as some other characteristics of the *EUA*.

The general sufficient condition for there to be an allocation with positive unemployment that dominates the EUA is that $\partial \mathscr{L}/\partial L_u > 0$, evaluated at the *EUA* or equivalently that

$$[S_u - \lambda_A a_u - \phi + \eta(U^A - U^u) - \lambda_M m_u]_{EUA} > 0. \tag{12}$$

This condition itself is not very illuminating. In what follows we consider first the Benthamite social welfare function, and then more general social welfare functions.

PROPOSITION 4.1. *If* $S(U) = U$ *(Benthamite Case) then there is an allocation with* $L_u > 0$ *that dominates the EUA if* $\eta_{EUA} > 1$.

Proof. At the *EUA*, $U^M = U^A$ and $L_u = 0$. From $(11\,a, b)$ these results imply that $\phi = U^M$, and (12) becomes $[(U^M - U^u)(\eta - 1) - \lambda_A a_u - \lambda_M m_u] > 0$. It is easily shown that $U^M > U^u$, $\lambda_A > 0$ and $\lambda_M > 0$, so that with $a_u \geqslant 0$, $m_u \geqslant 0$, it is impossible to satisfy (12) with $\eta \leqslant 1$. Thus $\eta_{EUA} > 1$ is necessary. But $\eta \geqslant 1$, $(9c)$

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[1] In the example of footnotes 3 on page 417 and 1 on page 418, unemployment could not be used to approximate the first best since with $U^u = U^A$ there would always be an incentive to migrate if $U^M > U^A$

and $(10c)$ together imply that $a_u = 0$, $m_u = 0$, so that (12) reduces to $\eta_{EUA} > 1$. From $(9a, b)$ and $S(U) = U$,

$$\eta_{EUA} > 1 \leftrightarrow \left. \frac{U_a^M - U_a^A}{\dfrac{L_M}{L_A} U_a^A + U_a^M} \right|_{EUA} > \beta. \qquad (13)$$

Now $[U_a^M / U_a^A]_{EUA} \sim \theta^\rho$, where ρ is the marginal propensity to consume m.[1] Using this approximation, (13) implies

$$\theta^\rho (1 - \beta) > 1 + \beta L_M / L_A. \qquad (13')$$

Thus, unemployment is more likely to be desirable, the higher is θ (the greater the cost differential between town and country), the higher is ρ (the higher the marginal propensity to consume m), the lower is β (the higher the percentage of patronage jobs), and the lower is L_M / L_A (the lower the proportion of workers in manufacturing). The higher is θ, the greater the inequality that can be generated from a given loss of output.

If the unemployed cannot be excluded from consumption of some of the m-good, say because it involves urban amenities available to everyone, then the condition for urban unemployment is less likely to be met. (In Proposition 4.1 the last step would not be valid, since $m_u > 0$ would be a constraint.) We now turn to the non-Benthamite ($S'' < 0$) cases.

PROPOSITION 4.2 a. *If $S'' < 0$ and $\eta_{EUA} > S'[U(0, 0)]$ then there is an allocation with $L_u > 0$ that dominates the EUA.*

Proof. If $\eta_{EUA} > S'[U(0, 0)]$, then $(9c)$, $(10c)$, $\lambda_M > 0$, and $\lambda_A > 0$ together imply that $a_u = m_u = 0$. Then (12) becomes $[S_u - S_M + \eta(U^M - U^u)]_{EUA} > 0$. Since $\eta_{EUA} > S'[U(0, 0)]$, then

$$S_u - S_M + \eta(U^M - U^u) > S_u - S_M + S'[U(0, 0)] (U^M - U^u) > 0$$

from the strict concavity of S. Because terms involving S' appear in $(9a, b)$ it is not possible to provide a simple analogue to equation (13) in the non-Benthamite cases.

In the general, non-Benthamite case, we can however, characterise the allocation by

PROPOSITION 4.3. *When $L_u > 0$ at the optimum:*
(i) $U^M > U^A > U^u$,
(ii) $a_A \gtrless a_M > a_u$,
(iii) $m_M > m_u \gtrless m_A$, *and* $m_M > m_A$.

Proof. (i) From (7) and the first-order conditions. (ii) By example. The last inequality by normality of a and $U_a^M / U_m^M = U_a^u / U_m^u$. (iii) Normality of m.

As a corollary, equations (9), (10), $S' > 0$ and $\eta > 0$ imply that $U_a^M > U_a^A$ and $\theta U_m^M > U_m^A$. The planner goes only part way to equating the marginal utilities of each good (adjusted by θ), as would be done in the first best, up to the point where

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[1] The elasticity of U_a with respect to U_m / U_a (or θ) with U held constant equals minus the marginal propensity to consume m.

the efficiency gain from making the marginal utilities more nearly equal is offset by the efficiency loss from higher unemployment. Of course it is still true that $U_a^M/U_m^M = \theta U_a^A/U_m^A = U_a^u/U_m^u$, the last equality holding only when $m_u > 0$ and $a_u > 0$.

We now specialise the model by assuming that $S(U) = U = a^\epsilon m^{1-\epsilon}$, $0 < \epsilon < 1$. The first-order conditions imply $a_u = m_u = U^u = 0$.[1] Solving the planner's problem without imposing the constraint that $L_u \geqslant 0$ yields

$$-L_u[\beta + (1-\beta)L_u] + \{\epsilon(\theta^{1-\epsilon}-1) - \theta^{1-\epsilon}[\beta + (1-\beta)L_u]\}\beta(1-L_u) = 0. \quad (14)$$

It can be shown (Appendix) that: (i) of the two roots for L_u in (14) only one can be between zero and one; (ii) when there is a root in this interval, it gives the optimal unemployment rate; and (iii) positive unemployment is desirable if and only if β is below a threshold value, β^T, where

$$\beta^T = \frac{(\theta^{1-\epsilon}-1)\,\epsilon}{\theta^{1-\epsilon}} < 1. \quad (15)$$

The value of β^T is strictly less than 1 because, at $\beta = 1$, the EUA, optimal unemployment is actually negative infinity, not zero. It is only the inequality constraint on L_u that leads to $L_u = 0$ in the EUA. Thus β must be distinctly less than 1 before the unconstrained L_u increases from negative infinity to zero.

As β increases from zero to one, for $\theta > 1$, the optimal unemployment rate first increases and then decreases. When β is near zero, there are few non-patronage jobs and rural dwellers have little incentive to move to the city and unemployment is low; while, with $\beta = 1$, unemployment creates a cost with no compensating benefit, so that the optimal unemployment rate is zero. For intermediate β, however, unemployment can generate desired inequality, and is necessary to do so. Note that $\partial\beta^T/\partial\theta > 0$. This result accords with the intuition that the condition for unemployment to be desirable is less stringent the greater the asymmetry in consumption possibilities between the two sectors.

Returning to the general, non-Benthamite model, we discuss conditions for decentralisation of the unemployment equilibrium, taking the agricultural good as numeraire. From (9) and (10), the manufactured goods prices should be λ_M/λ_A in the city and $\theta\lambda_M/\lambda_A$ in the country. From (11), the income of agriculturalists, the urban employed and the urban unemployed are, respectively,

$$y_A = \alpha - (\phi - S_A)/\lambda_A, \quad (16a)$$

$$y_M = [\lambda_M + S_M - \phi - \eta\beta(U^M - U^A)]/\lambda_A, \quad (16b)$$

$$y_u = [S_u - \phi + \eta(U^A - U^u)]/\lambda_A. \quad (16c)$$

These incomes will just allow the purchase of the consumption bundles at the appropriate consumer prices. Since ϕ is average social utility, (16a) implies that agriculturalists are taxed on their initial income of α. The urban employed

[1] Although our choice of utility function leads to a corner solution for a_A and m_A in the first-best allocation, it creates no problems in the unemployment allocation. Note that at $\beta = 1$ (9) can no longer be satisfied. Up until this point, a homogeneous-of-degree-one utility function can be consistent with bounded consumption of each good by each type of individual. It is because the analysis of the model is so intractable that we adopt this Cobb–Douglas utility function.

always receive a subsidy, as can be shown by rewriting the η constraint in (8) to involve only L_A and L_u and rederiving ($16b$) with redefined but positive λ_M and λ_A. This subsidisation of the urban employed can be interpreted as similar to an urban minimum wage policy. The total package necessary to sustain a decentralisation of the unemployment allocation does, however, require more than just a minimum wage. If $y_u > 0$, the urban unemployed also receive an income transfer, or unemployment compensation.

Cost benefit analysis of public projects requires a formula for the opportunity cost of labour. In the unemployment allocation, this cost depends on whether these jobs are created in the urban or rural sectors, and if in the urban sector, whether employment is open or closed to migrants. If the jobs are urban, and a fraction γ are open to migrants, then minus the shadow wage in terms of the agricultural good is

$$\frac{1}{\lambda_A}\frac{\partial \mathscr{L}}{\partial L_p} = \frac{S_M - \phi - \eta\gamma(U^M - U^A)}{\lambda_A} - y_M = -\frac{\lambda_M + \eta(\gamma - \beta)(U^M - U^A)}{\lambda_A}, \qquad (17)$$

where L_p project jobs are created.[1] If $\gamma = 0$ so that no project jobs are available to migrants, the shadow wage is less than y_M since $S_M > \phi$ When $\gamma > 0$, however, the shadow wage may exceed y_M, a particular case being $S = U$, $\gamma = 1$, and $U^u = m_u = a_u = 0$. Also, the shadow wage is greater (smaller) than the value of the marginal product in manufacturing as γ is greater (smaller) than β. In general, the shadow wage rises with γ since more unemployment is induced. On the other hand, when the project is located in the rural sector and gives project workers U^A, ($11b$) implies that the shadow wage equals α, the marginal product in agriculture which exceeds y_A the income of agriculturalists.

V. CONCLUSION

The central theme that we have stressed is the dilemma for policy-makers caused by the relatively high cost of providing certain goods and services to residents of rural areas. This one asymmetry between the two sectors results in a social incentive to discriminate against rural residents. Since rural residents will try to leave the sector to avoid this discrimination, the first best cannot be attained unless migration can be restricted. The creation of urban unemployment can, under certain circumstances, provide a means of discouraging migration. Consequently there may be a positive optimal unemployment rate.

Because unemployment derives from the desire of migrants to obtain an urban job, it is important in the shadow costing of labour in government projects to know whether migrants view employment with the project as possible or not. Depending on migrants' access to project employment, the shadow wage can either exceed or fall below the value of the marginal product of urban sector employees or the income of employed individuals in the urban sector. These results contrast sharply with assertions made by others that the shadow wage

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[1] Derived by substituting $L_M + L_p$ for L_M in all places in (8) except: (i) following η where $\beta L_M + \gamma L_p$ is substituted for L_M and (ii) following λ_M where L_p is only substituted following m_M since the L_p workers do not contribute to the output of the m-good.

should be equal to zero (because there is unemployment), the expected wage in the urban sector (and therefore always less than income of employed urban residents) or the wage of the urban employed.

The theoretical framework suggests some priorities for empirical research. As already noted, there is little knowledge about relative costs of providing different goods and services to the two sectors. There is also very little information on the importance to the migration decision of access to goods and services at relatively low prices. Some public goods may be virtually unavailable in the countryside. Effects should be made to document how migrants obtain jobs and the advantages held by urban-born job seekers.

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APPENDIX

Analysis of the Roots of Equation (14)

Equation (14) is a quadratic in L_u:

$$aL_u^2 + bL_u + c = 0,\qquad\qquad\qquad (A\ 1)$$

where $a \equiv (1-\beta)(\theta^{1-\epsilon}\beta - 1)$, $c \equiv [\beta\theta^{1-\epsilon}(\epsilon-\beta)] - \epsilon\beta$ and $b \equiv -(a+c+1)$. The two roots of (A 1) are R^+ and R^- with $R^+ > R^-$ found by applying the quadratic root formula to (A 1). Denote the β in (0, 1) for which $a = 0$ as β_a, etc. Then it can be proved that β_a and β_c are less than β_b but bear no necessary relation to each other.

By inspection of the graphs of each coefficient against β (not shown) there are five possible cases

$$
\begin{aligned}
&\text{I:} \quad b > 0, \quad a > 0, \quad c < 0; \quad R^+ > 0, \quad R^- < 0, \\
&\text{II:} \quad b < 0, \quad c > 0, \quad a < 0; \quad R^+ < 0, \quad R^- > 0, \\
&\text{III:} \quad b < 0, \quad c < 0, \quad a < 0; \quad R^+ < 0, \quad R^- < 0, \\
&\text{IV:} \quad b < 0, \quad c > 0, \quad a > 0; \quad R^+ > 0, \quad R^- > 0, \\
&\text{V:} \quad b < 0, \quad c < 0, \quad a > 0; \quad R^+ > 0, \quad R^- < 0.
\end{aligned}
$$

Case III can immediately be ruled out since both roots are negative.

The other cases can be examined by making the substitution $L_u = 1 - \hat{L}_u$ in (14) to yield

$$
\hat{a}\hat{L}_u^2 + \hat{b}\hat{L} + \hat{c} = 0 \tag{A 2}
$$

where $\hat{a} \equiv a$, $\hat{b} \equiv (2 - \beta - \beta\epsilon) + \beta(\epsilon - 1)\theta^{1-\epsilon}$ and $\hat{c} \equiv -1$. Note that $\hat{b}$ is linear in β, and the $\beta_{\hat{a}} > \beta_{\hat{b}}$. By inspection of the graphs of these coefficients against β, there are three cases:

$$
\begin{aligned}
&1: \quad \hat{b} > 0, \quad \hat{a} < 0, \quad \hat{c} < 0; \quad \hat{R}^+ > 0, \quad \hat{R}^- > 0, \\
&2: \quad \hat{b} > 0, \quad \hat{a} > 0, \quad \hat{c} < 0; \quad \hat{R}^+ > 0, \quad \hat{R}^- < 0, \\
&3: \quad \hat{b} < 0, \quad \hat{a} > 0, \quad \hat{c} < 0; \quad \hat{R}^+ > 0, \quad \hat{R}^- < 0,
\end{aligned}
$$

where $\hat{R}^+$ and $\hat{R}^-$ are defined analogously to R^+ and R^- but with $\hat{a}$, $\hat{b}$ and $\hat{c}$ replacing a, b and c. A negative root to equation (A 2) implies $L_u > 1$, an unacceptable value.

Putting these cases together we have

$$
\begin{aligned}
&\text{I: 2 or 3:} \quad R^+ > 1, R^- < 0, \\
&\text{II: 1:} \quad\quad R^+ < 0, 1 > R^- > 0, \\
&\text{IV: 2 or 3:} \quad R^+ > 1, 1 > R^- > 0, \\
&\text{V: 2 or 3:} \quad R^+ > 1, R^- < 0.
\end{aligned}
$$

Hence only cases II and IV satisfy the condition that at least one root be positive but less than one. In both cases it is R^- that is acceptable. These cases imply $\beta < \beta_c$ and we define $\beta^T \equiv \beta_c$.