General Equilibrium with Parallel Markets for Goods and Foreign Exchange: Theory and Application to Ghana

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Summary. — This paper presents a simple model of a distorted economy with parallel markets which is motivated by the Ghanaian experience. The model has simultaneous illegal trade in goods and foreign exchange markets. Its general equilibrium determines the black market foreign exchange rate and the price of consumer goods. After determining some comparative statics, the predictions are tested on Ghanaian monthly data from 1982–83. The empirical results are shown to be consistent with those from the theoretical model.

1. INTRODUCTION

Until recently, many governments in African countries (and elsewhere) have attempted to control their balance of payments by imposing exchange controls and quantitative restrictions (QRs) upon imports. Such policies have been justified either by concerns with the balance of payments and mounting foreign indebtedness, or by the choice of an inward-looking development strategy (or both). In most cases, QRs are imposed initially on imports of consumption goods, leaving capital goods imports relatively free for as long as possible. This paper will focus upon some consequences of rationing consumer goods on the peasant economy, and is motivated directly by the experience of Ghana.

Parallel markets are common as a response to shortage, i.e., as a means of venting excess demand. In particular, incentives are created to acquire foreign exchange through illegal channels, in order to supplement allocations of scarce imported consumer goods. Smuggling is one means by which illegal foreign exchange acquisitions are made. Ghana and Nigeria are among the most famous places in Africa for unrecorded and illegal trade in goods and currency across their borders with their neighbors, who belong to the CFA franc zone.¹ The incentives to smuggle are often fueled additionally by the fact that the producer price of cash crops is kept below the world price in order to tax producers.

The present paper analyzes a theoretical model of parallel trade in goods and foreign exchange, with an econometric application to Ghana. We test a model of the simultaneous determination of the black market exchange rate (BME) of the cedi to the CFA franc, and of the consumer price index (CPI). We are by no means the first to analyze smuggling in general and in Ghana in particular. However, our approach is somewhat different to those taken before. Other aspects of smuggling in Ghana have been studied previously by Franco (1981), May (1985) and Pinto (1987). The theory of smuggling was considered by

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Bhagwati and Hansen (1973) and developed by Pitt (1981), among others. Our approach differs from theirs since general equilibrium consequences are paramount in our model.

We demonstrate how spillovers from one rationed market to another may be important. There are interdependencies between the demands for consumer goods and foreign exchange when the former are in short supply. Our model also, crudely, captures some intertemporal effects. We emphasize that smuggling is a two-way activity: export crops are smuggled out, while consumer goods are smuggled in. Of course, parallel trade need not balance, and we also consider the asset demand for foreign exchange, which yields claims to future imported consumer goods. We postulate that because of its two-way nature, smuggling benefits from economies of scope. Increased smuggling out of export crops lowers the costs at the margin of smuggling consumer goods into the country.

Economies of scope might arise for a number of reasons. First, they may represent gains accruing through reductions in transportation costs; having found a vehicle to smuggle out the export crop to a neighboring country, it is relatively cheap to bring it back full with consumer goods. Second, if smuggling is possible because officials are bribed, then having an official in one’s employ may assist in smuggling both ways.

When such economies are large, the incentive to smuggle back in consumer goods immediately rather than to hold foreign exchange is enhanced. If economies of scope are significant, we shall demonstrate that they may have striking implications for some properties of the economic equilibrium. For example, if consumer goods imported through official channels are rationed, and sold at an artificially low price on the official market, there may be an incentive to smuggle some of the export crop abroad in order to purchase more consumer goods. The incentives to do this depend upon, and in turn determine, the price of illegally imported consumer goods and the black market exchange rate. We present a theoretical model in which these are determined together by the requirement that the market for consumer goods clears along with the market for “illegal” foreign exchange.

In order to analyze some of the interdependencies that a full-blown general equilibrium model should take into account, while at the same time keeping the model simple, we model the behavior of a representative agricultural household (see, e.g., Singh, Squire and Strauss, 1986) that transacts on parallel markets as a seller of a cash crop which it cannot consume, and as a buyer of consumer goods which it cannot produce. We also incorporate some intertemporal aspects of the problem. These underpin the asset demands for domestic and foreign currency whose properties are important below.

Our model seems appropriate for the Ghanaian experience, where cocoa has been smuggled in order to purchase consumer goods which are in short supply (see Azam and Besley, forthcoming). Hence, using our theoretical model as a guide, we present some empirical results for a model of the simultaneous determination of the black market exchange rate and the consumer price index in Ghana. We then compare our findings with the comparative statics of the theoretical model. Subject to the data limitations which we face, the results we find are reasonably encouraging although anything more than tentative conclusions must await a more comprehensive test.

The structure of the paper is as follows. In the next section, we set up the theoretical model. Section 3 shows how the equilibrium price level and black market exchange rate are determined, while Section 4 presents the empirical results for Ghana, based on monthly data from January 1982 to December 1983. In section 5, we conclude.

2. A SIMPLE MICROECONOMIC MODEL OF PEASANT BEHAVIOR WITH PARALLEL MARKETS

There are three goods and two assets in our model. There is an imported consumer good and a cash crop, which is produced by peasant farmers and exported. The third good is leisure which, as usual in this type of model, serves as a catch-all for the nontraded goods which are either produced and consumed within the household, or which are traded on a perfectly competitive (flex price) domestic market (Azam and Besley, 1988). There are two assets in the model: domestic and foreign currency.

(a) Notation

The representative household purchases a quantity $C$ of consumer goods from three sources:

(i) $C_F$ is the quantity bought on the official market, at a price $p_F$. As we are modeling an economy where parallel markets coexist for the three goods, we typically assume that the price $p_F$ is below $p$ (there is an implicit subsidy). This would be the case if official and unofficial
importers faced the same world price denominated in foreign currency, while official importers get an allocation of foreign exchange at the overvalued official exchange rate. The quantity \( C_F \) can be thought of as being rationed to a few lucky consumers who get it at the official price.

We assume that the rationing scheme is costless and cannot be manipulated (see Benassy, 1982; Jones and Roemer, 1987). Hence, \( C_F \) is a policy-determined variable and exogenous for our purposes.

(ii) \( C_S \) is the quantity which is smuggled into the country, at the price \( e p_S \), where \( e \) is the black market exchange rate (BME) and \( p_S \) the world price of the good, expressed in foreign currency units.

(iii) \( C_B \) is the quantity of consumer goods which is traded on the black market at price \( p \), but not smuggled into the country (i.e., it is the leakage from the official market into the parallel market). In the real world, the black market is fueled by such leakages. However, we do not model the process governing this here and we shall treat \( C_B \) as exogenous throughout. Aggregate consumption of the consumer good is

\[
C = C_F + C_S + C_B. \tag{1}
\]

We denote the quantity of "leisure" consumed by \( N - L \) (where \( N \) is the total labor endowment and \( L \) is labor supply), and we assume that the quantity of output \( X \) is produced with constant returns to scale with labor \( L \) as the only input. We choose units such that:

\[
X = L. \tag{2}
\]

This cash crop can either be sold on the official market, at the official price set by the government and denoted by \( q_F \), or smuggled out of the country, at a price \( e q_S \), where \( q_S \) is the given world price of the crop and \( e \) is the black market exchange rate. It is clear that if the black market exchange rate exceeds the official exchange rate and the price set by the government is less than the world price, then there may be an incentive to smuggle (depending upon the precise costs of doing so). As before, the subscript " \( F \)" is used for official and " \( S \)" for smuggled, giving us the following expression for total output:

\[
X = X_F + X_S. \tag{3}
\]

Smuggling is a costly activity. It entails transportation costs, bribes and fines, search costs, etc. We do not analyze the smuggling "technology" in great detail. Instead, we postulate a cost function for smuggling expressed in domestic currency units and denoted by:

\[
Z(C_S, X_S), \tag{4}
\]

satisfying the following assumptions (subscripts denote partial derivatives):

\[
Z_{C}, Z_{X}, Z_{CC}, Z_{XX}, Z_{CC} - (Z_{CX})^2 > 0 \\
\text{and } Z_{CX} < 0. \tag{5}
\]

It is the assumption \( Z_{CX} < 0 \) which we refer to as economics of scope, since the marginal cost of smuggling consumer goods is lower, the greater the level of smuggling of the export crop and vice versa. Jones and Roemer (1987) and Pitt (1981) use a somewhat different cost-of-smuggling function, in which the risk of getting caught depends upon the level of legal transactions which give a "cover" for illegal ones. Although this has some merit as a specification of smuggling costs, we do not consider it here; the extension would be fairly simple.

Finally, we denote by \( M \) and \( D \) the end-of-period quantities of domestic and foreign currency, which can be converted into each other at the BME. Total assets, \( A \), at the end of the period are thus:

\[
A = M + eD \tag{6}
\]

with corresponding initial balances: \( A_o, M_o \) and \( D_o \). Putting together the above ingredients, we have the budget constraint faced by the representative agent:

\[
A + p_F C_F + p_C B + e p_S C_S = A_o + q X_F + e q_S X_S - Z(C_S, X_S). \tag{7}
\]

We can express the balance of trade on the parallel market \( T \) as:

\[
T = q S X_S - p S C_S = D - D_o \tag{8}
\]

i.e., the value of imports less the value of exports at the price vector \((p_S, q_S)\). The representative agent’s utility function is given by:

\[
U(C, N - L, D, M, v^e) \tag{9}
\]

where \( v^e \) denotes the vector of expected future values of all the relevant exogenous variables, including prices and future rations of official consumer goods. We treat these expected values as exogenous in this short-run model, and hence they will be kept implicit from here on. We will assume that \( U(\cdot) \) is twice continuously differentiable, strictly increasing in its arguments, and strictly quasi-concave. Including \( M \) and \( D \) in the utility function, as we do here, is a well-known simplification which takes advantage of the recursive nature of dynamic programming exercises (see, for example, Benassy, 1982). It allows one to handle the intertemporal aspect of the problem at hand, while only dealing explicitly with current variables, given the expected paths
of the relevant exogenous variables, and the
optimal choice of all the future control variables.

A utility function such as \( U(\cdot) \) is sometimes
referred to as “semi-indirect.”

To solve this problem in a convenient way, we
will take advantage of its recursive nature (see
the discussion in Singh, Squire and Strauss,
1986). We first analyze the part of the house-
hold’s behavior which can be dealt with as a
model of a smuggling firm.

(b) The smuggling firm

Let \( r_C \) and \( r_X \) denote the premiums which can
be earned by smuggling in and out, defined
respectively as:

\[
\begin{align*}
    r_C & = p - e p_S, \\
    r_X & = q e S - q.
\end{align*}
\]

Using these, we can now define the profit
function from smuggling by:

\[
R(r_C, r_X) = \max_{C_S, X_S} (r_C C_S + r_X X_S - Z(C_S, X_S)).
\]

It will have all of the usual properties of profit
functions: \( R(\cdot) \) is twice continuously differenti-
able, convex and increasing in its arguments. The
demand for smuggled consumer goods and the
supply of the smuggled export crop can be
derived from the profit function using Hotelling’s
lemma, i.e.,

\[
R_C = C_S(r_C, r_X) \quad \text{and} \quad R_X = X_S(r_C, r_X),
\]

where \( R_C \) and \( R_X \) denote the partial deriva-
tives of \( R(\cdot) \) with respect to \( r_C \) and \( r_X \),
respectively. For future reference, it is useful to note
that profit maximization leads to the following arbi-
trage conditions (ruling out corner solutions):

\[
p = e p_S + Z_C \quad \text{and} \quad q = e q_S - Z_X.
\]

These say that in equilibrium, the premiums on
smuggling out and smuggling in are set equal to
their respective marginal costs. It is straightforward
to check that under the assumption of
economies of scope in smuggling, the functions
\( C_S(\cdot) \) and \( X_S(\cdot) \) are increasing in both of
their arguments. This means, for example, that an
increase in the premium on smuggled consumer
goods into the country, which might be brought
about by domestic inflation, yields an incentive
to smuggle more of the export crop out of the
country. Keeping \( p_S \) and \( q_S \) as implicit pa-
neters, we can write the smuggling flow func-
tions as:

\[
C_S(e, p, q) \quad \text{and} \quad X_S(e, p, q)
\]

The signs of the influences of \( p \) and \( q \) are fairly
straightforward to determine: an increase in \( p \)
and a decrease in \( q \) have a positive impact on the
two premiums, providing incentives for increasing
both types of smuggling flows. The effect of
an increase in the black market price of the
foreign currency is ambiguous: it increases the
premium on smuggling out, while reducing the
premium on smuggling in. If economies of scope
are not too strong, then one may expect a
negative impact on \( C_S \) and a positive impact on
\( X_S \), which is conveyed by the signs given after the
question marks.

Defining the balance of trade on the parallel
market as in equation (7) above, and using the
assumption of economies of scope in smuggling,
an increase in the BME has a positive impact on
the balance of trade on the parallel market:

\[
T_e = q_S \quad \frac{\partial X_S}{\partial e} - p_S \quad \frac{\partial C_S}{\partial e} > 0
\]

where the subscript \( e \) denotes the relevant partial
derivative. The impact of \( p \) and \( q \) on \( T \) are
ambiguous in sign, under the assumption of
economies of scope economies. If, however, the posited
economies are not too strong, then the signs given in
the following equation are reasonable:

\[
T(e, p, q).
\]

Hence, given the two endogenous prices \( e \) and \( p \),
and the policy-determined producer price \( q \),
the analysis of the smuggling firm has enabled us
to determine the trade flows and the current
account on the parallel market. Next, we need to
embed these results in the complete model. To
this end, we first define the full wealth of the
representative agent and consider the wealth
effects of parameter changes.

(c) The wealth effects of market controls

We shall define the full wealth of the represen-
tative household as:

\[
W(e, p, q) = A_e + q N + (p - p_r) C_r + R(e, p, q),
\]

where we have suppressed a number of variables
from the notation for convenience. The first two
terms in equation (17) are familiar, being the sum
of the initial balances of domestic money and
foreign currency, and the market value of the
labor endowment. The last two terms are more
specific to the present model, and pinpoint the two channels of influence for trade and exchange controls on the wealth of the representative agent. We have already defined \( R(\cdot) \) as the profit from smuggling. The term \((p - p_s)C_F\) represents the subsidy implicit in the provision of an amount \(C_F\) of consumption goods at the official price \(p_F\) which is below the market price \(p\). Hence the policy of providing \(C_F\) at the official price is just like a lump sum subsidy. As long as there is rationing of consumer goods, their price and quantity have only inframarginal effects.

Most of the effects of changing the arguments of the function \(W(\cdot)\) are straightforward. Nevertheless, some of them are worth a comment. An increase in the official price of the crop \(q\) has a positive effect on wealth, as the resulting reduction in the smuggling profit is more than offset by the increased value of the labor endowment, provided that some leisure is consumed in positive quantity, and some output is sold on the official market. Nominal wealth \(W\) is as well an increasing function of \(p\), via the smuggling profit and the lump sum subsidy. The effect of the BME is positive as well:

\[
W_e = q_s X_S - p_s C_S + D_e = D > 0.
\]

(18)

Finally, an increase in \(C_F\) or a decrease in \(p_F\) has a positive effect of wealth. We can now give a complete account of the functions describing the representative agent’s behavior.

(d) The behavioral functions

A complete description of the representative agent’s behavior is derived from maximization of the utility function (8), subject to the budget constraint:

\[
W(\cdot) \geq p C + q(N - L) + M + eD.
\]

(19)

Assuming away corner solutions, this exercise yields the agent’s demands for the two assets and consumer goods and his supply of labor which we summarize as:

\[
C(e, p, q, W),
\]

\[
X(e, p, q, W),
\]

\[
D(e, p, q, W) \text{ and}
\]

\[
M(e, p, q, W).
\]

(20a)

(20b)

(20c)

(20d)

The relevant price of consumer goods at the margin is \(p\), and not the administered one \(p_F\), for reasons that we explained above. Few comments are required to justify the assumed signs. We have assumed that all four goods are normal, hence the wealth effect acts negatively on output. We have also assumed that the substitution effect of a rise in \(q\) dominates the income effect, in the demand for leisure function. To understand the effects of \(p\), beside the substitution effect between consumption and leisure, one must remember that this model is a short-run one, and assumes that the expected paths of all the relevant prices are given. Hence, an increase in \(p\) entails an intertemporal substitution effect: it is both an increase in the current price of the good and a reduction in expected inflation. Thus, it has a positive impact on savings. We have nevertheless assumed that the substitution effect of the cut in the real producer price is the dominant effect in the output supply function. Finally, an increase in the BME has both a substitution effect (away from \(D\)), an income effect and an intertemporal substitution effect.

The positive effect of \(e\) on \(C\) might seem counterintuitive at first sight, as the consumption good is imported. It must be understood as the partial effect of the domestic price of the foreign currency, given the price \(p\) of the consumption good. An increase in \(e\) is regarded here as an increase in the price of foreign currency as an asset, while its indirect impact through \(p\) will only be taken into account in the general equilibrium analysis below. As \(D\) becomes more expensive when \(e\) rises, the demand for the three other goods is increased under reasonable assumptions about the static and the intertemporal substitution effects. On the other hand, there is a negative income effect, which we assume to be dominated by the two substitution effects. This explains the positive impact of \(e\) on the demand for the consumption good and the demand for leisure.

We now possess all the ingredients for analyzing the general equilibrium of the model, which determines \(e\) and \(p\) as endogenous variables.

3. GENERAL EQUILIBRIUM AND COMPARATIVE STATICS

At this point, it is useful to draw diagrams which give a better intuitive understanding of how the markets for the consumer good and for the cash crop clear. Figure 1 illustrates the determination of the equilibrium price \(p^*\) and the equilibrium quantity consumed \(C^*\), where the demand and supply curves intersect at the partial equilibrium of the consumption good market, assuming, for simplicity, that \(C_B = 0\). The arbitrage equation (13) has been used to draw the supply curve, and \(C(\cdot)\) is the demand function.
(20a). The quantity of consumer good smuggled is, of course:
\[ C^*_k = C^* - C_F, \]
which can be seen in the diagram. The implicit subsidy due to the procurement of the quantity \( C_F \) of the consumption good at below equilibrium price is the rectangle \( p^* p B A \) in the diagram. Part of the profits from smuggling is given by the area of the curvilinear triangle above the supply curve, below \( p^* \), and between \( C_F \) and \( C^* \).

These diagrams also give an approximate picture of the welfare loss due to market controls. These are represented by the areas of the triangles to the right of \( C^* \) and \( X^* \), below the \( C(\cdot) \) curve between \( p^* \) and \( p k \), and above the \( X(\cdot) \) curve, between \( qk \) and \( q \) in Figure 2. The picture given by these is only approximate, as the \( C(\cdot) \) and \( X(\cdot) \) curves would shift if market controls were removed. This will be made clearer below. We have not specified the nature of the costs of smuggling \( Z(\cdot) \). These costs must be added to the welfare loss in the present setting since they represent a real resource cost.

We turn next to the simultaneous determination of the equilibrium values of \( e \) and \( p \). Given all of the exogenous variables, this requires us to analyze simultaneous clearing of the markets for consumer goods and for foreign exchange. In this four-good model, we therefore characterize the general equilibrium by determining only two prices \((e^*, p^*)\), as the other two markets have fixed price \( q \) for the cash crop, while we have normalized on the price of domestic currency. The market for \( X \) clears in the way illustrated in Figure 2, while the market for \( M \) clears by adjustment of the end-of-period money supply, which is determined by the government budget constraint, not modeled here. Walras's law ensures that we have equilibrium in the fourth market when the other three clear.

(a) The general equilibrium of the model

The market for consumer goods clears, assuming for simplicity that \( C_B = 0 \), when:
\[ C(e, p, q, W) = C_F + C_Z(e, p, q), \]
\[ D(e, p, q, W) = T(e, p, q) + D_e, \]
\[ D(e, p, q, W) = T(e, p, q) + D_e, \]
\[ D(e, p, q, W) = T(e, p, q) + D_e, \]
\[ D(e, p, q, W) = T(e, p, q) + D_e, \]
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\[ D(e, p, q, W) = T(e, p, q) + D_e, \]
\[ D(e, p, q, W) = T(e, p, q) + D_e, \]
\[ D(e, p, q, W) = T(e, p, q) + D_e, \]
\[ D(e, p, q, W) = T(e, p, q) + D_e, \]
\[ D(e, p, q, W) = T(e, p, q) + D_e, \]
demand for foreign exchange to the right of the ee locus, which is corrected by depreciation of the local currency and is illustrated by the vertical arrows. The relative slopes of the ee and pp loci are obtained by imposing stability of the equilibrium point \((e^*, p^*)\) for simultaneous adjustments of \(e\) and \(p\), governed by the \(tatonnement\) process described above. This is a prerequisite for the comparative statics exercises performed below.

(b) Comparative statics

The model can be used to perform some simple policy experiments, which highlight some of the choices open to governments of economies with parallel markets. They are summarized in Table 1.

Consider first the impact of an increase in the official producer price \(q\). It results in more cash crop being sold through official channels, and less through the border. At the same time, fewer consumer goods are smuggled into the country, and the demand for foreign currency increases. Hence, under the assumptions stated above, the ee locus shifts upward, i.e., a depreciation of the local currency is needed to remove the excess demand for foreign currency created at the initial value of \(p\). Similarly, the pp locus shifts to the right, as excess demand for the consumption good increases at the initial equilibrium. Therefore, the new general equilibrium following the increase in the producer price is characterized by a depreciated domestic currency on the black market, and hence by an increased price level, as few consumer goods are smuggled into the country.

An increase in the allocation of consumer goods through official channels leads unambiguously to an upward shift of the ee locus operating through the wealth effect. By contrast, the impact on the pp locus is ambiguous: there is a supply effect which pushes this locus to the left, and a wealth effect which tends to pull it back to the right. If one assumes that the wealth effects are weak as compared to the supply effect, one would predict a fall in \(e\) and \(p\). But, if the wealth effects are strong enough, one may get \(e\) up and \(p\) down, or even \(e\) and \(p\) up, the latter being a somewhat pathological case.

An increase in the official price of the consumption good is deflationary and leads to an appreciation of the local currency on the black market. This result is entirely due to the wealth effect of the reduced subsidy, implying a reduction of the demand for consumer goods and foreign exchange (ee goes down and pp up).

We have not explicitly represented the official exchange rate in this model, but we can try to capture some effects of a devaluation by analyzing the impact of a proportional increase in \(p_F\) and \(q\). On the goods market, this tends to create excess demand, shifting the pp locus to the right, assuming that the negative wealth effect of the increased \(p_F\) (representing a cut in the subsidy) is offset by the other effects. Similar reasoning applies to the black market for foreign exchange and the ee locus shifts upward. Hence, the devaluation leads to a depreciation of the BME and an increase in the CPI. Notice that a cut of \(M_c\) might be used to offset these effects.2

It is rather difficult to analyze the impact of a liberalization policy using this model. Most probably, such a policy would consist of a devaluation, accompanied by dismantling of market controls. As emphasized in the previous section, the latter would entail wealth effects, which are theoretically removed by the lifting of market controls. In the present framework, this is a deflationary influence, opposing on both markets the effect of the devaluation. One would, however, need a more sophisticated

<table>
<thead>
<tr>
<th>Policy change</th>
<th>(p)</th>
<th>(e)</th>
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<tr>
<td>Increase in the official producer price ((q))</td>
<td>+</td>
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<tr>
<td>Increase in the allocation of consumer goods ((C_P)) (\text{with weak wealth effect})</td>
<td>-</td>
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<tr>
<td>Increase in the official consumer price ((p_c))</td>
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</table>
model to go any deeper into this analysis, especially on the supply side. Using our theoretical model as a guide, we shall now test some of its predictions on Ghanaian data.

4. ECONOMETRIC APPLICATION

It is impossible, given the data constraint, to estimate a structural model of the behavioral functions derived above. Our aim is only test some of our comparative static predictions regarding the black market exchange rate and the rural price level. It is not, unfortunately, possible to estimate the two equilibrium loci (23) and (24), because they are not identified. Hence, one must be content with a reduced form model for each variable.

It is rather difficult to put together a consistent set of data on which to test the model presented above, because the Ghanaian data are rather poor. One needs, in particular, to use some rather rough proxies for the quantity of consumer goods made available through official channels. Data availability has thus determined our sample choice, consisting of monthly data for 1982 and 1983.

(a) The data

To capture the extent of consumer goods' shortage, we have used the level of import duties levied by the customs in nominal terms. To our knowledge, there were no substantial rate changes in the tariff structure during our period. We use $ID$ to denote the logarithm of this variable. The variable has been used undeflated for lack of an appropriate deflator. Within the two-year period of the sample, it is probable that the official prices of these goods at the customs have not changed much, as compared with the CPI. According to the Central Bureau of Statistics in Accra, the latter is predominantly based on observed free market prices, and not on administered ones. It therefore corresponds roughly to our theoretical $p$ and not to $p_r$. Deflating by this CPI would have probably overstated the extent of shortage.

The official exchange rate remained fixed over the most of the period under study, i.e., until October 1983. A dummy variable called $DD$ has been used to capture the effects of this change over the last three months of the sample. Coincidentally or not, it happens that the official price of cocoa was changed as well at that date, after having been kept constant for some years. Therefore, the $DD$ dummy captures simultaneously the devaluation of the official exchange rate and the increase in the official price of the main export crop.

Another important policy change which occurred during these two years (following the return of Lt. Lt. Rawlings to power in December 1981) was the Economic Recovery Program, which was launched in April 1983. A dummy variable called $DR$ has been included in the equations to take care of this event. To capture the linkage of the consumer goods market in Ghana with its neighbors, we have used the CPI in Côte d'Ivoire. It is an index of the prices of the consumer goods for African families in Abidjan, published by the Banque Centrale des États d'Afrique Ouest (BCEAO). We denote its logarithm by $PA$ in the equations below.

To represent the cash balance effect, we have used a narrow definition of the quantity of money, namely “currency held outside the banking sector,” from the Quarterly Digest of Statistics (QDS) published by the Bureau of Statistics. Although we suspect that some of the big traders on the parallel market are in fact using checking accounts in the banks, we believe that the narrow definition is appropriate for “the representative agent.” We use $M$ to denote its logarithm.

Finally, the dependent variables to be explained are the “rural CPI” (again from the QDS) and the black market exchange rate of the cedi to the CFA franc, computed from Pick's Currency Yearbook. The logarithms of these two variables are denoted by $P$ and $E$, respectively.

(b) The results

Our preferred equation for the CPI is:

$$P = -11.7 + 0.47 P (-1) + 0.55 M + \frac{1.80 PA (-1) - 0.09 ID (-1) + 0.14 DR + 0.09 DD}{(2.26)} \frac{(2.13)}{(2.40)} \frac{(2.37)}{(2.37)} \frac{(2.37)}{(2.37)} (25)$$

$N = 21, R^{2a} = 0.99, FLM = 0.24.$

All the variables (except the dummies) are in logarithms, and $N$ is the number of observations, $R^{2a}$ is the usual coefficient of determination corrected for the degrees of freedom, and $FLM$ is the $F(8,11)$ version of the LM test for residual autocorrelation up to the second order, which is not biased by the presence of the lagged endogenous variable in the equation (see Breusch
and Pagan, 1980; Pagan, 1984). The numbers in parentheses below the estimated coefficients are the usual t-ratios.

The fit is excellent, and the residuals are serially uncorrelated. The coefficient of the lagged endogenous variable shows that this equation is dynamically stable. All the explanatory variables have the expected sign. Notable are the significant influence of the CPI in Abidjan, as well as the negative impact of import duties, with a one-month lag (ID(−1)), which represents, according to our theoretical model, the supply effect of an improved availability of consumer goods through official channels. The two dummy variables show that the devaluation and the launching of the Recovery Program had an inflationary impact.

The preferred equation for the BME is:

\[ E = -1.00 \ E(-3) + 1.26 \ P(-3) - \]
\[ 0.99 \ DR + 1.64 \ M(-3) + 1.95 \ M(-12) - \]
\[ 4.14 \ PA(-3) - 0.44 \ ID(-1) - \]
\[ 0.36 \ ID(-3) \]
\[ (7.08) \quad (5.83) \quad (6.80) \quad (3.47) \quad (8.38) \quad (6.41) \quad (5.39) \]

\[ N = 21, \ R^2 = 0.91, \ FLM = 2.99 \]

The fit is less impressive than above, and the test of serial independence of the residuals is passed with a narrow margin. Nevertheless, the equation is satisfactory, as all the coefficients have the predicted sign. Notice that the Economic Recovery Program seems to have had a positive impact on the value of the cedi on the black market. The impact of the quantity of money is strong and long lasting, as shown by the size and the significance of the coefficient of \( M(-12) \). The CPI in Abidjan has, once again, the predicted sign, as does the import duty variable. These results are rather comforting for our theory. Nonetheless, the empirical model is somewhat crude and should be regarded as essentially illustrative.

5. CONCLUSIONS

We have investigated a small general equilibrium model with parallel markets from both theoretical and empirical perspectives. Parallel markets are quite common as responses to market controls and, as such, are worth investigating. One consequence is that the official exchange rate becomes a fiscal device to tax those unlucky exporters who get trapped in the official segment of the market, and to subsidize some privileged imports. Our model has also emphasized that market controls create two kinds of wealth effects, through smuggling profits and through the provision of consumption goods at below equilibrium prices. As well as fiscal effects and wealth effects there are costs in real resources which depend upon the nature of the smuggling technology.

The model presented here has been used to analyze the simultaneous determination of the black market exchange rate and the price of consumer goods. As well as providing a characterization of the equilibrium, we have given some illustrative comparative statics predictions. The assumption of economies of scope plays some part in these results. For example, a reduced premium on smuggling cash crop products out of the country is an incentive to reduce smuggling of consumer goods into the country, for a given premium on the latter, with inflationary consequences.

Some of our model’s predictions have been tested on monthly data for 1982 and 1983, with a reduced form model of the black market exchange rate and the consumer price index in Ghana. Although a structural analysis of the whole model was not possible, the reduced form estimation performed suggests that some important relationships may actually be captured by our model. The significant influence of the CPI in Abidjan on both the price level in Ghana and the black market exchange rate illustrates, vividly, the informal integration of the regional markets. The negative impact of our indicator of the extent of rationing of consumer goods similarly underlines the possible relevance of our theory.

NOTES

1. We do not distinguish between smuggling and cross-border trade. For some discussion of the distinction and its consequences see Johnson (1987).

2. In Ghana, such a cut was indeed attempted in the form of a demonetization of the 50 cedi note.
REFERENCES


