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Housing, Land Prices, and Growth

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We consider the effects of land for housing on the growth process within an overlapping generations model. Our original interest was to enquire whether the introduction of land into a growth model might account for a "virtuous" circle in which saving-up for land (or housing) generates growth and higher land prices, generating further increases in saving, and so on. Such an account is sometimes proposed for high saving rates in East Asia, where mortgage markets are limited or absent. Our analysis does not support such a story. The user cost of land reduces the resources available for consumption of reproducible goods, so that the introduction of intrinsically valuable land into a growth model lowers the equilibrium stock of capital and raises the equilibrium interest rate. On the asset side, the presence of land causes life-cycle savings to be reallocated away from productive capital towards land. The social optimum in such a model is for land to be nationalized and provided at zero rent. Land markets, far from generating saving and growth, are inimical to capital formation.

Keywords: growth, saving, land, overlapping generations

JEL classification: D91, O11, R14

1. Introduction

This paper is concerned with the effects of housing and land on the growth process. Within an overlapping-generations model of economic growth, we ask how the existence of a fixed supply of land for housing, for which consumers must save prior to ownership, and whose price is endogenously determined by the growth process, affects the steady state growth path of the economy.

The analysis was originally inspired by anecdotal evidence on the role of housing in personal saving decisions in Korea and Taiwan. When people in their twenties and thirties were asked why they were saving a third of their incomes, a common response was that they were saving for a house or an apartment, that even a "starter" apartment cost many multiples of income, and that mortgage finance was either unavailable or severely restricted. Household surveys in Taiwan (as elsewhere) often show substantial saving rates among the elderly, contrary to simple life-cycle accounts. When asked, the elderly often give the same response as their children, that they are saving for housing, not for themselves, but for their children. Of course, such accounts raise as many questions as they answer, and are not even obviously internally consistent, at least in equilibrium. Nevertheless, consumers in the Asian tigers save large fractions of their incomes, they face high prices for housing relative to their incomes, and there are poor or non-existent mortgage markets. It is therefore worth investigating whether the presence of a limited amount of housing land can act as a catalyst in a virtuous circle of saving, growth, and rising land prices.

We investigate these issues by developing an overlapping generations growth model which can be used to compare a world with and without housing land. There is a fixed rate of population growth n, or equivalently of labor augmenting technical progress. We focus on land, rather than on housing itself because, in contrast to land, which we assume is in fixed supply, housing is a produced good. Land combines with housing goods to generate housing services, so that in a model where housing is not distinguished from other goods, land and consumption of goods are the arguments of the utility function. In the baseline model, developed in Section 2, if land exists it has no intrinsic worth. The younger generation works for wages using the capital owned by the older generation, and saves or consumes the proceeds. Their saving is used to purchase land from the older generation and to form the capital stock in next period, during which the currently young live on the income from capital and the proceeds from land sales. With these (and other) assumptions, the model will always have as a stationary solution the Golden rule with the rate of interest equal to the rate of population growth, provided that there is a rich enough financial structure. In Section 3, we allow land to be useful, yielding a flow of services to consumers. Long run stationary equilibria are only possible in this setting with rather restrictive forms of the utility function, but within this class we show that the Golden rule is not possible once land is useful. When land generates a flow of services, there still exists at least one equilibrium, but all equilibria have associated capital stocks smaller than in the Golden rule, and a real rate of interest larger than the rate of population growth. The market for land lowers consumption and welfare in the long run stationary equilibrium compared with the Golden Rule allocation which could be attained by nationalizing land and "renting" it out at no charge.

These results are very different from those for which we set out to look, and clearly provide no support for the "virtuous circle" of growth, saving, and land prices. Indeed, we show that it is possible to have a stationary equilibrium whose essential features, the capital stock and the level of output per head (or per efficiency unit), are independent of the endowment of land. There is therefore no necessary relationship between a country's endowment of land and its long run interest rate, capital stock, or output per head. Nevertheless, within the class of utility functions that we consider, increasing the taste for land (or the amount required to generate housing services) will usually decrease the long-run level of capital stock and output per head. Credit restrictions play no role in our analysis. In the case where land is useful, we show that the economy cannot be in debt with respect to the outside world, and without such debt, there is no possibility of borrowing and lending between the generations. We consider only situations in which financial markets are well-functioning, so that consumers (and firms) can borrow and lend freely at prevailing rate of interest.

Our results demonstrate that, within a simple overlapping generations model which seems well adapted to addressing these questions, the presence of a market for land tends to depress the accumulation of other productive capital. This effect is likely always to be present in economies in which the capital stock is supported by life-cycle saving. The introduction of credit restrictions on consumers, like the introduction of useful land, change the amount of saving available for the accumulation of physical capital, see Jappelli and Pagano (1994).

Credit constraints typically increase capital accumulation, while useful land decreases it, so that it possible that the high saving rates in Taiwan and Korea can be explained in terms of their credit restrictions, as Jappelli and Pagano argue, even though the presence of useful and valuable land is actually working in the opposite direction.

Our results are closely related to earlier results in the literature, particularly the paper by Drazen and Eckstein (1988). Drazen and Eckstein develop a dual economy model of economic development, in which land is used by the "backward" agricultural sector as a factor of production. They show that, in a fully competitive equilibrium with land markets, the stock of capital can be lower than when land is not traded but passed on from one generation to the next. The mechanism is the same as in the current paper; capital accumulation is a by-product of life-cycle retirement saving, and saving in the form of land crowds out capital. The same mechanism was earlier invoked by Feldstein (1977) to argue that taxation of land need not result in a decrease in its price by the capitalized value of the tax. Our own model is set up differently from that of Drazen and Eckstein. Because we are concerned with housing, land appears in the utility function, not as a factor of production. Moreover, we have a single production sector, not two. Nevertheless, the differences are more in the labeling than in the substance.

2. An Overlapping Generations Model of Saving and Growth

We start with the specification of a simple OLG economy. People live for two periods, so that there are two generations alive at any given time, and population growth (or labor augmenting technical progress) takes place at rate n. The young work for wages with capital owned by the old, which they accumulated when they were young. Capital lasts for one period and generates profits for the older generation who do not work. There is an asset called "land" whose treatment will differ between this section and the next. In this section, land does not bring any direct utility or services, but lasts for ever, and is passed from one generation to the next so that, if it has value, it is a pure bubble in the sense of Tirole (1985). In the next section, land will have some intrinsic value, appearing in the utility function through its role in providing housing services. We proceed informally, attempting to convey the essentials of the argument. Assumptions and proofs are gathered together in the Appendix.

For a person who is young in period t, the utility function to be maximized is written as

$$u(c_{1t}) + v(c_{2t+1}, h_t), \tag{1}$$

where c is consumption, $u(.)$ and $v(.)$ are the youth and old utility functions respectively, and the first suffix indicates age, and the second, time period, so that, for example, c_{2t+1} refers to the consumption of the old (2) in period $t+1$, i.e. of those who are currently young. The argument h_t is the amount of land that is combined with goods to generate second period utility. For the moment, it is a dummy argument whose presence in the utility function has no effect on the analysis. Each consumer inelastically supplies one unit of labor when young and none when old. We work in per capita terms, so that the budget constraint of each young person in period t is written as

$$c_{1t} + s_{t+1} = y_{1t} + w_t, \tag{2}$$

where s_{t+1} is overall saving, positive or negative, and comprises purchases of new capital stock, as well as the net payments to the older generation in return for receiving their land, debt, or other assets. The labeling of saving with the suffix $t + 1$ emphasizes that saving in the current period funds next period's capital stock. Because there is no uncertainty, and because markets are assumed to work perfectly, all assets yield the same rate of return r_{t+1}. The wage is w_t and y_{1t} is exogenous income ("manna") received by the young independently of work. In period 2, the same individuals will face the new constraint

$$c_{2t+1} = (1 + r_{t+1})s_{t+1} + y_{2t+1} \tag{3}$$

where y_{2t+1} is the "manna" received by the old. The presence of the unearned incomes y_{1t} and y_{2t+1}, both of which are assumed to be positive, is required for largely technical reasons (in the proofs in Lemma 1), and it will often be possible (for example for specific utility and production functions) to dispense with them. They can be justified by the assumption that there is always some possibility of receiving positive income without participating in the formal sector. Equations (2) and (3) can be combined to yield the familiar form

$$c_{1t} + \frac{c_{2t+1}}{1 + r_{t+1}} = y_{1t} + \frac{y_{2t+1}}{1 + r_{t+1}} + w_t \tag{4}$$

which is the standard two period intertemporal budget constraint that constrains the present value of outgoings to the present value of incomings.

The production technology is represented by a constant returns to scale production function

$$Y_t = F(K_t, L_t) \tag{5}$$

for capital K and labor L. The arguments of (5) are given by

$$L_t = (1 + n)^t \tag{6}$$

where the initial labor force has been normalized to unity, and

$$K_t = (1 + n)^t k_t \tag{7}$$

so that total capital is capital per head for the current generation's young workers k_t, multiplied by their total numbers. The feasibility constraint for the economy as a whole says that total consumption plus investment should sum to production plus manna. Writing this constraint per head of the younger generation alive at t, we have

$$c_{1t} + \frac{c_{2t}}{1 + n} + (1 + n)k_{t+1} = y_{1t} + \frac{y_{2t}}{1 + n} + f(k_t). \tag{8}$$

The last term on the right hand side is output per head of the young generation and $f(.)$ is the "intensive" production function for output per head in terms of capital per head.

Factors are paid their marginal products, so that the capitalists, who are the retirees, each get the marginal product of capital, so that

$$1 + r_t = f'(k_t) \tag{9}$$

while the younger generation, who are workers, receive the wage

$$w_t = f(k_t) - k_t f'(k_t) \tag{10}$$

We first look at utility maximizing choices of consumption, and solve for the prices and capital stocks that satisfy the feasibility constraint. We leave for a final stage questions of portfolio allocation and matching the demand and supply of capital and other assets, including land. For the generation born at t, (4) is the budget constraint against which they maximize utility (1); the solutions are the levels of consumption, which we write in the standard way:

$$c_{1t} = g_1\left(y_{1t} + \frac{y_{2t+1}}{1 + r_{t+1}} + w_t, (1 + r_{t+1})\right) \tag{11}$$

$$c_{2t+1} = g_2\left(y_{1t} + \frac{y_{2t+1}}{1 + r_{t+1}} + w_t, (1 + r_{t+1})\right). \tag{12}$$

In this paper, we concern ourselves only with *stationary* solutions, defined as those for which capital per head is constant, and in which the environment is constant, so that $y_{1t} = y_1$ and $y_{2t} = y_2$. If the stationary capital per head is k then we can write (11) and (12), after substitution using (9) and (10), as

$$c_1 = \xi_1(k, y_1); \qquad c_2 = \xi_2(k, y_1) \tag{13}$$

where the dependence on y_1 is carried forward for use in the next section. From (13), the feasibility restriction (8) will hold provided that the function $G(k, y_1)$ is zero, where

$$G(k, y_1) \equiv \xi_1(k, y_1) + \frac{\xi_2(k, y_1)}{1 + n} + (1 + n)k - y_1 - \frac{y_2}{1 + n} - f(k) \tag{14}$$

Figure 1 sketches one particular case for the function $G(k, y_1)$. The general shape of $G(k, y_1)$, which goes to infinity as k tends to zero, or as k tends to infinity, comes from the effect of k on the marginal productivity of capital and the associated patterns of intertemporal substitution. Because the functions $\xi_1(k, y_1)$ and $\xi_2(k, y_1)$ inherit the properties of the functions $g_1(.)$ and $g_2(.)$ in (11) and (12), as proved in Lemma 1 in the Appendix, second period consumption goes to infinity as k tends to 0, and first period consumption goes to infinity as k tends to infinity, and in both cases, $G(k, y_1)$ goes to infinity. The function must therefore have the general shape as shown, with either no or an even number of solutions (unless there is a tangency with an associated single solution.)

The Golden Rule capital stock k^* is the stationary value of capital that maximizes the resources available for consumption which, from (8), is the value of k that maximizes the difference $f(k) - (1+n)k$. It is therefore the capital stock that sets the marginal productivity of capital, or the rate of profit, equal to the rate of population growth. It turns out the model

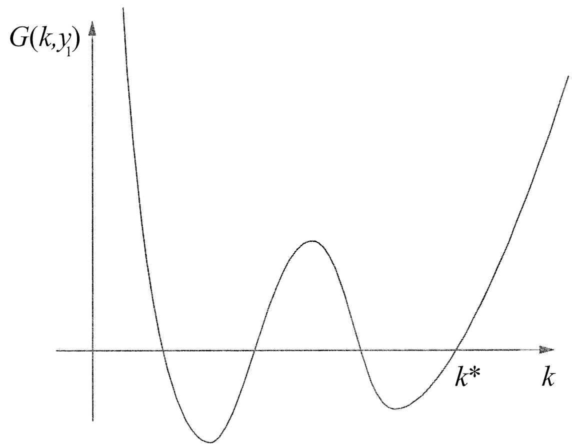

Figure 1. Stationary solutions for capital per head.

has a stationary equilibrium with this capital stock. Start from the definition; k^* is defined as the solution to

$$f'(k^*) = 1 + r = 1 + n. \tag{15}$$

At the Golden Rule value of k^*, the wage is given by

$$w^* = f(k^*) - k^* f'(k^*) = f(k^*) - (1+n)k^*. \tag{16}$$

The intertemporal budget constraint is

$$c_1 + \frac{c_2}{1+r} = y_1 + \frac{y_2}{1+r} + w, \tag{17}$$

so that, substituting n for r, and using (16) for the wage, (17) becomes

$$c_2 + \frac{c_1}{1+n} + (1+n)k^* = y_1 + \frac{y_2}{1+n} + f(k^*) \tag{18}$$

which is the feasibility constraint (8). In consequence, if we start from k^*, the demand functions (13), which by construction satisfy the budget constraint (4), will automatically satisfy the feasibility constraint, so that the function $G(k, y_1)$ is zero at k^*. The Golden rule solution is illustrated in Figure 1 as the largest solution, though this need not be the case in general.

Note that we have not so far discussed the supply and demand of assets, and there is nothing in the analysis that guarantees that the Golden rule capital stock k^* will match the amount of saving supplied by each generation. Golden rule saving, from (2) and (13), is given by

$$s^* = y_1 + w^* - \xi_1(k^*, y_1). \tag{19}$$

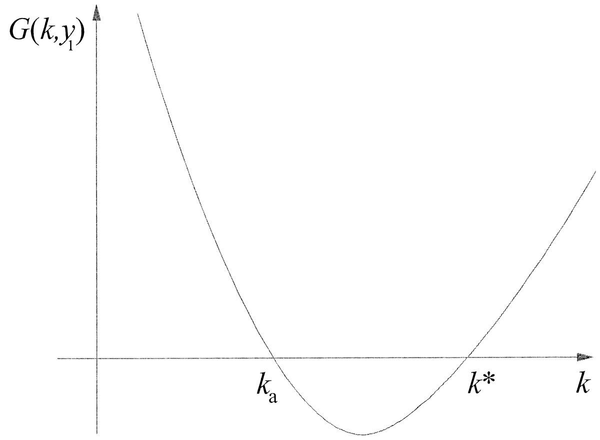

Figure 2. Golden rule and one other solution.

If $s^* > (1+n)k^*$, land, although without intrinsic value, may have a positive price, rising at the rate of interest, and the value of this "bubble" can help fill the gap between Golden rule saving and the Golden rule capital stock. If $s^* < (1+n)k^*$, the Golden rule allocation can only be supported by the presence of some extrinsic debt to the outside world that is passed down the generations. If the capital stock is the only asset in the economy, the Golden rule allocation will only come about by chance because it requires that $s^* = (1+n)k^*$.

An important case is illustrated in Figure 2. Here there are two solutions, the greater of which is the Golden Rule. At the other, k_a, where capital per head is less than at the Golden Rule, the interest rate is larger than the rate of population growth. Just as the Golden Rule is one of the stationary solutions, the shape of $G(.)$ shows that (in the absence of a tangency) there is another solution which turns out to be sustainable with the stock of capital as the only asset, i.e. without either land or external debt. Since there are no transfers between generations, this is often referred to as the "autarkic" equilibrium. To see what it looks like, write the two consumption levels in the form

$$c_1 = y_1 + w - (1+n)k; \qquad c_2 = k(1+n)(1+r) + y_2, \tag{20}$$

assuming that saving and capital are identical. Given (20), and the wage and interest rate conditions, it is straightforward to check that the feasibility constraint is satisfied. The equilibrium is a value k_a of the capital stock, such that when w and r are given by (9) and (10) with $k = k_a$, the consumer's problem

$$u[y_1 + w - k(1+n)] + v[y_2 + k(1+n)(1+r), h]. \tag{21}$$

has its maximum at k_a. As a result, an autarkic equilibrium (k_a, c_1, c_2) is a solution of the system of equations

$$u'(c_1) = f'(k_a)v'(c_2, h) \tag{22}$$

$$c_1 = y_1 + f(k_a) - k_a f'(k_a) - (1+n)k_a \tag{23}$$

$$c_2 = y_2 + (1+n)f'(k_a)k_a. \tag{24}$$

The existence of such an equilibrium is guaranteed by the shape of $G(k, y_1)$ and the fact that any solution of $G(k, y_1) = 0$ such that $k \neq k^*$ is an autarkic equilibrium, see Appendix Lemma 2.

3. Growth, Saving, and Land

Given the baseline model of Section 2, we can examine what happens if land is actually useful, yielding housing services in the utility function. We think of the land as being used for housing; the single good in the model is combined with land according to some technology that generates shelter. In the utility function (1), h_t is the amount of land that is combined with goods to generate second period utility. Individuals do not own land in the first period; we think of them as living with their parents, or in infinitely high tower blocks, which provide accommodation without land. Consumers are required to purchase their second period land in the first period, although they do not get to use it until later; the treatment of land therefore parallels the treatment of capital, which must be saved in the first period, but yields no return until the second period. However land, unlike capital, never deteriorates, and so is purchased in the first period from the older generation, who, in spite of living on the land, can thus use its sales to finance second period consumption. (Richer structures than this are clearly possible, but would require more than a two-period model. But a critical element in any story of land and housing is to ensure that, if someone is saving to buy land, or is being forced to save because of rising land prices, someone else is receiving the purchase, and can consume the capital gains.)

It is now necessary to separate land purchases from other saving in the accounting. If we denote other saving by $\tilde{s}_{t+1}$ and let p_t be the price of land, then the budget constraints in the two periods are

$$c_{1t} + p_t h_t + \tilde{s}_{t+1} = y_{1t} + w_t \tag{25}$$

for the first period, and

$$c_{2t+1} = p_{t+1}h_t + (1 + r_{t+1})\tilde{s}_{t+1} + y_{2t+1} \tag{26}$$

in the second period.

Feasibility requires the same condition on output as before, equation (8). In addition, total land is fixed, so that the land purchased by each person of the younger generation must be falling at rate n,

$$h_t = \frac{h_0}{(1+n)^t}. \tag{27}$$

If, as before, and ignoring irrelevant corner solutions, we eliminate s between the first and second period budget constraints, (25) and (26), we get a new version of the intertemporal budget constraint

$$c_{1t} + \frac{c_{2t+1}}{1 + r_{t+1}} + \left(p_t - \frac{p_{t+1}}{1 + r_{t+1}}\right) h_t = y_{1t} + \frac{y_{2t+1}}{1 + r_{t+1}} + w_t. \tag{28}$$

The term multiplying h_t in (28) is, of course, the user cost of land, which we denote π_t, so that

$$\pi_t = \left(p_t - \frac{p_{t+1}}{1 + r_{t+1}}\right). \tag{29}$$

The first-order conditions for the maximization of (1) subject to (28) are

$$u'(c_{1t}) = (1 + r_{1+t})\nu'_1(c_{2t+1}, h_t) \tag{30}$$

$$\pi_t u'(c_{1t}) = \nu'_2(c_{2t+1}, h_t). \tag{31}$$

In any stationary equilibrium of this model, we require that the value of land per head, $p_t h_t$, be constant, which, in view of (27), requires that p_t increase at rate n. In equilibrium, the user cost π_t must be strictly positive, otherwise the demand for land would be unbounded. It then follows from (29) that the rate of interest must be strictly larger than the rate of growth of the population. In consequence, the equilibrium capital stock will always be less than its Golden Rule value. In a slightly different context, this is McCallum's (1987) result that the presence of useful land rules out dynamic inefficiency, see also Tirole (1985) and Rhee (1991). Note also that, by the result proved in the Appendix, Lemma 2, non-Golden rule equilibria have saving equal to the capital stock plus purchases of land so that, in equilibrium, total net external assets (or debts) must sum to zero.

The existence of a stationary equilibrium also requires further restrictions on preferences. If the price of land p_t rises at rate n, and the real interest rate is constant, then by (29), the user cost π_t must also rise at rate n, in which case it takes the form

$$\pi_t = p_0(1 + n)^t \left(\frac{r - n}{1 + r}\right). \tag{32}$$

Since consumption per head is constant in a stationary equilibrium, (31) implies that $\nu'_2(c_{2t+1}, h_t)$ grows at rate n. This typically holds only if, for some function $\varphi(.)$,

$$\nu'_2(c_2, h) = \varphi(c_2)/h, \tag{33}$$

so that, integrating,

$$\nu(c_2, h) = \varphi(c_2) \ln h + \mu(c_2). \tag{34}$$

But, by the other first-order condition, (30), the first derivative of $\nu(c_2, h)$ is constant in stationary equilibrium, and thus independent of h, so that (34) can be specialized further to

$$\nu(c_{2t+1}, h_t) = \upsilon(c_{2t+1}) + \gamma \ln h_t, \tag{35}$$

and life-cycle preferences take the additively separable form

$$u(c_{1t}) + v(c_{2t+1}) + \gamma \ln h_t. \tag{36}$$

We can look for solutions for the model with land in the same way that we looked for solutions for the model without land. We solve the consumer's intertemporal maximization problem, maximizing (36) subject to the intertemporal budget constraint (28), and then substitute the solutions into the feasibility constraint that consumption and investment should not exceed production, together with the new feasibility constraint that the demand for land should equal its fixed supply. Because utility is additive, the solution to the maximization problem can be straightforwardly related to the corresponding solutions in section 1 when land was not intrinsically useful. Rewrite the intertemporal budget constraint (28) in equilibrium as

$$c_1 + \frac{c_2}{1+r} = y_1 - \pi_t h_t + \frac{y_2}{1+r} + w. \tag{37}$$

In equilibrium, π_t is given by (32) and grows at rate n, and h_t is decreasing at the same rate so that $\pi_t h_t$ is constant at its original value $\pi_0 h_0$; in order to find a solution, we need then only solve for the initial price of land p_0 as well as the capital stock per head k. The maximization of (36) subject to (37) gives, see (13),

$$c_1 = \xi_1(k, y_1 - \pi_0 h_0); \qquad c_2 = \xi_2(k, y_1 - \pi_0 h_0). \tag{38}$$

The two feasibility conditions are thus given by, cf. (14),

$$\begin{aligned} G(k, y_1 - \pi_0 h_0) \;\equiv\; & \xi_1(k, y_1 - \pi_0 h_0) + \frac{\xi_2(k, y_1 - \pi_0 h_0)}{1+n} \\ & + (1+n)k - y_1 - \frac{y_2}{1+n} - f(k) = 0. \end{aligned} \tag{39}$$

and for land,

$$\gamma = h_0 \pi_0 u'(c_1) \tag{40}$$

which equates the marginal utility of land to its user cost multiplied by the marginal cost of the numeraire. Using (32), (40) can be written in terms of the initial price of land p_0

$$\gamma = h_0 \pi_0 u'(c_1) = h_0 p_0 \left(\frac{r-n}{1+r}\right) u'(\xi_1(k, y_1 - \pi_0 h_0)). \tag{41}$$

A solution of the model is then a value for k and for π_0, or equivalently p_0, which solve (39) and (41) simultaneously.

The Appendix provides a proof that at least one equilibrium exists in the model with land. Figure 3 provides an illustration based on Figure 2 and shows the $G(.)$ function in (39). Note that this is only one of the two equations that defines the equilibrium—the other is (41)—but it is nevertheless informative. The higher curve is the original function which cuts the horizontal axis twice, at the autarkic and Golden Rule equilibria. Once land is

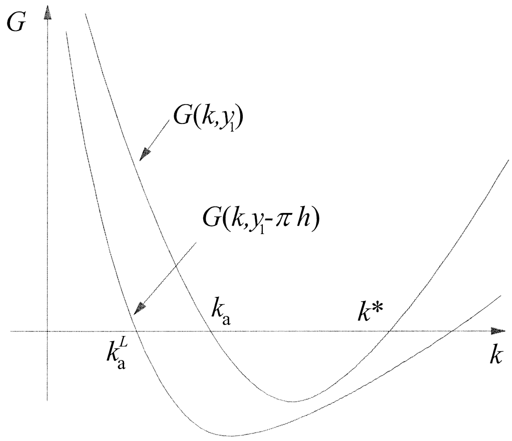

Figure 3. Golden rule and autarkic solutions with and without land.

useful, its user cost is deducted from y_1, and since income is normal for consumption in both periods, the consumption levels in both periods are lower for any given value of k, and thus so is the value of the $G(.)$ function. In consequence, if the autarkic equilibrium lies below the Golden Rule equilibrium as shown, the new $G(.)$ function will cut the horizontal axis once below the original autarkic equilibrium, at k_a^L, and once above the Golden Rule. The latter cannot be an equilibrium, because the real rate of interest is below the rate of population growth, so that user cost would be negative, and the demand for land infinite. In consequence, in the case of Figures 2 and 3, the introduction of land removes the Golden Rule equilibrium, and the new autarkic equilibrium has a lower level of capital per head than did the original autarkic equilibrium. If Figure 2 had been such that the two equilibria were ordered so that the Golden Rule equilibrium had lower capital per head than the (inefficient) autarkic equilibrium, the introduction of land would have eliminated the autarkic equilibrium, because the user cost is negative above the Golden Rule. The new, unique equilibrium would lie to the left of the original Golden Rule equilibrium, once again with lower capital per head. In both these cases, the introduction of land decreases the level of capital per head compared with the original equilibrium.

The situation is more complex when there are multiple equilibria of the sort shown in Figure 1. In that case, moving the graph down will decrease the capital stock at the first equilibrium, increase it at the second, decrease it at the third, and eliminate the fourth (Golden Rule) equilibrium altogether. At equilibria where the G curve cuts the k-axis from below, and the introduction of land increases the capital stock, we obtain the "virtuous" mechanism from land to capital stock, the search for which originally inspired our work. What separates this sort of equilibrium from the equilibria where the introduction of land reduces the capital stock, and how can we judge which is more likely in practice? The crucial issue here is the effect on consumption in equilibrium of an increase in the capital stock. Higher capital stock is associated with higher production, and therefore income, but also with a lower interest rate, with indeterminate consequences for saving, and thus for consumption. In what we might label the "standard" case, higher capital stock means more consumption, the G function cuts the k-axis from above, and the introduction of land decreases both consumption and the capital stock. In the "non-standard" case, where the lower interest rate decreases consumption by more than the increase in production from the higher capital stock, the introduction of land *increases* the capital stock. This outcome seems implausible on empirical grounds.

Given our initial concerns about the effects of land endowments on the process of economic growth, it is interesting to compare equilibria with different initial amounts of total land h_0. Inspection of the conditions that define the solution, equations (39) and (41), shows that h_0 appears only through the products $\pi_0 h_0$ or $p_0 h_0$. In consequence, any solution can be reproduced with a lower initial value of land and a proportionately higher price of land and user cost. This result is a consequence of the additive and logarithmic way that land appears in the utility function (36), a formulation that is necessary to get stationary solutions. While perhaps not very realistic, it is important because it demonstrates that different endowments of useful land need not have any fundamental effect on the nature of the equilibrium path of income and capital accumulation.

Instead of changing the quantity of land, we can analyze the effects on the equilibrium of the taste for land (or the technology of land use) by varying the parameter γ in (36). (This is perhaps of limited interest, if only because we have no obvious way of measuring γ, for example for a cross-section of countries.) When $\gamma = 0$, we have the model of section 1 where land is not useful; more generally, the preference for land increases with γ. Analyzing the effects of changes in γ requires handling both (39) and (41) simultaneously. In the Appendix, we investigate how changes in γ affect the capital stock. We derive conditions under which it is true that, at the equilibrium with the smallest capital stock, the capital stock is declining in the taste parameter γ; the derivative of the capital stock with respect to the taste for land, γ, is negative. More generally, we show that whenever the derivative of $\xi_1(k, y_1 - \pi_0 h_0)$ with respect to k is positive, the result carries through. When these conditions are met, the stronger is the taste for land, the lower is the capital stock and the level of (non-land) output her head.

4. Concluding Remarks

Our original interest in this paper was to enquire whether the introduction of land into a growth model might help account for a "virtuous" circle in which saving-up for land (or housing) helped generate growth and higher land prices, which would generate further increases in saving, and so on. But our simple overlapping generations model does not support this conclusion, except in what appear implausible circumstances. In the long run, the user cost of land reduces the resources available for consumption of the reproducible goods, all other things equal. Production, together with the associated stock of productive capital, adjusts to demand, which in turn is affected by the new level of production. The interest rate moves with the marginal productivity of capital. The new equilibrium is typically obtained with a lower stock of capital and a higher rate of interest. This is the effect identified by Feldstein (1977) and by Drazen and Eckstein (1988), and which was first explored by Allais (1948). On the asset side, the presence of land causes a portfolio reallocation away from capital towards land. The social optimum here is for land to be nationalized and not to be allocated through markets, but passed down from one generation to the next through customary rules, such as division among the heirs, or by entail, as in Drazen and Eckstein. Land markets, far from generating growth, are inimical to capital formation.

We note that the negative effect of land on capital is not the only theoretical possibility. If consumption increases by enough in response to an increase in the interest rate, it is possible for the introduction of land, or for an increase in the taste for land, to increase the capital stock, the virtuous circle effect with which we began. But without empirical support for such an apparently implausible outcome, we do not regard this theoretical outcome as providing much support for the original hypothesis.

The results in this paper are derived under a number of special assumptions, and it is important to consider their robustness. Note first that all of these results concern long-run equilibria, and it would be an interesting exercise to consider the dynamics to see whether transitional paths might be able to give a different account of the stylized facts. It should also be noted that the preferences we have used are very special, particularly in the case where there is land. Additive preferences over the two consumption levels and land are analytically convenient, but are not plausible, if only because land requires complementary goods to construct housing, which is what people care about. However, the results in Section 3 are driven by the fact that, when people care about land, they must make room in their budgets to purchase it and in their portfolios to hold it. This mechanism would carry through into more complex models, and would be supplemented by the effects of non-separability, in particular the substitution effects on the consumption profile of the shrinking supply of land per head. One might suppose that, if construction can substitute for land—ever taller buildings, for example—there would be an additional motive to save in the first period, and a higher capital stock. We have not attempted to work through such a model in detail, because it seems unlikely to account for the stylized facts with which we began.

Another issue is whether the results would change in a model where the representative agent lives for more than two periods so that, for example in a three period model, there could be saving for housing in the first period, the purchase of a house in the second period, which is sold in the last period. But the basic mechanism works as before. Consider a long run equilibrium where aggregate consumption plus investment is equal to production net of capital depreciation, and assume an increase in the taste for land at the margin. At a fixed stock of capital and constant interest rate, the only consequence of such a change would be an increase in the user cost of land supported over the life time, and a corresponding reduction in the intertemporal income that can be spent on the reproducible good. Under normal circumstances—the "standard" case—equilibrium would be restored at lower levels of production and capital stock.

The final topic that needs to be discussed is bequests. There are many different ways of modeling bequests, and we have considered only one, "Barro preferences," by which the utility of each generation depends on the utility of the next. For example, we would modify (36) to read

$$U_t = u(c_{1t}) + v(c_{2t+1}) + \gamma \ln h_t + \beta U_{t+1} \tag{48}$$

where U_t is the utility of the generation born at t and β is a discount factor less than to one. The repeated substitution of future utilities in (48) gives an infinite discounted sum of present and future subutilities so that, as usual with this form of bequest motive, we have moved from a model with an infinite number of overlapping generations to a model with essentially one consumer, albeit an infinitely lived dynasty. In the case without land, or $\gamma = 0$, this model has the general structure of a standard infinite horizon optimal growth model, which has a unique efficient equilibrium. In general, the capital stock per head will not tend to the Golden Rule level except when there is no discounting of intergenerational utility, which takes place as β tends to unity. When γ is positive, and land matters, the equilibrium will not change. Land is valued at the discounted value of its service flow, but the dynamics of consumption and the capital stock are the same as in the model without land. In this sense, and provided bequests are modeled in Barro's form, the results in this paper are not robust to the introduction of bequests. This should not be surprising. In the single agent dynasty of Barro's model, there is no room for the divergence between social and private optima that is at the heart of our account of land and growth, indeed this is exactly the argument used by Calvo, Kotlikoff and Rodriguez (1979) to eliminate Feldstein's (1977) result on the shifting of a tax on land. The results in this paper crucially depend on a life-cycle story of saving.

Appendix

5.1. Technical Assumptions on Utility and Production in the Model without Land

The subutility functions $u(.)$ and $\nu(.)$ in (1) are increasing, concave, and continuously differentiable on $\mathbb{R}_+$, and satisfy

$$\lim_{c_1 \to 0} u'(c_1) = +\infty \qquad \lim_{c_2 \to 0} \nu'(c_2) = +\infty.$$

We require $k_t \geq 0$, $y_{1t} > 0$, $y_{2t+1} > 0$, $p_t \geq 0$, $w_t \geq 0$, $r_t \geq -1$. For the production function,

$$f(0) = 0, \quad \lim_{k \to \infty} f(k) = \infty, \quad \lim_{k \to 0} f'(k) = +\infty, \quad \lim_{k \to \infty} f'(k) = 0.$$

5.2. Lemma 1

Let

$$W(k) = f(k) - kf'(k), \qquad R(k) = f'(k)$$

Then

$$\lim_{k \to \infty} g_1\left(y_1 + \frac{y_2}{R(k)} + W(k), R(k)\right) = \infty$$

$$\lim_{k \to 0} g_2\left(y_1 + \frac{y_2}{R(k)} + W(k), R(k)\right) = \infty$$

Proof: The consumer demands are determined by the first-order condition associated with the income constraint:

$$u'(c_1) = R(k)v'(c_2)$$
$$c_1 + \frac{c_2}{R(k)} = y_1 + \frac{y_2}{R(k)} + W(k).$$

When k goes to infinity, $R(k)$ goes to zero. If c_1 were to stay finite (implying that $u'(c_1) \geq \underline{u}$ for some $\underline{u} > 0$), the budget constraint would imply that $c_2 \geq y_2 - R(k)c_1$, which tends to y_2. Hence the limit of c_2 is greater than $y_2 > 0$, so that the limit of $v'(c_2)$ would be smaller than $v'(y_2)$ which is finite. But this makes the limit of the right hand side of the first-order condition zero, which is a contradiction.

Similarly, when k goes to zero, $R(k)$ goes to infinity. If c_2 were to stay finite (implying that $v'(c_2) \geq \underline{v}$ for some $\underline{v} > 0$), the budget constraint would imply that $c_1 \geq y_1 > 0$, so that $u'(c_1)$ would be smaller than $u'(y_1)$, again contradicting the first-order condition. ■

5.3. *Lemma 2*

Any solution in k of the equation $G(k, y_1) = 0$ has either $f'(k) = 1+n$ or $y_1 + W(k) - c_1 = (1 + n)k$, i.e. if it is not the Golden rule, it is an autarkic equilibrium.

Proof: By definition:

$$G(k, y_1) = c_1 + \frac{c_2}{1+n} + (1+n)k - y_1 - \frac{y_2}{1+n} - f(k),$$

where (c_1, c_2) maximize the utility function under the intertemporal budget constraint:

$$c_1 + \frac{c_2}{f'(k)} - y_1 - \frac{y_2}{f'(k)} - W(k) = 0.$$

Now subtracting the budget constraint multiplied by $f'(k)/(1+n)$ from G implies that any solution of $G(k, y_1) = 0$ satisfies:

$$\left(1 - \frac{f'(k)}{1+n}\right)(c_1 - y_1) + (1+n)k - f(k) + \frac{f'(k)}{1+n}W(k) = 0.$$

Since $f(k) = W(k) + kf'(k)$, this equality can be rewritten as:

$$\left(1 - \frac{f'(k)}{1+n}\right)(c_1 - y_1 - W(k) + (1+n)k) = 0,$$

which yields the desired result. ■

5.4. *Theorem 1*

There exists an equilibrium of the economy with land, with a smaller capital stock than the Golden Rule.

Proof: We look for a stock of capital k and a user cost of land π at date 0 (which will give the price of land at date 0), which correspond to an equilibrium. The proof builds a mapping whose fixed points are equilibria.

We define demand functions for good, given some couple (k, π) as the solutions of:

$$\begin{cases} \max_{c_1,c_2} u(c_1) + v(c_2) \\ c_1 + \dfrac{c_2}{R(k)} = \max\left[0, y_1 + \dfrac{y_2}{R(k)} + W(k) - \pi h_0\right] \\ c_1 \leq \bar{c} \quad c_2 \leq \bar{c}, \end{cases}$$

where $\bar{c}$ is a large enough number so that no equilibrium can occur at $c = \bar{c}$, e.g.

$$\bar{c} = y_1 + \frac{y_2}{1+n} + \max_k[f(k) - (1+n)k] + 1.$$

We let $c_1 = \xi_1(k, y_1 - \pi h_0)$, $c_2 = \xi_2(k, y_1 - \pi h_0)$ be the solutions of the program (note that they are well defined for all $k \geq 0$, $\pi \geq 0$, including $k = 0$, where $R(k) = +\infty$, i.e. $\xi_1(0, y_1 - \pi h_0) = \max[0, y_1 - \pi h_0]$, $\xi_2(0, y_1 - \pi h_0) = \bar{c}$). For $\pi = 0$, the demand functions are those of the economy without land. Moreover, from separability of the utility function, the demands are increasing in income so that both $\xi_1(k, y_1 - \pi h_0)$ and $\xi_2(k, y_1 - \pi h_0)$ are decreasing in π. The excess demand for good is defined as:

$$G(k, \pi) = \xi_1(k, y_1 - \pi h_0) + \frac{\xi_2(k, y_1 - \pi h_0)}{1+n} + (1+n)k - y_1 - \frac{y_2}{1+n} - f(k).$$

The demand for land is determined by the first order condition:

$$\frac{\gamma}{h} = \pi u'(c_1),$$

so that the user cost of land that equalizes demand to availability in period 0 is:

$$\Pi(k, \pi) = \frac{\gamma}{h_0} \frac{1}{u'[\xi_1(k, y_1 - \pi h_0)]}.$$

We now define the mapping that gives the equilibria. Let:

$$\bar{\pi} = \left[y_1 + W(k^*) + \frac{y_2}{1+n}\right] / h_0 + 1$$

be an upper bound for the user cost of land, where k^* is the Golden Rule capital stock. We build a convex valued upper hemi continuous correspondence from $[0, k^*] \times [0, \bar{\pi}]$ into itself by associating with any (k, π) the set $K(k, \pi) \times \min[\Pi(k, \pi), \bar{\pi}]$, where

$$\begin{cases} K(k, \pi) = 0 & \text{if} \quad G(k, \pi) < 0 \\ K(k, \pi) = [0, k^*] & \text{if} \quad G(k, \pi) = 0 \\ K(k, \pi) = k^* & \text{if} \quad G(k, \pi) > 0. \end{cases}$$

By Kakutani's fixed point theorem, the correspondence has a fixed point (k, π). We show that it is an equilibrium and that $k \neq k^*$.

1. k is different from zero, since $\xi_2(0, y_1 - \pi h_0) = \bar{c}$ implies $G(0, \pi) > 0$.
2. π is such that the uncommitted income of the consumer program is positive ($y_1 + \frac{y_2}{R(k)} + W(k) - \pi h_0 > 0$). Suppose not: then $\xi_1(k, y_1 - \pi h_0) = 0$, which implies $\Pi(k, \pi) = 0$. Since $\pi = \Pi(k, \pi)$, uncommitted income would be strictly positive, a contradiction.
3. Finally, k is different from k^*. If $k = k^*$, using the budget constraint as above (recall that $W(k^*) = f(k^*) - k^* f'(k^*) = f(k^*) - (1+n)k^*$), we would have:

$$G(k^*, \pi) = -\pi h_0 < 0,$$

and therefore, by the construction of K, $k^* = K(k^*, \pi) = 0$, a contradiction.

Consequently, since k belongs to the interior of $[0, k^*]$, from the definition of K we have:

$$G(k, \pi) = 0,$$

and, furthermore, since π is smaller than $\bar{\pi}$ (otherwise the uncommitted income would equal zero):

$$\pi = \frac{\gamma}{h} \frac{1}{u'(c_1)},$$

and we have found an equilibrium with $k < k^*$. ∎

To study how the stock of capital depends on γ at the equilibrium with the smallest capital stock, we differentiate the equilibrium equations.

Consider the consumer program:

$$\begin{cases} \max_{c_1, c_2} u(c_1) + v(c_2) \\ c_1 + \dfrac{c_2}{R(k)} = y_1 + \dfrac{y_2}{R(k)} + W(k) - \pi h_0 \equiv T(k, \pi). \end{cases}$$

From the definitions of W and R:

$$W'(k) = -kf''(k) > 0,$$
$$R'(k) = f''(k) < 0,$$

so that $T'_k > 0$ and $T'_\pi < 0$.

The derivatives of $\xi_1(k, y_1 - \pi h_0)$ can be computed using the familiar bordered determinant:

$$\begin{pmatrix} u'' & 0 & -1 \\ 0 & v'' & -\frac{1}{R(k)} \\ -1 & -\frac{1}{R(k)} & 0 \end{pmatrix} \begin{pmatrix} dc_1 \\ dc_2 \\ d\lambda \end{pmatrix} = \begin{pmatrix} 0 \\ -\lambda \frac{R'}{R^2} dk \\ -c_2 \frac{R'}{R^2} dk - T'_k dk - T'_\pi d\pi \end{pmatrix},$$

where λ is the marginal utility of income. The determinant Δ of the matrix is positive, and:

$$\begin{pmatrix} dc_1 \\ dc_2 \\ d\lambda \end{pmatrix} = \frac{1}{\Delta} \begin{pmatrix} -\frac{1}{R^2} & \frac{1}{R} & v'' \\ \frac{1}{R} & -1 & \frac{u''}{R} \\ v'' & \frac{u''}{R} & u''v'' \end{pmatrix} \begin{pmatrix} 0 \\ -\lambda \frac{R'}{R^2} dk \\ -c_2 \frac{R'}{R^2} dk - T'_k dk - T'_\pi d\pi \end{pmatrix},$$

which gives:

$$\xi'_{1\pi} = \frac{1}{\Delta}(-v'' T'_\pi) < 0,$$

(the familiar income effect) and

$$\xi'_{1k} = \frac{1}{\Delta}\left[-\frac{\lambda R'}{R^3} - v'' c_2 \frac{R'}{R^2} - v'' T'_k \right].$$

The sign of ξ'_{1k} is a priori not determined. There are two positive terms on the right hand side, the first one corresponding to the substitution effect (an increase in k lowers the interest rate, and makes future consumption more expensive relative to consumption today) and the last one corresponding to the wealth effect (an increase in k increases life cycle income). On the other hand, the middle term runs in the opposite direction: this is the income effect on second period consumption, which says that to maintain consumption in the second period when k increases, one needs to save more, and therefore to reduce current expenditure. The overall sign is positive when the substitution effect is large (working on the first two terms, this happens for sure when the intertemporal elasticity of substitution $-v'/c_2 v''$ is greater than 1), or when the income effect is small (combining the last two terms, this is typically the case when the user cost of land is small).

Proceeding under the assumption that ξ'_{1k} is positive, the comparative statics of the equilibrium come from differentiating the two equations;

$$\begin{cases} G(k, \pi) = 0 \\ \pi u'(\xi_1(k, y_1 - \pi h_0)) = \frac{\gamma}{h_0}. \end{cases}$$

The second equation yields:

$$u' \frac{\partial \pi}{\partial \gamma} + u'' \left[\xi'_{1k} \frac{\partial k}{\partial \gamma} + \xi'_{1\pi} \frac{\partial \pi}{\partial \gamma} \right] = \frac{1}{h_0},$$

or:

$$[u' + u'' \xi'_{1\pi}] \frac{\partial \pi}{\partial \gamma} + u'' \xi'_{1k} \frac{\partial k}{\partial \gamma} = \frac{1}{h_0}. \tag{A1}$$

The coefficient of $\partial \pi / \partial \gamma$ is positive, that of $\partial k / \partial \gamma$ is negative. The first equation gives:

$$G'_k \frac{\partial k}{\partial \gamma} + G'_\pi \frac{\partial \pi}{\partial \gamma} = 0.$$

Now G'_π is negative, from the income effects. At the equilibrium with the lowest capital stock, G'_k is also negative (since G tends to infinity when k goes to zero). It follows that $\partial\pi/\partial\gamma$ and $\partial k/\partial\gamma$ are of opposite signs, and by (A1), we must have:

$$\frac{\partial k}{\partial \gamma} < 0 \qquad \frac{\partial \pi}{\partial \gamma} > 0.$$

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