EXCHANGE RATE BEHAVIOR WITH
CURRENCY INCONVERTIBILITY

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Received February 1981. revised version received July 1981

The paper analyzes exchange rate dynamics in a world where the domestic currency is inconvertible into foreign currency and there is a 'black market' for foreign exchange. The official exchange rate is thus set by policy but the black market rate is determined by portfolio balance. The stock of domestic money changes with the reported current account and the stock of foreign assets of the private sector changes with smuggling and other unreported current account transactions. The effects of capital and current account disturbances as well as the effects of changes in the level and the rate of crawl of the official rate on the black market premium and on the currency substitution ratio are analyzed in variants of the basic model.

1. Introduction

In the Articles of Agreement of the International Monetary Fund, convertibility means absence of 'restrictions on the making of payments and transfers for current international transactions' (Art. VIII, Section 2), and, to date, only 50 of the 140 member countries have accepted to refrain from imposing such restrictions while convertibility for capital account transactions is unrestricted only in 33 member countries.1 Nevertheless, the development of private international financial intermediation has provided channels for individuals and organizations to build up foreign exchange balances, so that they can diversify their portfolios across assets denominated in different currencies. In response, many governments have attempted to recapture foreign exchange by offering special advantages to certain types of transactions, namely the ones where evasion of exchange controls would be easier, like tourist services and migrants' remittances. These incentives notwithstanding, 'black' markets for foreign

*Earlier versions of this paper were presented in seminars at Princeton, PUC/RJ (Brazil) and Minnesota. Comments from the participants, the members of my Ph.D. committee and the referee are gratefully acknowledged, as well as financial assistance from Michael G. Porter in the Fall of 1978, when the first draft was written. Errors are my own.

1Acceptance of the Article VIII obligations, furthermore does not imply that the country cannot directly restrict trade since payments restrictions have to be lifted only for authorized current account transactions. See Annual Report, table I.1 and Annual Reports on Exchange Restrictions, analytical appendix, for the list of country practices.
exchange have developed in many countries with inconvertible currencies to the point that the relative price between domestic and foreign money that is determined in these markets may affect private capital flows to a greater degree than the official exchange rate, with obvious repercussions on balance of payments adjustment.

Awareness of the importance of illegal transactions in international trade and payments is not new: it has been credited to Beccaria (1764–65). In general, one should expect trade restrictions to encourage both smuggling and factor movements. For example, illegal imports require foreign exchange so we can expect a black market premium higher than the tariff to discourage them. Nevertheless, factor movements are ruled out in recent analyses of the relative welfare effects of smuggling and tariffs. Similarly, the monumental work on trade and exchange liberalization directed by Bhagwati (1978) and Krueger (1978) views black market exchange rates through the ‘popular’ Bickerdike–Robinson–Metzler (BRM) model, according to which the exchange rate is determined by flow supply of and demand for foreign exchange. Recently, however, black market rates have been used in estimating monetarist models of exchange rate determination.

This paper applies the portfolio approach to exchange rate determination to the black market exchange rate, as if there were a dual foreign exchange market, where current account transactions go through the official market and capital account transactions go through the black market. Unlike the models in Flood (1978) and Cumby (1979), however, the two markets are linked by smuggling, which makes up the unreported current account.

The paper begins with a brief critique of monetary models of the black market rate (section 2). In section 3 a Kouri (1982) type of dynamic partial equilibrium model of the two markets is presented, under an official crawling peg regime. Section 4 contains a general equilibrium model of the determination of the black market rate, given the rate of crawl of the official rate. The major findings are outlined in a brief conclusion.

2. A monetary approach to currency inconvertibility

According to the monetary approach, there is no portfolio diversification and no trade in assets because domestic and foreign money are perfect substitutes. Under a fixed exchange rate and given demand for real balances, domestic

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2 See Mundell (1968; ch. 6) and Girton and Roper (1981).
3 See the contributions in Bhagwati (1974). Beccaria is quoted and discussed by Bhagwati and Hansen in chapter 1, footnote 1. The positive relation between the amount of smuggling and the tariff rate is observed in the case of Indonesia in ch. 13 by Cooper. Chapters 5 and 6 by Winston and Bhagwati, respectively, deal with balance of payments effects. This literature is related to the partial equilibrium graphics of black markets in Sheikh (1976).
4 A model of the dual exchange market in this spirit is in Macedo (1978).
5 See Macedo (1979a) for a critical survey of the black market literature.
inflation is given by foreign inflation and the balance of payments deficit equals the real increase in domestic credit. Similarly, if the exchange rate floats, domestic credit, now the only factor of monetary expansion, determines prices, which, with perfect goods arbitrage, determine the exchange rate. Blejer (1978) models exchange rate behavior under currency inconvertibility by grafting a flow black market for foreign exchange and a non-traded good into a monetary model of the balance of payments where the official rate of crawl is set at a fraction of the inflation differential. As a consequence, the rate of change of the black market rate is equated to a fraction of domestic credit creation at foreign prices. Using the balance sheet of the central bank, though, the real black market rate, \( r \), can be expressed as a function of the given official real rate, \( \bar{r} \), and the change in the foreign monetary base. Using 'hats' for proportional changes and 'dots' for time derivatives,

\[
\dot{r} = [c(1 - \tau) + \tau]\dot{\bar{r}} - (1 - c)(1 - \tau)\dot{\bar{r}}/M, 
\]

where:

\( R \) = central bank reserves in domestic currency;
\( M = C + R \), the money stock, \( C \) being domestic credit;
\( c \) = the relative elasticity of supply of foreign exchange to the black market; and
\( \tau \) = a function of the weight of traded goods in the domestic price level.

Clearly, if all goods are traded and \( \tau = 1 \), the black market premium is constant. The same happens when the flow supply of foreign exchange does not respond to the premium and \( c = 1 \). Also, the role of the given official real rate in (1), even when \( c = 0 \), raises doubts on the appropriateness of making black market flow demand a function of the real black market rate. More importantly, there is no reason for the black market rate to vary inversely with central bank reserves when there are no capital movements. When there are capital movements, on the other hand, the rate of change of the black market rate could only be a function of the current account (in foreign currency) in the presence of portfolio diversification, as shown in the next section. This inconsistency is apparent when the postulated reaction function of the authorities is such that it fixes \( \bar{r} \), because then \( r \), which becomes the black market premium, only depends on reserves. Yet this is precisely the implication of Blejer's empirical testing of the model, if the solution is interpreted over the two years needed for an ex ante excess supply of money to be fully transmitted to the black market rate. Then, the premium is fixed, the model collapses into a unified float with no change in reserves, the exogenous money stock determines prices, these determine the exchange rate, and the black market is irrelevant.

The log of the flow supply of foreign exchange is a linear function of the log of the black market premium with coefficient \( C_1 \). The log of the flow demand is a linear function of the level of the real black market exchange rate with coefficient \(-C_2\). We define \( c = C_1/C_1 + C_2 \).
3. Exchange rates and the current accounts under inconvertibility

The theory of exchange rate determination which regards the decisions of domestic and foreign residents about the composition of their portfolios as the primary determinants of exchange rates in the short run, given expectations about the fundamental determinants of the long-run equilibrium value of the exchange rate, seems a more promising framework for the analysis of black markets for foreign exchange than the monetarist construction outlined in the previous section.

While this framework stemming from the Tobin approach to monetary theory, usually includes several domestic and foreign assets — as in Branson (1977) or Tobin and Macedo (1980) — its essential features can be captured by models where domestic residents only hold non-interest-bearing domestic and foreign money and foreigners do not hold domestic money. These models of ‘currency substitution’, as they are sometimes called after Girton and Roper (1981), imply that the current account determines the rate of change of the exchange rate. The level of the exchange rate, in turn, determines the current account, like in the traditional ‘elasticities’ approach to the foreign exchange market. When there is a black market, however, the BRM condition that a weighted sum of the demand and supply elasticities of imports and exports be positive is not sufficient to determine the long-run equilibrium value of the two rates. In fact, an increase in the black market rate, given the official exchange rate, is likely to increase export supply as well as the share of exports channeled through the black market via under invoicing or smuggling, thus increasing flow supply of foreign exchange to the black market. Conversely, import demand is likely to decrease, as well as the share of imports channeled through the black market through over invoicing or smuggling, which will in turn decrease flow demand for foreign exchange in the black market. We therefore write the reported and unreported current accounts as follows:

\[ B_r = \alpha(e)\bar{X}(\bar{e}, e) \bar{P}_X - \beta(e)\bar{M}(\bar{e}, e) \bar{P}_M. \]  

\[ B_u = (1 - \alpha)\bar{X} \bar{P}_X - (1 - \beta)\bar{M} \bar{P}_M. \]

where \( \bar{e}(\bar{e}) \) is the official (black market) exchange rate, \( \bar{X}(\bar{M}) \) is the quantity of exports (imports), \( \bar{P}_X(\bar{P}_M) \) is the foreign currency price of exports (imports), and \( \alpha(\beta) \) is the share of the value of exports (imports) going through the official market.

When there are no authorized private capital account transactions, we can equate the reported current account with the change in central bank reserves \( F^G \). Similarly, when the central bank stays out of the black market, the unreported

\(^7\text{See Kouri (1982), Rodriguez (1980) and Dornbusch (1980; ch. 13). A similar model with inflation and two-way currency substitution is developed and tested for Portugal using quarterly data in Macedo (1979b) while Dornbusch et al. (1981) have a model featuring the Brazilian black market for dollars in the same spirit.} \)
current account equals the change in the stock of foreign assets of the private sector \( F \) so that we can rewrite (2) and (3) compactly as

\[
B_A(e, \bar{\varepsilon}) = F^G, \quad (2')
\]

\[
B_A(e, \bar{\varepsilon}) = \dot{F}. \quad (3')
\]

Now, portfolio balance implies that at each instant the domestic currency value of the stock of foreign assets is equal to a desired proportion of private wealth \( W \). Neglecting wealth effects and valuing foreign assets at the black market rate, we can write portfolio balances as

\[
eF = A. \quad (4)
\]

Since the official rate is not freely floating, but rather determined by the authorities, we need their reaction function to close the model. One polar case is the pegged official rate, or an exogenous rate of crawl, accompanied by sterilization of reserve changes, so that the domestic money stock is given and the partial equilibrium analysis of Kouri (1982) could be applied to the black market. Here we assume instead that the monetary authorities keep reserves in domestic currency constant so that

\[
\varepsilon F^G = R. \quad (5)
\]

According to (5), the foreign currency value of reserves is an instrument rather than a target and it is presumed that \( F^G \) is ‘large enough’.

Then, differentiating (5) and substituting from (2'), we express the official rate of crawl as

\[
\dot{\varepsilon} = - \frac{B_A(e, \bar{\varepsilon})}{F^G}, \quad (6)
\]

so that the official rate appreciates when the current account is in surplus and depreciates when it is in deficit as the result of central bank policy. In the black market, though, the same occurs as the result of participants’ behavior. In fact, differentiating (4), given \( A \) and substituting from (3'), we get Kouri’s ‘acceleration’ result:

\[
\dot{\varepsilon} = - \frac{B_A(e, \bar{\varepsilon})}{F}. \quad (7)
\]

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8The problem of the choice of indicators for a crawling peg is discussed in Branson and Macedo (1980). Also, as shown by Krugman (1979) in a similar model, under perfect foresight there may not exist an initial reserve stock that is ‘large enough’. See footnote 16 below.

9The more general case where the size of the market differs and the full derivation of the premium elasticities is in Macedo (1980, appendix I).
The system described by (6) and (7) has a linear approximation around the steady-state values $\bar{e}^*$ and $\bar{z}^*$ such that $B_r = B_u = 0$ and $e^* = \bar{e}^*$ by choice of units. If the steady-state level of imports through each market as a proportion of the respective stock is the same (and set to unity by choice of units), we can write the system as

$$
\begin{bmatrix}
\dot{e} \\
\dot{e}
\end{bmatrix} =
\begin{bmatrix}
-\mu_r - z & \mu_r \\
\mu_u - z & -\mu_u
\end{bmatrix}
\begin{bmatrix}
\bar{e} - \bar{e}^* \\
\bar{e} - \bar{e}^*
\end{bmatrix},
$$

where $z > 0$ is the usual BRM condition for a devaluation of the official rate to improve the reported current account; $\mu_r = \mu_x + \mu_M + \bar{Z}$ is the sum of the premium elasticities of the shares of the official market in exports and imports (respectively $\mu_x = -\hat{z}/\hat{p}$ and $\mu_M = \hat{\beta}/\hat{p}$) and the premium elasticities of export supply and import demand weighted by the BRM elasticities, $\bar{Z}$, and $\mu_u = [\alpha/(1 - \alpha)]\mu_x + [\beta/(1 - \beta)]\mu_M - \bar{Z}$ is the corresponding expression for the unreported current account.

The eigenvalues of the Jacobian matrix in (8) are $-\lambda_1 = z$ and $-\lambda_2 = \mu_x/1 - \alpha + \mu_M/1 - \beta$. Thus the system is stable provided that the BRM condition holds and that the weighted sum of the share elasticities is positive. Notice that because the ratio of imports to asset stocks is the same in both markets, the direct effect of the premium on supply and demand cancels, even though a large enough positive value of $\bar{Z}$ could make $\mu_u < 0$ so that the unreported current account would improve with an official devaluation and deteriorate with an increase in the black market rate and conversely for the large enough negative value of $\bar{Z}$. In any event, stability requires that the cross-effects be smaller (in absolute value) than the ‘own’ effects. In other words, an official devaluation has to improve the reported current account by an amount greater than that by which the same devaluation in the black market rate would deteriorate (or improve) the reported current account.

The nature of the interaction between the two foreign exchange markets when wealth effects are neglected is apparent in fig. 1, where negative cross effects and an initial situation of long-run equilibrium are assumed. In the fourth quadrant we have temporary equilibrium at $e_0$ with the private stock of foreign assets at $F_0$. In the second quadrant we depict the reaction function of the monetary authorities: given reserves in foreign currency at $F_0^r$, the official rate is $\hat{e}_0$. This implies a premium given by a ray through the origin in the first quadrant. The ray has slope $\hat{e}_0/e_0 = 1/p_0$ at the point of long-run equilibrium, where the two upward sloping loci where $B_r = 0$ and $B_u = 0$ intersect. Below the $B_r = 0$ locus, the reported current account is in surplus and above it is in deficit. Also, the slope of the $B_u = 0$ locus, which has the ‘own’ effect on the numerator, will be larger than the slope of the $B_r = 0$ locus, so that when the official rate is devalued, there is an unreported

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deficit and a decrease in the stock of foreign assets held by the private sector, which will push up the black market rate (horizontal arrows). The vertical arrows, on the other hand, imply that a decrease in the stock of foreign assets of the central bank motivated by an increase in the black market rate leads the authorities to devalue the official rate. Conversely, if the slope of the $B_r$ schedule were larger than the slope of the $B_u$ schedule, the unreported current account would be in deficit when the official rate is depreciating, which would increase the unreported deficit, whereas the reported current account surplus would also increase, so that the two rates would depreciate without limit. Note also that if $\mu < 0$ — which, ignoring $\tilde{Z}$, can happen with $\mu > 0$ as long as the ‘perverse’ share elasticity is offset by the larger initial share of the official market in the good with the ‘normal’ share elasticity — then the $B_r = 0$ locus will slope downward, while if $z > \mu > 0$, it is the $B_u = 0$ locus which slopes downward. If both loci were downward sloping, the $B_u = 0$ locus would still have to be steeper. Coming back to fig. 1, we can depict in the third quadrant the sum of the given domestic currency value of $F^G$ and $F$ by choosing units so that $\tilde{e}_0$ and $e_0$ are unity: the downward
sloping line \( W = R + A \) represents national wealth when there is no domestic credit, as in a stylized gold standard regime.\(^{11}\)

Consider now the effects of an increase in the demand for foreign assets, \( A \) goes up in (4). Since at a point in time \( F \) is fixed, the upward shift of the portfolio balance locus implies a proportional increase in \( a \) to \( a_1 \) (we are ignoring the wealth effect of the increase in \( e \)). This depreciation of \( e \) puts the reported current account in deficit and the unreported current account in surplus. As a consequence, \( F \) gradually increases leading to an appreciation of \( e \) and \( F_0 \) gradually decreases leading to a temporary depreciation of \( \tilde{e} \) to \( \tilde{e}_1 \), followed by an appreciation back to \( \tilde{e}_0 \). When \( F \) reaches \( F_1 \), in effect, the two rates are back at their original level, and so are the reserves of the central bank, but national wealth has increased. Therefore, the steady-state effects of capital account disturbances are limited to asset stocks and do not change the equilibrium values of the exchange rates. Fig. 2 traces the effects of an increase in reported trade, say an exogenous increase in \( \tau \) in (2) and (3). The reported current account improves and the unreported current account deteriorates from \( A_0 \) to \( A_1 \). As a consequence the official exchange rate appreciates and the black market rate depreciates so that the premium increases to \( p_1 \), central bank reserves increase to \( F_1 \) and the private stock of foreign assets decreases to \( F_1 \). While initially the premium remained at \( p_0 \), over time there was a full adjustment to the current account disturbance. Since \( R \) and \( A \) did not change, however, the wealth line does not change either, so that current account disturbances have no effect on the domestic currency value of asset stocks. Therefore, while the initial stock of foreign assets determines the black market rate at each instant, trade in goods completely determines the steady-state values of \( e \) and \( \tilde{e} \), in this case the combination of the two rates that keeps official and unofficial current accounts in balance.\(^{12}\) If expectations were endogenous, or wealth effects were not neglected, the strong dichotomy would not hold, as will be shown in the next section.

To sum up this partial equilibrium analysis, as well as the corresponding gold standard general equilibrium model, we found that, unlike the unified regime and when the official rate of crawl is determined by the reported current account, the BNM elasticities condition is not sufficient for stability.' In addition, we need that the sum of the premium elasticities of export and import shares weighted by their black market share be positive. This is, however, compatible with a negative elasticity in the market with the larger black market share. For example, if most exports initially go through the black market and an increase in the premium perversely decreases the black market share of exports, the system

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\(^{11}\)The model where (3) gives the rate of change of domestic money valued at the official exchange rate is analyzed in Macedo (1980).

\(^{12}\)How long the long run depends on the elasticities and on the relative magnitude of the current and capital accounts which was taken to be the same for the two markets. See numerical examples in Kouri (1982).
Fig. 2. Effect of an increase in reported trade.

will nevertheless be stable if most imports go through the official market and the increase in the premium increases this share further. Assuming stability and negative own effects, we illustrated the dynamic adjustment to a capital account and a current account disturbance, retrieving the usual result in models of this type, that only the latter have steady-state effects on the exchange rate while only the former have steady-state effects on asset stocks.

Now that partial equilibrium analysis has illuminated some important features of currency inconvertibility, general equilibrium effects can be introduced.

4. Currency inconvertibility, money and relative prices

Instead of an import and an export good channeled through the two markets in response to the black market premium, we now fix the terms of trade and interpret the premium as the price of the smuggled traded good in terms of the
good traded through the official market.\(^{13}\) We also incorporate the monetary effects of reported current account imbalances and do not require the rate of crawl to follow the ‘acceleration hypothesis’. As a consequence of the introduction of wealth effects in consumption, as in Kouri (1975) and Calvo and Rodriguez (1977), furthermore, we do not require that the effect of the official rate be stronger on the reported current account and conversely. Nevertheless, stability will turn out to depend on the effect of relative prices on the unreported current account being stronger than the wealth effect.

Assuming full employment of the factors of production, we can express supply as a function of relative prices. Consumption, in turn, is a function of prices, income and wealth, so that the current accounts are the excess supply of the two goods and the equation equivalent to (2) and (3) in the previous section can be written as

\[
B_s = X_r(P, \bar{P}) - C_r(P, \bar{P}, X_u P + X_r \bar{P}, W),
\]

\[
B_u = X_u(P, \bar{P}) - C_u(P, \bar{P}, X_u P + X_r \bar{P}, W),
\]

where \(X_r, C_r, P, (X_u, C_u, P)\) is the supply, demand and price of the officially traded (smuggled) good respectively.

Consumption is homogenous of degree zero in prices and income and because of the full employment assumption the income effect cancels and the total demand effect has the same sign as the compensated effect.\(^{14}\) Using the official traded good as the numeraire and setting foreign currency prices at unity, we have:

\[
B_s(p, \bar{W}) = F^o,
\]

\[
B_u(p, \bar{W}) = \tilde{F},
\]

where

\[
\bar{W} = W/\bar{e}.
\]

Wealth is composed of two assets, domestic money \(M\) and foreign money \(F\). Portfolios are always balanced and the asset demand functions depend on the relative returns, captured by the (exogenous) expected change in the black market exchange rate, \(\pi\), on goods prices and on wealth:

\[
M = M(\pi, P, \bar{P}, W),
\]

\(^{13}\)A similar model with non-traded goods is derived in Macedo (1980, appendix II).

\(^{14}\)This is shown in Macedo (1980).
\[ eF = F(\pi, P, \bar{P}, W), \]  
\[ W = M + eF. \]  
(12)  
(13)

Asset demands are homogeneous of degree one in prices and wealth. Defining them in terms of the official rate and eliminating (11) by the wealth constraint in (13), we write portfolio balance as

\[ pF = f(\pi, p, \bar{W}). \]  
(14)

Taking the money multiplier as given, the change in the money stock valued at the official rate can be written as

\[ \dot{M} = \tilde{F}_g + \delta \tilde{M}, \]  
(15)

where \( \tilde{M} = M/\tilde{\pi} \) and \( \delta = \tilde{C}/M - \tilde{\alpha} \) is the difference between domestic credit creation as a proportion of the money stock and the exogenous rate of crawl.

The model therefore consists of the portfolio balance condition (14), which, given expectations and asset supplies, determines the temporary equilibrium value of the black market premium, of the domestic money supply equation given by (15), and of the unreported current account equation given by (10) above. These equations together determine the dynamic adjustment to steady-state equilibrium where the stock of foreign assets is constant and the black market rate and the domestic money stock grow at the rate of crawl so that the premium and real balances are constant. Totally differentiating (14), we find:

\[ \dot{p}(1 - \alpha\eta - \varepsilon) - (1 - \alpha)\eta \dot{\tilde{M}} - (1 - \alpha\eta)\dot{F}, \]  
(16)

where \( \alpha = pF/\bar{W} \) is the share of foreign assets in wealth, \( \eta = (\partial f/\partial \bar{W})/\alpha \) is the wealth elasticity of the demand for foreign assets, and \( \varepsilon \) is the premium elasticity of the demand for foreign assets.

We now assume that

\[ \psi = 1 - \alpha\eta - \varepsilon > 0, \]  
(17)

or that the effect of wealth on the demand for domestic money is larger than the premium elasticity of demand for foreign assets. If this effect is given by the average shares (\( \eta = 1 \)), then (17) says that if the premium increases by 10% and domestic money is 90% of the wealth,\(^{15}\) the demand for foreign assets has to
increase by less than \(9\%\). Then substituting for \(p\) in (9) and (10), we can express the two equations of motion for the state variables \(\dot{M}\) and \(\dot{F}\) as

\[
\begin{align*}
\dot{M} &= B_u(\pi, \bar{M}, F) + \delta \bar{M}, \\
\dot{F} &= B_r(\pi, \bar{M}, F).
\end{align*}
\]  

(18)  

(19)

Since portfolio balance obtains at all times, we know that \(B_u = 0\) implies that \(B_r = 0\) as well. In the steady state, therefore, domestic credit creation as a proportion of the money supply is equal to the rate of crawl, \(\delta = 0\). Taking a linear expression around the steady-state equilibrium, we write the model compactly as

\[
\begin{bmatrix}
\dot{M} \\
\dot{F}
\end{bmatrix} = \begin{bmatrix}
-\bar{\omega} \eta - \bar{\sigma}(1-\varepsilon) & \bar{\omega} \eta + \bar{\sigma} h \\
\omega \eta - \theta(1-\varepsilon) & -\omega \eta + \theta h
\end{bmatrix} \begin{bmatrix}
1 - \alpha \\
\frac{\psi}{\eta}
\end{bmatrix}
\begin{bmatrix}
\bar{M} - \bar{M}^* \\
\bar{F} - \bar{F}^*
\end{bmatrix},
\]

(20)

where \(\bar{\omega}, \bar{\sigma}(\alpha, \theta)\) are the positive semi-elasticity of the reported (unreported) currency account with respect to the black market premium, and real wealth respectively, e.g., \(\bar{\sigma} = -\bar{W} B_r / \partial \bar{W}\); and \(h = \alpha \eta / (1 - \alpha \eta)\) is the marginal currency substitution ratio.

The determinant of the Jacobian in (20) is given by \((\omega \bar{\sigma} + \bar{\sigma} \theta)(1 - \alpha) / \psi\), and is positive when condition (17) holds. Stability requires in addition that the relative price elasticities of the current account be larger than the wealth elasticities. Even if \(\alpha\) is at the upper bound given by condition (17) and \(\eta - 1\), the trace will be negative when

\[
\bar{\omega} + \bar{\sigma}(1 - \alpha) + \omega \geq \alpha \theta.
\]

(21)

Since foreign currency prices are given, \(\bar{\omega}\) and \(\omega\) are likely to be quite large relative to \(\bar{\sigma}\) and \(\sigma\). Furthermore, \(\alpha\) is likely to be smaller than \(\frac{1}{2}\). Note also that stability always obtains when \(\varepsilon \leq 0\), which would happen if expectations of changes in the black market rate responded negatively to the black market premium.\(^{16}\) Even

\(^{16}\) Under perfect foresight, solve (14) for the rate of change of the black market exchange rate \(\dot{e} = f^{-1}(p, \bar{M}, F)\). Then the Jacobian of the system given by (14), (15) and (10) will have one positive eigenvalue, the system will exhibit saddle-point stability and \(\epsilon\) will be indeterminate. Expectation could also be adaptive \(\bar{e} = \rho \hat{e}\). If \(\rho < 0\) they are regressive with respect to the black market rate (and extrapolative with respect to the official rate) and stabilize the model by lowering the total response of \(\alpha\) to the premium. If \(\rho > 0\) expectations would destabilize the model.
though, unlike the model in Section II, here the response of the reported and unreported current accounts to the official and black market rates is the same (so that the continued devaluation of $\hat{e}$ is compatible with balance in the current accounts provided the premium is constant), conditions (17) and (21) are analogous to the dominance of the 'own' effect, this time between demand for goods and demand for assets.\textsuperscript{17}

Given condition (17) and $\omega > ax$ — which is stronger than condition (21) — the two loci where $\hat{M} = 0$ and $\hat{F} = 0$ will be positively sloped. Furthermore, since the determinant in (20) is positive, the $\hat{F} = 0$ locus is steeper than the $\hat{M} = 0$ locus. The portfolio balance locus, along which the premium is constant, is a ray through the origin with a slope of $p/h$ higher than the slope of the $\hat{M} = 0$ locus but flatter than the slope of the $\hat{F} = 0$ locus, as depicted in fig. 3. As shown by the arrows, above the $\hat{M} = 0$ locus the reported current account has a deficit larger than domestic credit creation and real money balances are decreasing, while below they are increasing. Above the $\hat{F} = 0$ locus the unreported current account is in surplus and the private stock of foreign assets is increasing, while below it is decreasing. The economy is always on a portfolio balance locus and at $A_0$ it is also in steady-state equilibrium.

Consider now the long run effects of an exogenous increase in the demand for foreign assets, because of an increase in the foreign interest rate or in $\pi$. Given

![Fig. 3. Steady-state effects of various disturbances.](image)

\textsuperscript{17}This condition carries over to the model with non-traded goods mentioned in footnote 13 above.
asset supplies $p$ goes up in proportion so that the portfolio balance ray remains the same. The increase in $p$, in turn, moves the reported current account into deficit and the $\hat{M} = 0$ locus shifts to the right because an increase in $F$ or a decline in $M$ are necessary to bring back balance. Similarly, the increase in $p$ moves the unreported current account into surplus and the locus where $\hat{F} = 0$ also shifts to the right. As shown in the Appendix, these changes in asset stocks bring back the premium to its original steady-state level, so that the portfolio balance locus shifts down and at the new steady state equilibrium $A_1$ the currency substitution ratio has increased. Now, from $A_1$, an increase in $\delta$ shifts the $\hat{M} = 0$ locus up because only an increase in $M$ or a decline in $F$ will deteriorate the current account to compensate for the increased domestic credit. The $\hat{F} = 0$ locus remains the same and the increase in $\hat{M}$ and decline in $F$ increases the premium, as that the portfolio balance ray shifts up. If the increase in $\delta$ is such that the $\hat{M} = 0$ locus has moved back to its original position, the new steady-state will be at $A_2$. The increase in $F$ is less than the increase in $\hat{M}$ but $\delta$ is larger at $A_2$ than at $A_1$.\(^{18}\) Note that, by the same argument, an increase in the rate of crawl would lead to a decline in steady-state $\delta$.

Consider now a once-and-for-all devaluation of the official rate from $A_1$. Because devaluation decreases the premium, demand for foreign assets goes down and the black market rate appreciates at the original asset stocks. This brings the reported current account into surplus and the unreported current account into deficit, so that $\hat{M}$ has to increase and $F$ has to decrease to restore balance. This shifts the $\hat{M} = 0$ locus up and the $\hat{F} = 0$ locus to the left, which increases the premium so that the economy will tend to move toward $A_0$. It can, however, be shown that the steady-state stock of foreign assets does not change while the premium and the stock of real balances increase in proportion to devaluation, so that the new steady-state equilibrium is at $A_3$, where $\delta$ is the same as at $A_1$ (see the Appendix). Unlike the crawling peg, devaluation has therefore no effect on the currency substitution ratio valued at the black market rate.\(^{19}\) Note that this applies even if the decline in the premium generates an expectation of depreciation of the black market rate and demand for foreign assets goes up, so that expectations of the black market rate are then extrapolative with respect to the official rate. In any case, the dichotomy of the previous section between current and capital account disturbances does not apply.

Consider finally the effects of exchange rate unification. If unification is accompanied by a floating official exchange rate, central bank reserves do not change, portfolio balance is described by (14) above with $p = 1$, domestic money is

\(^{18}\)See Macedo (1979a) for results in a model where there is a part of domestic credit creation that is induced by the increases in the net worth of the central bank due to devaluations and revaluations of gold and of foreign exchange reserves.

\(^{19}\)Calvo (1979) elaborates on the differences between a once and for all devaluation and an increase in the rate of crawl.
given and the change in the stock of foreign assets is equal to the aggregation of the reported and unreported current account, so that we have a standard portfolio model, in which the increased demand for foreign assets increases the steady-state currency substitutions ratio. If, however, unification is accompanied by a fully convertible pegged exchange rate, under stationary expectations domestic and foreign money become perfect substitutes. Given money demand, the money multiplier and prices, the supply of base money is given and we are back at the standard reserve flow equation, according to which an increase in credit creation is offset by a decline in reserves. Our model represents an intermediate case where unification is not an institutional feature but rather obtains in the steady state. Then the premium is constant and the same is true for the stock of foreign assets and real balance since domestic credit creation as a proportion of the money stock grows at the rate of crawl.

5. Conclusion

This paper has analyzed regimes of currency inconvertibility using the portfolio approach to exchange rate determination according to which the exchange rate depreciates when the current account is in deficit and appreciates when it is in surplus. A monetarist model of the black market for foreign exchange was criticized in section 2. Then, two portfolio models were developed, with and without wealth effects. In both cases the dominance of an 'own' effect was seen as a condition for stability. The 'own' effect referred to the elasticities in the BRM-type model of section 3 while the 'own' effect referred to the effect of relative prices on goods demands relative to their effect on asset demands in the general equilibrium model of section 4. In this framework it was shown that the effect of an increase in the rate of crawl decreases the currency substitution ratio while it is invariant with respect to a once-and-for-all devaluation. To the extent that the monetary authorities wish to decrease the currency consumption of private financial wealth, a crawling peg rather than a devaluation is the appropriate instrument. The models present were also contrasted with the conventional portfolio model of a unified (flexible) exchange rate and the monetary approach to the balance of payments when the official rate is pegged but fully convertible.

Appendix

Differentiate (14), (18) and (19) totally, taking into account that (aside from increases in \( \pi \) and \( \delta \)), there is an exogenous devaluation of \( k \%). Then the system can be written as

\[
\begin{bmatrix}
1 - \alpha \eta - \epsilon \\
\omega + \alpha \theta \\
-(\omega - \alpha \theta)
\end{bmatrix}
\begin{bmatrix}
-(1-\alpha)\eta \\
(1-\alpha)\theta - \delta \tilde{M} \\
(1-\alpha)\theta
\end{bmatrix}
\begin{bmatrix}
\frac{\delta}{\tilde{M}} \\
\frac{\epsilon}{\tilde{M}} \\
\frac{\delta}{\tilde{M}}
\end{bmatrix}
= \begin{bmatrix}
\frac{\epsilon_\pi + (1-\eta - \epsilon)k}{\delta \tilde{M} \delta + (\omega + \theta - \delta \tilde{M})k} \\
\frac{\epsilon_\pi + (1-\eta - \epsilon)k}{\delta \tilde{M} \delta + (\omega + \theta - \delta \tilde{M})k} \\
\frac{\epsilon_\pi + (1-\eta - \epsilon)k}{\delta \tilde{M} \delta + (\omega + \theta - \delta \tilde{M})k}
\end{bmatrix}
\]
The determinant is given by
\[ \Delta = (1 - \alpha) (\omega \bar{\theta} - \bar{\alpha} \theta) - (\eta^{-1} - \alpha) (\omega \bar{\eta} - \theta \bar{\eta}) \delta \hat{M} > 0. \]

The effect of an increase in \( \pi \) on the three endogenous variables is
\[ (\Delta / \varepsilon_{\pi}) \hat{p} / \hat{\pi} = - \alpha \theta \delta \hat{M}, \]
\[ (\Delta / \varepsilon_{\pi}) \hat{M} / \hat{\pi} = - \alpha (\bar{\omega} \theta + \omega \bar{\theta}), \]
\[ (\Delta / \varepsilon_{\pi}) \hat{Y} / \hat{\pi} = (1 - \alpha) (\bar{\omega} \theta + \omega \bar{\theta}) - (\omega - \alpha \theta) \delta \hat{M}, \]
so that \( \hat{p} = 0 \) if \( \delta = 0 \). Also \( (\Delta / \varepsilon_{\pi}) \hat{h} / \hat{\pi} = \bar{\omega} \theta + \omega (\bar{\theta} - \delta \hat{M}) \).

The effect of an increase in \( \delta \) is
\[ (\Delta / \delta \hat{M}) \hat{p} / \hat{\delta} = (1 - \alpha) \theta, \]
\[ (\Delta / \delta \hat{M}) \hat{M} / \hat{\delta} = (\eta^{-1} - \alpha) (\omega \bar{\eta} - \theta \bar{\eta}), \]
\[ (\Delta / \delta \hat{M}) \hat{Y} / \hat{\delta} = (1 - \alpha) (\omega \bar{\eta} - \theta (1 - \varepsilon)), \]
so that \( (\Delta / \delta \hat{M}) \hat{h} / \hat{\delta} = \omega (\eta - 1) + \theta \varepsilon \).

The effect of devaluation is
\[ \hat{p} / k = \hat{M} / k = 1, \]
\[ \hat{Y} / k = 0, \]
so that \( \hat{h} / k = 0. \)

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