THE ESTIMATION OF THE TWO-GAP MODEL

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Two-gap and related theoretical models imply that different constraints on growth may be binding at different times. This paper develops and applies an econometric methodology suitable for estimating this type of model and determining which constraints hold at different times. The empirical results suggest that growth is more often constrained by savings than by foreign exchange and that these models can be used to interpret the behavior of LDCs during the oil crisis.

1. Introduction

The publication of a detailed theoretical and empirical paper on two-gap models by Chenery and Strout (1966) precipitated a considerable controversy over the possibility of import constrained growth.\(^1\) Debate over these models continues among trade and development economists, while at the same time recent work [Dixit (1977)] has discussed anew the relation between rationing and the balance of payments.

The primary goal of this paper is to assess the prospects for estimating a two-gap model of the type originally formulated. An econometric methodology is discussed and applied which solves the major conceptual problems of estimation and classification in this literature. These methods are valuable in shifting the debate on import constrained growth toward formal econometric evidence and away from casual empiricism. For the particular examples analyzed here the neoclassical regime of savings constrained investment is more likely. The import constrained regime, emphasized by two-gap theorists, is, however, quite frequently observed.

This paper also suggests how models with many potential constraints on growth can be used to interpret the effects of oil price changes during the 1970s. For non-oil LDCs, sudden increases in their import bills yield the expectation of import constrained growth, a problem that can be investigated with a traditional two-gap model. On the other hand, the oil rich LDCs may find their growth

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\(^1\)Among the most prominent critics of this approach were Fei and Ranis (1968) and Bruton (1969).
limited by aspects of absorptive capacity, suggesting constraints on growth not emphasized in the two-gap model. For instance, Nigerian port capacity was an often mentioned impediment to obtaining imported investment goods in the 1970s. The development of the Arab oil exporters may be limited by the availability of skilled labor. I briefly indicate how these other constraints could be formulated for estimation purposes, although the countries which are most susceptible to them also have data availability problems precluding estimation at present.

Thus the confluence of the continuing controversy on two-gap models and new developments in both theory and country experience suggest the importance of evidence on international constraints on domestic growth. Section 2 presents a formulation of the two-gap model emphasizing issues relevant to estimation. Section 3 discusses the econometric theory for this type of model with special attention to the classification of observations between the savings and import constrained regimes. Section 4 presents the estimates of the two-gap model for a number of cases. Section 5 returns to the econometric methodology and discusses the incorporation of alternative constraints.

2. The two-gap model

This section presents a skeletal discussion of a two-gap model emphasizing only points relevant to estimation. Weisskopf (1972a) provides a fuller description of the model using a notation which I have adopted in this paper where possible.

Required imports, measured in physical units \((M^*)\), are given by

\[
M^* = \alpha + \beta Y + \gamma I + \nu,
\]

(1)

where \(Y\) is a... index of the physical quantity of production, \(I\) is the physical quantity of investment and \(\nu\) is an error term. The notion is both that production of output requires imports and that there may be a differential requirement for investment goods, represented by \(\gamma\).

Maximum savings in the same units as investment \((S^*)\) is given by

\[
S^* = a + \ell \frac{P_Y}{P_I} Y + c \frac{P_M}{P_I} F + d \frac{P_E}{P_I} E + u,
\]

(2)

where \(F\) is the foreign transfer defined in the same units as the import, and \(E\) is exports in physical units. The \(P_I\) denote the money price of the respective goods. ²

²I depart from the earlier work of Weisskopf and Blomqvist in emphasizing a distinction between physical and nominal quantities. These authors used only a general price deflator rather than sector specific deflators as in section 4. Using specific deflators to yield quantities in eq. (1) is more appropriate to a discussion when relative prices change, for instance, during the oil price hikes. This approach more closely approximates the physical relationships stressed by two-gap theorists.
Eq. (2) assumes that a constant marginal share of general income $\rho Y$ is reserved for savings while income from exports may have a differential effect and the foreign transfer has a separate effect.

Three accounting identities involve actual imports ($M$), actual savings ($S$) and consumption ($C$):

$$F = M - \frac{P_E}{P_M} E,$$

$$P_M M + P_Y Y = P_C C + P_I I + P_E E,$$

$$P_Y Y - P_C C = P_I S.$$

Exports and the foreign transfer are assumed exogenous.\(^3\)

$$E = \bar{E}$$

and

$$F = \bar{F}.$$

Production is less than or equal to capacity output ($Y*$):

$$Y \leq Y*.$$  \(8\)

Capacity output, $Y*$, may vary between different countries or between different years for the same country, but is assumed exogenous for each country in any year. Note that $Y$ is still potentially endogenous despite the exogeneity of $Y*$.\(^4\)

Actual imports are at least equal to required imports

$$M \geq M*$$

and actual savings are at most equal to maximum savings

$$S \leq S*.$$  \(10\)

\(^3\)The exogeneity of the foreign transfer is suggested by the fact that most LDCs are likely to be credit constrained in their borrowing from private capital markets. For theoretical and empirical discussions of this point, see Eaton and Gersovitz (1980 and 1981).

\(^4\)It would be preferable to endogenize capacity via a production function but this approach is infeasible because capital stock data for LDCs are not easily available. If this approach could be pursued, capacity would be a function of the maximum available inputs of domestically produced capital, imported capital and different types of labor. A production function embodying fixed proportions is in the two-gap tradition and is implicit in the assumption that investment can only be undertaken if the fraction $(\beta P_I / P_Y + \gamma)$ [substituting (4) into (1)] of the investment goods are imported. The parameters of a fixed coefficient production function are related therefore to the parameters of the import function and estimation of the production function at the same time as the rest of the model could increase the efficiency of all parameter estimates via cross-equation constraints.
Eqs. (1)–(7) and any two of conditions (8)–(10) holding as equalities solve the system and, in particular, determine values for \( I \). Let \( I_1 \) denote the value of \( I \) when (8) and (10) are chosen:

\[
I_1 = a + b \frac{P_Y}{P_I} Y + (1 + c) \frac{P_M}{P_I} \bar{F} + a \frac{P_E}{P_I} \bar{E} + v.
\]

Let \( I_2 \) denote the value of \( I \) when (8) and (9) hold as equalities:

\[
I_2 = -\frac{\alpha}{\gamma} \frac{\beta}{\gamma} Y + \frac{1}{\gamma} \bar{F} + \frac{1}{\gamma} \frac{P_E}{P_M} \bar{E} - \frac{1}{\gamma} v.
\]

Finally, \( I_3 \) is determined if (9) and (10) hold as equalities:

\[
I_3 = \lambda + \mu \bar{F} + \nu \bar{E} + w,
\]

where the \( \lambda, \mu, \nu \) and \( w \) depend on \( a, b, c, d, \alpha, \beta, \gamma \), the \( P_i \)'s and \( v \) and \( u \) but not on \( Y \) or \( Y^* \).

Using ex post data only one of the \( I_i \)'s is observed. Under the usual assumption that \( b, \beta \) and \( \gamma \) are all positive, fig. 1 can be used to investigate the model further. Since \( b \) is positive, \( I_1 \) is an increasing function of \( Y \). \( I_2 \) is a decreasing function of \( Y \). \( I_3 \) does not depend on \( Y \) and is a horizontal line through the intersection of \( I_1 \) and \( I_2 \). The region of feasible combinations of \( Y \) and \( I \) given implicitly by inequalities (8) and (9) is shown by the area \( 0abc \).

The location of \( Y^* \), a vertical line in \( (Y, I) \) space determines a third constraint. If \( Y^* \) lies to the left of point \( b \) (e.g. \( Y_1^* \)) then the region is further restricted to area \( 0adY^* \). In this case equilibrium will be at point \( d \) on the assumption that more of both \( Y \) and \( I \) is preferred. If \( Y^* \) lies to the right of point \( b \) (e.g. \( Y_2^* \)) then the region is restricted to area \( 0abeY^*_2 \). Again, assuming that more of both \( Y \) and \( I \) is preferred, a point along \( be \) will be chosen. The government therefore may choose to reduce activity before (8) holds as an equality at point \( e \). In this case total output is traded off for investment and future output along \( be \). So long as the policy choice of \( Y \) is viewed as exogenous to the model estimation can proceed as discussed below. If, however, the government were viewed as behaving endogenously, a reaction function would need to be specified. Under the above assumptions, equilibrium will fall somewhere along \( abc \), depending on the value of \( Y^* \). The unique, ex post value of investment will be given by

\[
I = \min (I_1, I_2),
\]

where \( I_1 \) and \( I_2 \) are evaluated at \( Y = Y^* \). Thus, so long as \( b, \beta \) and \( \gamma \) are positive, \( I_3 \)
3. Econometrics for the two-gap model

The dispute over the estimability of two-gap models arises from the critical role of the inequality constraints embodied in eq. (14). Estimation must allow for the possibility that different constraints are binding at different times and for the fact that the researcher is unlikely to know a priori which situation prevails when. Previous attempts at the estimation of these models have failed to meet these criteria. The original Chenery–Strout contribution assumed that, during the period of estimation, the savings and trade gaps were equal. Thus all constraints were simultaneously binding so that the economy was always assumed to be at

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5This analysis agrees with that of Blomqvist but not with the methodology employed by Weisskopf, and it appears that Weisskopf unnecessarily investigated the possibility of a third regime.
point $b$ in fig. 1. As Fei and Ranis aptly noted (1968, p. 190) '... an essential aspect of the Chenery–Strout thesis — i.e., the existence of a gap between the two gaps — must be sacrificed!' Recent contributions by Weisskopf (1972a, b) have assumed that each country is under only one of three possible regimes during the period of estimation. Blomqvist (1976) assumes only two possible regimes, a position shared by this paper, but follows Weisskopf in the assumption of uninterrupted membership in a single regime. These studies then estimate all possible regimes for each country, establish which regime gives the best fit for a particular country, and classify each country's experience accordingly. No allowance is made for the possibility that any individual country may have experienced more than one regime during the period of estimation. As Blomqvist notes:

...the tests [performed by himself and Weisskopf] will be invalid for any country which switched from one case to another during the time period for which the sample was taken. Weisskopf's results as well as our own must therefore be regarded as conditional on the validity of this assumption.

This section discusses an econometric methodology which overcomes these restrictions.

The estimation of eqs. (11)–(14) is similar to the estimation of markets in disequilibrium [Maddala and Nelson (1974)]. Consider the model:

\begin{align}
  y_1 &= A_1 x_1 + \omega_1, \\
  y_2 &= A_2 x_2 + \omega_2, \\
  y &= \min(y_1, y_2),
\end{align}

where $x_i$ is a set of exogenous variables, $A_i$ is an unknown coefficient matrix and $\omega_1$ and $\omega_2$ are normally distributed random variables.

The crucial aspect of the system (15) is that only $y$ is observed. It is not known a priori whether any $y$ is a $y_1$ or a $y_2$, yet it is possible to estimate the parameters of both equations. Let the joint likelihood of $y_1$ and $y_2$ be denoted by $g(y_1, y_2)$, and the likelihood of $y$ be $h(y)$. Then

\[ h(y) = \int_y^\infty g(y, y_2) \, dy_2 + \int_y^\infty g(y_1, y) \, dy_1. \]

The first part of (16) refers to the situation $y_1 = y$ and $y_2 \geq y$ while the second part refers to $y_1 \geq y$ and $y_2 = y$. Now assume that $\omega_1$ and $\omega_2$ are independently
distributed with variances $\sigma_1^2$ and $\sigma_2^2$. Let

$$f_i(y) = \frac{1}{\sqrt{2\pi} \sigma} \exp \left( -\frac{1}{2\sigma^2} (y - A_i x_i)^2 \right),$$

(17a)

$$F_i(y) = \int_y^\infty f_i(y) \, dy_i.$$

(17b)

Then

$$h(y) = f_1 F_2 + f_2 F_1.$$

(18)

Maximizing this likelihood yields the estimates of the $A_i$ and $\sigma$. If the $\omega_i$ were not assumed independent, the general formulation of eq. (16) could be used to form the likelihood of the sample.\textsuperscript{6}

The two-gap problem of eqs. (11), (12), and (14) is analogous to model (15). Assume that $u$ and $v$ are independent, normally distributed random variables with variances $\sigma_u^2$ and $\sigma_v^2$. In this case $\omega_1$ has variance $\sigma_1^2 = \sigma_u^2$ and $\omega_2$ variance $\sigma_2^2 = \sigma_v^2 / \gamma^2$ which must be used in forming the $f_i$'s and $F_i$'s.

An important issue in the two-gap literature is the identification of when different countries have been in the different regimes. Two measures of regime membership may be of interest [Gersovitz (1980]). Denote an actual observation generated by the model (15) by $(y, x_1, x_2)$. First there is the probability that any observation with the given independent variables $x_1, x_2$ will come from regime 1:

$$P(1|x_1, x_2) = \int_{-\infty}^{(A_2 x_2 - A_1 x_1)/\sigma} \frac{1}{\sqrt{2\pi}} e^{-v^2/2} \, dv,$$

(19)

where $\sigma^2 = \sigma_1^2 + \sigma_2^2$. Second, there is the likelihood that the particular observation $(y, x_1, x_2)$ did come from regime 1:

$$P(1|y, x_1, x_2) = f_1 F_2 / (f_1 F_2 + f_2 F_1).$$

(20)

Eq. (20) has the intuitive rationale of the fraction of the likelihood of the particular observation contributed by the first regime.

In general, these probabilities are different since eq. (20) uses the realization of the actual $y$. The unconditional probability (19) is analogous to a prediction in ordinary least squares while the conditional probability is analogous to the examination of the actual residuals. For the two-gap problem, eq. (20) seems most

\textsuperscript{6}Computation of the estimated parameters would be more difficult, and most studies using this type of likelihood function have made the independence of errors assumption. One goal of future research in this area could be the relaxation of this assumption.
relevant since the interest of past studies has been the classification of actual historical experience.

4. Estimation of the model and classification of the observations

Only for very few LDCs do standard UN, IMF or OECD publications report consistent and reliable annual series on the necessary variables for very many years. Virtually no African or Asian countries could be included using these data sources especially if countries involved in serious wars and partitions are ruled out. The requisite time series are available for a number of Latin American countries for nearly thirty years providing the best opportunity to estimate the model. Estimation using cross-section data from LDCs, while providing many more observations, assumes a similarity of parameters which is questionable.

Table 1 presents estimates of the parameters for five data sets. The a priori expectation for $b$, the effect of income on savings, is a positive fraction, small in magnitude. The incremental influence of export income on savings is represented separately by $d$, often thought to be a positive fraction on the presumption that exporters are somewhat richer individuals. The coefficient $d$ may even exceed $b$ by a considerable amount. The total effect of an increase in exports on savings is $(b + d)P_e/P_i$. A fraction of any foreign transfer is consumed, thus lowering domestic savings by a corresponding amount, $c$, a negative fraction. The parameters of the import function are $\beta$, the income coefficient which should be a small positive fraction and $\gamma$, the incremental effect of investment on import demand, also a positive fraction.

As is evident from table 1, the estimated coefficients for Argentina, Colombia, Ecuador, Guatemala and Peru are largely consistent with the theoretically

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7Monte Carlo results [Gersovitz (1980)] indicate that the disequilibrium estimation technique can be applicable with samples of as small as 30 observations.

8A cross-section of 77 countries for the year 1967 was used to complement the time series investigation. The coefficient estimates, for the slopes, with asymptotic $t$ statistics in brackets, were consistent with the theoretical model, being $b = 0.196 (15.86)$, $c = -0.820 (6.23)$, $d = 0.168 (1.90)$, $\beta = 0.004 (0.25)$ and $\gamma = 0.519 (3.82)$. Only five countries had an appreciable probability of being in the import constrained regime. However, all of these countries were relatively large (Argentina, Brazil, India, Mexico and Turkey) leading to the suspicion that the model was merely picking up the well-known tendency for large countries to be less reliant on trade, and therefore seemingly import constrained. As a consequence, cross-sectional analysis in this context seems relatively unjustified.

9The likelihood function (18) was maximized using the GRADX option [Goldfeld et al. (1966)] of the GQOPT program written in double precision FORTRAN at Princeton University with a relative convergence accuracy of $10^{-7}$.

10In the case of Ecuador, only a general CPI deflator was available to construct real values. For precise data definitions see the appendix.

11In the case of Guatemala, the variances of $u$ and $v$ were constrained to be equal. When this constraint was not imposed, the ratio of $\sigma_u/\sigma_v$ behaved erratically.

This problem potentially arises in all maximum likelihood estimations of models with min conditions because the likelihood function is globally unbounded [Quandt (1977)]. Intuitively, this problem is analogous to fitting a regression to a sample with fewer observations than coefficients. The likelihood is the sum of two regressions (the $f_i$'s) appropriately weighted (by the $F_i$'s). Consequently, it
Table 1

Coefficient estimates.*

<table>
<thead>
<tr>
<th></th>
<th>Argentina</th>
<th>Colombia</th>
<th>Ecuador</th>
<th>Guatemala</th>
<th>Peru</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a$</td>
<td>-14.4</td>
<td>-160.00</td>
<td>-0.388</td>
<td>-32.8</td>
<td>4450</td>
</tr>
<tr>
<td>(2.03)</td>
<td>(5.56)</td>
<td>(16.59)</td>
<td>(0.45)</td>
<td>(3.84)</td>
<td></td>
</tr>
<tr>
<td>$b$</td>
<td>0.152</td>
<td>0.168</td>
<td>0.273</td>
<td>0.141</td>
<td>0.126</td>
</tr>
<tr>
<td>(6.98)</td>
<td>(6.32)</td>
<td>(22.41)</td>
<td>(0.91)</td>
<td>(9.12)</td>
<td></td>
</tr>
<tr>
<td>$c$</td>
<td>-0.190</td>
<td>-0.355</td>
<td>-0.596</td>
<td>0.87</td>
<td>-0.467</td>
</tr>
<tr>
<td>(1.85)</td>
<td>(2.11)</td>
<td>(13.63)</td>
<td>(1.01)</td>
<td>(5.56)</td>
<td></td>
</tr>
<tr>
<td>$d$</td>
<td>0.701</td>
<td>0.466</td>
<td>-0.008</td>
<td>0.148</td>
<td>-0.055</td>
</tr>
<tr>
<td>(3.09)</td>
<td>(3.20)</td>
<td>(0.21)</td>
<td>(0.26)</td>
<td>(0.49)</td>
<td></td>
</tr>
<tr>
<td>$x$</td>
<td>95.4</td>
<td>4640</td>
<td>0.069</td>
<td>4.02</td>
<td>-2396</td>
</tr>
<tr>
<td>(3.05)</td>
<td>(2.95)</td>
<td>(0.73)</td>
<td>(0.39)</td>
<td>(5.30)</td>
<td></td>
</tr>
<tr>
<td>$\beta$</td>
<td>-0.177</td>
<td>-0.192</td>
<td>0.064</td>
<td>0.044</td>
<td>0.058</td>
</tr>
<tr>
<td>(1.69)</td>
<td>(3.50)</td>
<td>(1.36)</td>
<td>(1.95)</td>
<td>(4.71)</td>
<td></td>
</tr>
<tr>
<td>$\delta$</td>
<td>0.937</td>
<td>2.76</td>
<td>0.705</td>
<td>0.952</td>
<td>0.835</td>
</tr>
<tr>
<td>(2.44)</td>
<td>(5.17)</td>
<td>(3.57)</td>
<td>(6.31)</td>
<td>(13.30)</td>
<td></td>
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<tr>
<td>Estimate of $\sigma_v/\sigma_a$</td>
<td>0.463</td>
<td>1.05</td>
<td>0.342</td>
<td>1.00</td>
<td>2.53</td>
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<tr>
<td>(2.05)</td>
<td>(2.98)</td>
<td>(2.64)</td>
<td>(imposed)</td>
<td>(2.34)</td>
<td></td>
</tr>
<tr>
<td>Estimate of $\sigma_u$</td>
<td>7.70</td>
<td>284</td>
<td>0.0270</td>
<td>17.1</td>
<td>1248</td>
</tr>
<tr>
<td>Sample size</td>
<td>26</td>
<td>29</td>
<td>28</td>
<td>26</td>
<td>29</td>
</tr>
<tr>
<td>Log of likelihood function</td>
<td>-97.5</td>
<td>-195</td>
<td>37.4</td>
<td>-110.1</td>
<td>-242</td>
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</table>

*Coefficients values are given by the upper figure; the lower, bracketed figure is the coefficient divided by the asymptotic standard error, i.e. an approximate $t$-statistic.

anticipated signs and magnitudes. The case of Honduras did not produce sensible coefficient estimates and, in the interests of space, these results are not presented. As two-gap theorists emphasize, the incremental demand for imports deriving from investment activity (represented by $\gamma$) is positive and may greatly exceed that generated by other economic activity (represented by $\beta$).

is possible to take zero as the estimate of one variance, in effect throwing all observations into the other regime and thereby yielding an infinite value for the likelihood function. The equal variance assumption obviates this problem. Although other restrictions on the variances would bound the likelihood, rough equality seems appropriate for the error terms of the import and savings functions. Note that this constraint cannot be rejected for any of the other cases presented in table 1.

An alternative approach to the global unboundedness problem is presented by Amemiya and Sen (1977). These authors prove that if a variance constraint is not imposed, a local maximum of the likelihood function provides consistent estimates of the parameters. Because this approach required less a priori restrictions, I preferred to follow it where possible, i.e. for the other four cases of table 1.

The case of Honduras resulted in poor performance of the algorithm as evidenced by difficulty in finding starting values and obtaining convergence. The coefficient estimates were almost entirely inconsistent with theoretical expectations about either sign or magnitude and would not lead to a configuration of the constrained regions as given in fig. 1.
Indeed, the latter coefficient is not very reliably estimated and is negative in two cases. Turning to the savings function, the marginal propensity to save, \( b \), is always a positive fraction. An increase in the foreign transfer decreases savings in most cases, while the incremental effect of export income on savings is positive in some cases and negative in others.

The probability of the savings constrained regime in each year for each country is given in Table 2. For approximately 55 percent of the observations the probability of the savings constrained regime exceeds 0.5. The behavior of Ecuador in the post-1974 period contrasts with that of the other countries. The only country with important oil exports, Ecuador moves from a long period of import constrained growth into a savings constrained situation. The behavior of the other four countries is quite mixed, consistent with the fact that some of them participated in the partial commodity booms of this period. None exported anything as favored as oil, however, which they themselves had to import.

<table>
<thead>
<tr>
<th>Year</th>
<th>Argentina</th>
<th>Colombia</th>
<th>Ecuador</th>
<th>Guatemala</th>
<th>Peru</th>
</tr>
</thead>
<tbody>
<tr>
<td>1950</td>
<td>0.92</td>
<td>1.00</td>
<td>n.a.</td>
<td>0.28</td>
<td>0.11</td>
</tr>
<tr>
<td>1951</td>
<td>0.66</td>
<td>1.00</td>
<td>0.82</td>
<td>n.a.</td>
<td>0.18</td>
</tr>
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<td>1952</td>
<td>0.52</td>
<td>1.00</td>
<td>0.71</td>
<td>n.a.</td>
<td>0.15</td>
</tr>
<tr>
<td>1953</td>
<td>0.71</td>
<td>1.00</td>
<td>0.64</td>
<td>0.34</td>
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</tr>
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<td>1954</td>
<td>0.81</td>
<td>0.19</td>
<td>0.90</td>
<td>n.a.</td>
<td>0.23</td>
</tr>
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<td>1955</td>
<td>0.83</td>
<td>0.15</td>
<td>0.82</td>
<td>0.16</td>
<td>1.00</td>
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<td>1956</td>
<td>0.00</td>
<td>0.44</td>
<td>0.64</td>
<td>0.16</td>
<td>0.79</td>
</tr>
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<td>1957</td>
<td>0.12</td>
<td>1.00</td>
<td>0.29</td>
<td>0.03</td>
<td>0.98</td>
</tr>
<tr>
<td>1958</td>
<td>0.66</td>
<td>1.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.79</td>
</tr>
<tr>
<td>1959</td>
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<td>0.05</td>
<td>0.00</td>
<td>0.97</td>
</tr>
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<td>1960</td>
<td>0.63</td>
<td>1.00</td>
<td>0.00</td>
<td>0.00</td>
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<td>1961</td>
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<td>0.99</td>
<td>0.00</td>
<td>0.00</td>
<td>1.00</td>
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<tr>
<td>1962</td>
<td>0.75</td>
<td>1.00</td>
<td>0.00</td>
<td>0.00</td>
<td>1.00</td>
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<tr>
<td>1963</td>
<td>0.10</td>
<td>1.00</td>
<td>0.00</td>
<td>0.00</td>
<td>1.00</td>
</tr>
<tr>
<td>1964</td>
<td>0.00</td>
<td>1.00</td>
<td>0.00</td>
<td>0.00</td>
<td>1.00</td>
</tr>
<tr>
<td>1965</td>
<td>0.94</td>
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<td>0.00</td>
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<td>1.00</td>
<td>0.88</td>
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The results of table 2 suggest that the Weisskopf–Blomqvist assumption that any country is only in one regime is not justified. Guatemala was classified as import constrained and Colombia as savings constrained by Weisskopf, while Blomqvist's method could classify neither. Argentina is classified as savings constrained by Blomqvist and is unclassified by Weisskopf's method. Peru is savings constrained according to Blomqvist and import constrained according to Weisskopf. Neither author considers Ecuador.

5. Other constraints on the investment process in LDCs

The preceding analysis has focused on the two most frequently discussed constraints on the investment process. In general, there may be many constraints on investment which could be represented by

\[ I = \min(I_1, \ldots, I_n). \]  \hfill (21)

If each of the \( I_i \) were determined by stochastic equations of the form of eqs. (11) and (12), this estimation problem could be represented by

\[ y_i = A_i x_i + \omega_i, \]  \hfill (22a)

\[ y = \min(y_1, \ldots, y_n). \]  \hfill (22b)

in the notation of section 3.

In the case of (22), the likelihood of \( y \) would be given by

\[ h(y) = \sum_{i=1}^{n} \int_{y}^{\infty} \int_{y}^{\infty} g(y_1, \ldots, y_{i-1}, y, y_{i+1}, \ldots, y_n) \]

\[ \times dy_1, \ldots, dy_{i-1}, dy_{i+1}, \ldots, dy_n, \]  \hfill (23)

where \( g(y_1, \ldots, y_n) \) is the joint likelihood of the \( y_i \).

Perhaps the third most frequently discussed constraint in development planning models concerns the availability of skilled, or technically trained or entrepreneurial individuals. This type of constraint is often mentioned in the context of manpower planning models [Blitzer (1975) and Taylor (1975)]. Unfortunately, those countries such as the Arab oil exporters which are most susceptible to this type of constraint lack the data on both manpower supplies and other variables necessary to test the model.

Suppose that the skilled manpower constraint is given by

\[ I_s = e + fL + w, \]  \hfill (24)
where \( e \) and \( f \) are constants, \( L \) is the number of skilled workers in the LDC and \( w \) is an error term. The corresponding inequality constraint would be

\[
I \leq I_4,
\]

(25)

and eq. (14) is replaced by

\[
I = \min(I_1, I_2, I_4).
\]

(14')

In terms of fig. 1, eq. (25) is represented by a horizontal line in \((Y,I)\) space, the height of which depends on \( L \) and \( w \). Should \( I_4 \) fall below \( I_3 \), the skill constraint is potentially binding.

The estimation problem posed by (14') is a special case of the general problem for \( n = 3 \) given by (22), and (23) yields the likelihood. Assuming that \( \omega, v \) and \( w \) are independent and defining \( f_i \) and \( F_i \) as in (17) yields the likelihood function

\[
h(y) = f_1 F_2 F_3 + f_2 F_4 F_3 + f_3 F_1 F_2.
\]

(26)

Maximizing this likelihood with the \( \omega_i, A_i \) and \( x_i \) suitably defined yields estimates of the parameters of (14').

The constraint potentially imposed on the investment process by skilled manpower is only one representation of absorptive capacity, the capacity of a country to plan and execute investment projects as determined by factors other than the availability of savings or foreign exchange: [Marris (1970)]. Other possible indicators of a country's absorptive capacity would be last year's investment or the highest level of investment attained within the recent past multiplied by a coefficient to reflect learning-by-doing. This type of constraint might be investigated in a detailed model of Venezuela, whose oil revenues have risen so dramatically as to suggest the possibility of an absorptive constraint.

6. Conclusion

One purpose of this paper has been to focus debate over import-constrained models of growth by developing an appropriate methodology for estimating the two-gap model and classifying observations into the different regimes. This estimation explicitly allowed for the possibility that investment is limited by two of the three constraints: capacity, savings or imports. The technique presented

\[\text{\footnotesize For the 1967 cross-section referred to in footnote 8, the World Bank (1976) gives information on the number of literate individuals for 56 countries, a proxy for skilled workers. These data were used to investigate the skilled labor constraint. Most (48 of 56) countries continued to be classified as savings constrained. Four countries including Libya were most likely to have been constrained by literate manpower and an additional five countries had a probability in excess of ten percent of being in this regime. These results suggest that a manpower constraint justifies further investigation with detailed time series if these can be obtained.}\]
here offers considerable advantages over past methods by allowing different observations to be generated by different regimes. The empirical results of the estimation suggest that investment is less likely to be constrained by imports than by savings. The basic method of estimation used for the two-gap model was extended in section 5 to incorporate other constraints.

Data availability for the LDCs continues to limit empirical applications of these models, but perhaps the estimation methods elaborated in this paper will encourage the collection and refinement of data suitable for the estimation of this type of model. Estimation will further the usefulness of these models in focusing development efforts on the mitigation of the particular constraint limiting any individual country's growth. In particular, it will be possible to use the estimated models to predict the effects of foreign transfers of both capital and skilled labor on investment and development. Another important use of these models is in understanding the behavior of both oil-rich and non-oil LDCs during the period of rapid changes in the relative price of oil. Finally, a most important direction for future research using this class of models is the incorporation of price effects. While the rigidities implied by the two-gap approach may be operative in the short-run or because policy-makers suppress the market mechanism, a more general modelling of LDC aggregate behavior necessitates attention to the endogenous adjustment of markets.

Appendix: Definition of the variables

For Argentina, Colombia, Guatemala, Honduras and Peru:

\( I \): gross domestic investment at constant prices.
\( Y \): gross domestic product at constant prices.
\( E \): exports at constant prices.
\( M \): imports at constant prices.
\( P_i \): obtained by dividing the nominal value of a variable by its constant price value.

Source of data: OECD (1968 and 1970) and United Nations (1957–1979) for Argentina, Colombia, Guatemala, Honduras and Peru.

For Ecuador:

The \( P_i \)'s were assumed equal and \( I, Y, E \) and \( M \) were obtained by dividing the corresponding nominal data by the CPI. (Note that this approach allows the purchasing power of exports to vary insofar as oil price hikes affect prices relative to the CPI.)

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