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COMPLEXITY AND ADJUSTMENT IN INPUT-OUTPUT SYSTEMS

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I. INTRODUCTION

Considering the importance of the notion of the complexity of an economic system, surprisingly little work has been done to develop empirical measures of it. All the measures that have been developed are based on the input-output table. Wong has suggested that the determinant of the (I-A) matrix of coefficients is a relative measure of the volume of 'net' production and thus complexity of the input-output system.¹ Chenery and Watanabe have suggested two measures of complexity: (1) the ratio of intermediate demand to total demand for a sector's (or the economy's) output, which they call W; and (2) the ratio of the value of a sector's (or economy's) demand for intermediate goods to the value of its output, which they call U.² Both the Chenery and Wong approaches are based on the idea that an economy is more 'complex' if it churns more to achieve a given final demand.³

Hirschman has developed another approach to defining complexity based on his idea of forward and backward 'linkages'.⁴ The simplest way to measure the number of 'links' in an input-output system is to count the number of non-zero elements in the matrix—a procedure which has been used.⁵ Yan and Ames have extended this approach by trying to measure indirect as well as direct links, and by equating the degree of complexity with the number of links.⁶

All of the preceding empirical measures seem plausible, but they involve no explicit theory of how the churning process works. It is therefore theoretically difficult to compare the measures or to use them to make meaningful inter-temporal or inter-country comparisons.⁷

Our approach is somewhat different. First, we develop a notion of adjustment costs in an aggregate economic system out of equilibrium which involves at the same time theoretical notions of 'complexity' which it would be interesting to measure. Second, we develop a fable of how the economy operates in disequilibrium. Third, we develop and discuss various empirical measures of complexity that arise naturally from the fable. Finally, we present empirical results for five countries and compare the various measures.

¹ See Wong (1954), pp. 330-341. If the economy is viable and the matrix is perfectly diagonal, the determinant equals one (the upper bound).

² Chenery and Watanabe (1958).

³ Chenery and Clark (1959) do discuss the degree of triangularity of the matrix. Various other measures based on the inverse of (I-A) are discussed by Bjerke and Rasmussen (1966).

⁴ Hirschman (1958).

⁵ Peacock and Dosser (1957).

⁶ Yan and Ames (1965).

⁷ The Chenery measures have been severely criticized by Kleiman (1970) who argues that Chenery and Clark failed to consider variations in the structure of final demand and of imports in making their inter-country comparisons.

II. EQUILIBRIUM AND ADJUSTMENT

The economic system is considered to have as its goal the production of a vector of final demand (including export and investment goods) and to require certain flows within the system to achieve this goal. In a market economy, the inter-sectoral flows are not known *a priori* but emerge as the system adjusts to producing the given final demand vector. The different sectors are visualized as being joined by links through which goods and information flow. Define a flow of goods and services through a link as a 'transaction', which is a broad concept that need not correspond to a purchase or sale in a market. On producing a given output, an economic system will incur a set of transactions costs for each set of inter-sectoral flows—one for each non-zero element in its input-output matrix.¹

Along with each transaction, there is an information flow. Each order that a sector receives (and fills by a transaction) gives that sector information about the state of demand for its output. If the only knowledge a sector has about the state of demand for its output comes from the orders it receives, then those orders contain valuable information. The timing and number of orders required to gain information about the magnitude of total demand would be very important to a sector which is required to adjust its production to a changing and uncertain total demand. One would postulate that the longer it takes, or the more transactions are required, for a sector to learn the magnitude of total demand for its output, then the more difficult and costly will it be for it to adjust production to meet the demand.

The emphasis here is on the flow of information as an important variable affecting the real costs a sector incurs in *adjusting* its production plans to meet demand. If the economy produces the same equilibrium output year after year, or is on a balanced growth path with a known growth rate, then every sector has exact information and there is no need to worry about adjustment costs. The analytic framework is therefore a system out of equilibrium.

The institutional framework must be such that transactions are the only instruments of information transfer in the system. In the fable presented below, we further simplify by specifying a quantity adjustment model and so assume that the only relevant information is the quantity ordered, with no reference to prices. This assumption is, of course, a great simplification but it may be a reasonable first-order specification for market economies and even some planned economies.²

The magnitude of adjustment costs for an economic system is related to at least four factors:

- (1) Size of the variation in total demand. The more extreme the change, the higher are the costs of adjustment.

¹ There is, of course, an aggregation problem because each non-zero element in the A matrix represents a collection of actual flows, and the number of flows per non-zero element may vary.

² See Kornai (1971) and Kornai and Martos (1971). Kornai argues that neoclassical economics has been far too occupied with only price adjustment mechanisms and has developed a more general framework for looking at systems out of equilibrium. Kornai and Martos further develop the implications of a quantity adjustment model.

- (2) Quality of linkage. The better are the links, the more sure and faster is the transmission of the effects of a change in demand throughout the economy. Railroads and telephones are better than wagons and the pony express.
- (3) Adjustment time. How long does it take for all the effects of a change to be transmitted throughout the system.
- (4) Number of transactions. How many transactions are required to transmit the effects of a change. How many links are used, and how frequently, before a new equilibrium is reached.

The first two factors are clearly important, but will not be considered in any detail here. They both can be better approached with analysis at a less aggregative level than is used here. Our intention in this article is to develop measures of the latter two factors: systems are more 'complex' when they require more time and transactions to reach a new equilibrium after some exogenous change.

The only shock to a system we consider is a change in the structure of final demand, which is not very restrictive. The effects of technological change as well as changes in preferences are transmitted through variations in demand. Because we deal with specified input-output tables, we do not explicitly consider the effect of changes in technology. The basic adjustment model is presented in the next section.

III. A FABLE OF ADJUSTMENT

In order to measure the effects of a change in demand, it is necessary to specify exactly how the various sectors adjust. This is done in the following fable. The workings of the economy are completely described by six rules. The first two rules describe the technology and the latter four are behavioural.

(1) The technology is specified by an input-output table with all the usual assumptions of fixed coefficients, constant returns to scale, and so forth.¹ The following notation will be used.

A : the $n \times n$ matrix of technical coefficients.

x : $n \times 1$ vector of total domestic production.

f : $n \times 1$ vector of final demand (including exports).

m : $n \times 1$ vector of imports.

We have the usual result that total demand equals total supply:

$f + Ax = x + m$. Imports will be discussed later, but for the moment they can be ignored, yielding the familiar result: $x = (I - A)^{-1}f$.

(2) Every sector has the same period of production which is defined as the time between making a production decision and actually producing the output. One can call this time period a 'month' so that production is planned on the first day and delivered on the last day. Once planned, production cannot be changed.

(3) Every sector holds enough inventories so that it never fails to deliver an order immediately, on demand. There are no production bottlenecks in the economy.

¹ See Chenery and Clark (1959), Chapter 2 for a discussion of the basic model.

(4) Every sector produces to order. Its production plan for any month is to produce the total orders received (and delivered out of inventories) in the previous month. This is a quantity adjustment model, with no consideration of prices.

(5) To start the adjustment process, assume that in month zero final demand is ordered and delivered. One can instead assume that some fraction of final demand is ordered each month since it turns out that all the measures developed below are independent of scale.¹

(6) To complete the adjustment process, each sector finishes off production in the month after it has achieved some large fraction of total output (say 98 per cent) and then closes down. Each sector thus finishes off with a small lump of production which it uses to continue delivering orders. At the end of the process, all sectors have the same inventories as they did before the beginning of the first month.²

The first five rules provide an algorithm by which the adjustment process for a given economy can be programmed. In the first month, f is produced and the sectors place a vector of orders Af on other sectors. In the second month Af is produced and $A^2 f$ is ordered, and so forth. The rules simply require that the economy produces according to the power series expansion of the inverse:³

$$x = (I - A)^{-1}f = \lim_{n \rightarrow \infty} [I + A + A^2 + \dots + A^n]f. \quad (1)$$

Each month can now be identified as a production round.

The sixth rule is required to make the series coverage in a finite number of rounds. We can rewrite the series as:

$$x^{(m)} = [I + A^{(1)} + A^{(2)}A^{(1)} + \dots + A^{(m)}A^{(m-1)} \dots A^{(1)}]f \quad (2)$$

In this series, $A^{(1)} = A$. $A^{(2)}$ equals $A^{(1)}$ except that all the elements in the columns of A for those sectors which finished production in the last round are set to zero. If the whole system finishes in n rounds, then $A^{(n+1)}$ is just a matrix of zeros.

Sector i finishes in the next round if the ratio $x_i^{(m)}/x_i$ exceeds the specified cut-off point (.98 is used here). If this occurs in the m 'th round, then the sector produces for $m+1$ rounds. It is easily seen that the number of rounds in which a sector produces is independent of the scale of final demand since multiplying f by any scale factor changes both numerator and denominator proportionately in the cut-off equation. Since the adjustment process as described in the fable is independent of scale, then the measures developed below cannot reveal anything about the effect of the absolute size of any changes in demand on the overall costs of adjustment.

¹ One can therefore also assume that the system is on a balanced growth path. Essentially, it is the system's response to producing a given structure of final demand that is being studied.

² Specifying that a sector produces a small lump of output in the last month is only a convenience for the computer. In theory, one certainly would not assume that the inverse of $(I-A)$, and thus total output, is known in advance.

³ The general conditions under which this series converges are well known. See, for example, Lancaster (1968), pp. 85-87.

Further Considerations

As it stands, the fable is based on fairly restrictive assumptions. The sensitivity of the results to some of them will be considered in the empirical section. It is worth discussing here two assumptions—lack of production bottlenecks and producing to order—which are not pursued in the empirical work.

It is clearly unrealistic to assume all sectors deliver orders on demand. During the course of production, particularly in the underdeveloped economies, bottlenecks do occur and consequently certain commodities cannot be produced as planned. In an input-output system, the inability of any sector to deliver will halt the production of all commodities if the matrix is indecomposable. In some cases this is what happens in the real world (e.g., the withdrawal of oil supplies), but more frequently some alternative system is evolved which may consist of rationing or using substitute materials in a different production plan. These latter situations cannot be adequately dealt with by using an input-output matrix where there is assumed to be no substitution. Given the information, however, it would be possible to programme specified lags into the round-by-round response of the individual industries. Such an approach has not been pursued here.

The assumption of producing to order is crucial in determining the pattern of signals received and generated by each sector. If any one sector follows a different production rule which involves the building up or the running down of stocks, then it is easy to see that the number of rounds any sector goes can be affected. A different production rule could come about if each sector pursued some decision-making rule under uncertainty and *had more information than just the orders received in any particular round*. In such a case, the factors involved in the decisions would include the storage costs and the fixed costs of production (including transaction costs) for the commodities produced by the sector, the reliability of the information about future orders, and the producer's particular risk preferences. In addition, since the expectations of the different sectors can easily be inconsistent, an adaptive or learning rule would have to be included which may in turn generate a stable, unstable, or cyclical pattern of production. The building of such an appropriate inventory model is a large task and might be a fruitful extension of our simple model. The problem would, of course, differ depending on whether the economy is a market or planned economy.

We will develop below a measure of adjustment time which is the number of rounds the system requires to reach final equilibrium. It might be tempting, especially in making comparisons, to assume that the length of time required for a round is the same either across countries or over time. Such an assumption is unjustifiable as one would assume that an economy that required more rounds would seek to develop a transactions technology that would reduce the time per round (and so reduce the total transactions costs). Less complex economies (on this measure) would have less incentive to use faster transactions technologies. In any case, there could well be technological differences across countries which imply very different chronological times for a production round in a given sector. Therefore, to compare the actual adjustment costs across countries, one would

need to know, in addition to the number of rounds, the actual times per round in different sectors across countries. These data are, in principle, available—there are data on the time-lag between orders and deliveries for individual sectors. Both the theoretical and data problems are similar to those of specifying different sectoral periods of production. We restrict ourselves here to examining the number of rounds, recognizing that this is only one argument that must enter an adjustment cost function.

IV. EFFECT OF IMPORTS

If a country starts importing various goods, then the structure of transactions in the domestic economy will change. Assume that of each order it receives, sector i only produces the proportion d_i domestically and the rest is imported.¹ $0 < d_i \leq 1$. Define a diagonal matrix D such that $D_{ii} = d_i$, $D_{ij} = 0$ for $i \neq j$.

In the first round, f is ordered and Df is produced. In the second round, ADf is ordered and $DADf$ is produced, and so forth. The power series expansion now looks like:

$$x = \lim_{n \rightarrow \infty} [I + DA + (DA)^2 + \dots + (DA)^n] Df \quad (3)$$

which can be seen to be equal to:

$$x = [I - DA]^{-1} Df = [\bar{I} - \bar{A}]^{-1} \bar{f} \quad (4)$$

where $\bar{A} = DA$ and $\bar{f} = Df$.

One common way to deal with imports is to assume that each sector imports a constant proportion of total domestic production.² Define these import ratios as $\mu_i = M_i/X_i$. Define a diagonal matrix M $M_{ii} = \mu_i$, $M_{ij} = 0$ for $i \neq j$.

The basic input-output equation becomes:

$$f + Ax = x + m = [I + M]x$$

Manipulating the equation for x involving D yields another equation for $f + Ax$: (Note that D has an inverse since $d_i \neq 0$ for all i)

$$f + Ax = D^{-1} x.$$

Therefore:

$$D^{-1}x = [I + M]x$$

and since $x \neq 0$:

$$D = [I + M]^{-1}$$

To adjust for imports, first calculate D then adjust A and f by premultiplying them both by D . The fable is unchanged, except one substitutes \bar{A} and \bar{f} for A and f .³

¹ If imports are 100 per cent of demand ($d_i = 0$) they must be non-competitive imports and have no place in the A Matrix in the first place. They are in fact just like any non-produced input. For example there would not be a motor-car sector if no motor cars were produced domestically. It is true, of course, that such exclusion would make the domestic economy simpler.

² See Chenery and Clark (1959), pp. 22-25.

³ Note that the adjustment to the A matrix is to divide all elements in a row by one plus the import ratio: $A_{ij} = A_{ij}/(1 + \mu_i)$. Chenery and Clark (1959), p. 214, also use an adjusted A matrix, except that they divide all elements in the j 'th column by $1 + \mu_j$. Their purpose is to replace production by total supply ($x + m$) in the various equations.

V. MEASURES OF THE ADJUSTMENT PROCESS

We want to measure two characteristics of the adjustment process: (1) the number of rounds each sector, and the system, requires to adjust fully, and (2) the number of transactions required. One can form a matrix which summarizes the structure of transactions that occurs during the adjustment process. Define the Transactions Rounds Matrix, T , such that:

$$T_{ij} = \begin{cases} t_j & \text{if } A_{ij} \neq 0 \\ 0 & \text{if } A_{ij} = 0 \end{cases}$$

where t_j is the integer number of rounds in which sector j produces a positive output. Each element in the i 'th row of T tells how many times sector i makes deliveries to every other sector. Each element in the j 'th column tells how many times sector j places an order with every other sector (which must either be zero or the number of rounds in which it produces).

One can construct some interesting summary measures from T :

$$T^{*j} = \sum_{i=1}^n T_{ij}$$

$$T^{i*} = \sum_{j=1}^n T_{ij}$$

$$T^{**} = \sum_i \sum_j T_{ij}$$

$$S_1^j = T^{*j}/T^{**}$$

$$S_2^i = T^{i*}/T^{**}$$

T^{*j} is the total number of orders placed by sector j , and is thus a measure of the demand information generated by the sector during the adjustment process. T^{i*} is the total number of deliveries sector i makes to all other sectors during the process and is thus a measure of the demand information received by the sector. T^{**} is the total number of transactions and measures the total amount of information required by the system to reach equilibrium (and is thus perhaps a good measure of the complexity of the system). S_1^j and S_2^i are the sectoral shares of total orders and deliveries respectively.

One can devise many possible summary measures of the number of rounds for the entire system, but various weighted averages seem most interesting. Define three: (1) R_1 where each sector's number of rounds is weighted by its share of total orders placed, S_1^j ; (2) R_2 where each sector's number of rounds is weighted by its share of total deliveries made, S_2^i ; and (3) R_3 where each sector's number of rounds is weighted by its share of the value of total domestic production. The use of both R_1 and R_2 requires the view that the number, rather

than the value, of transactions is important (implicitly, that information flow has a fixed cost component). The last average reflects the value of transactions, but requires the use of prices to add up the output from different sectors.

The fable sets forth a quantity adjusting model without reference to prices so that the use of R_3 really requires additional assumptions to justify the use of value weights. One must specify a market economy or use calculated shadow prices. In the fable, a delivery is viewed as a passive response to an order and contains no information other than there is no production bottleneck (which has already been assumed). The model concentrates on the information *generating* characteristics of the economy. Thus the first measure, R_1 , would seem to be the theoretically most appropriate and interesting measure to use of the average number of rounds.

VI. EMPIRICAL RESULTS

The empirical results are presented in Tables I–VI.¹ The data for India are from Eckaus and Parikh (1968), Tables 3.2, 3.3, and 3.4. They refer to the year 1960–61. They adjusted the official Indian input-output table for various reasons, but unfortunately their adjusted table does not represent a consistent set of accounts—it simply does not add up. We have assumed the final demand and inter-sectoral flows are correct and have ignored their total domestic output figures. Their aggregated matrix (11x11) of coefficients was used without change. The data for Italy, Japan, and the United States are from Chenery and Clark (1959), Tables 8.6, 8.7 and 8.9. They refer to the years 1950, 1951, and 1947 respectively. The data for Korea are from the Bank of Korea, input-output section, and refer to the years 1960 and 1966. The Korean tables were aggregated from 117 sectors to almost the same 29 sectors as for the tables in Chenery and Clark. The only exception was sector 29, petroleum and gas, which did not exist in Korea, so a construction sector was used instead.

In Table I, a number of different aggregate measures are presented. For purposes of comparison, Chenery W, Chenery U, and the determinant of the (I–A) matrix are presented in addition to the measures discussed in the previous section. Tables II and III indicate the structure of production by rounds. Tables IV to VI give various sectoral measures for Korea, Italy, Japan, and the USA. In all the tables, the cut-off point for a sector to finish production in one more round was set at 0.98.

Varying the Fable

Setting the cut-off point at 0.98 is quite arbitrary. Experiments were done on some of the matrices, varying the cut-off point from 0.9 to 0.999. The total number of transactions and the average number of rounds changed quite a lot

¹ Some of the matrices are rather out-of-date. The Korean and Indian ones were, however, the most recent available at the time that the empirical work was done. In choosing the others, we were constrained to some extent by the problem of the comparability of sector definitions, and we chose those matrices that required the least adjustment as far as sector comparability was concerned.

(R_3 , of course, was unaffected), but the cross-country and inter-sectoral comparisons were virtually unaffected. If the data were available, it would be easy and interesting to specify different cut-off points for different sectors.

The fable assumes that all sectors have the same period of production, which is clearly invalid when one considers sectors such as agriculture or services. Varying production periods could easily be specified, but this was not done because there are no data with which to specify the sectoral differences. Some experiments were done: for example, the agricultural sector in the India (11x11) matrix was made to produce all its output (and thus place all its orders) in one round instead of its usual four rounds. This had no effect at all on any other sector. It seems likely that the results would be fairly insensitive to changes in the specification of production periods.

Aggregation

The effects of aggregation can be seen by comparing the results for the Indian eleven and thirty-two sector matrices in Table I. The larger system has a much greater total number of transactions, but only a slightly different average number of rounds. (R_1 and R_2 increase, R_3 hardly changes.) In Table II, there is virtually no difference in the distribution of total production by rounds for the two Indian matrices, this insensitivity of the speed of the convergence to aggregation is rather surprising, and is not reflected in all the other measures. The Chenery measures are nearly constant, but the percentage of non-zero elements doubles, and the determinant of (I–A) goes up by half.

Comparison Over Time

The effect of economic growth on the various measures can be seen by comparing the results for the two Korean matrices for 1960 and 1966. The various measures are quite similar for the two years. R_1 and R_2 are slightly lower when the adjustment is with its own import structure and slightly higher when no imports are allowed. The latter reflects the greater importance of imports to the Korean economy in 1966. The total number of transactions, T^{**} , increases somewhat reflecting the greater density of the later matrix.

The lack of change in the complexity characteristics of the Korean economy over time is interesting considering that the country grew quite rapidly during the period. The annual rate of growth of GNP went from 2.3 per cent in 1960 to 13.4 per cent in 1966 and averaged around 7 per cent. There was substantial structural change as well. One would expect that the more developed countries would exhibit less change over time.

Structure of Final Demand and of Imports

The effects of changes in the structure of final demand were examined by interchanging final demand vectors among the countries.¹ Examination of Table I indicates that the differences are not very great. Changing the structure

¹ The U.S. final demand had to be adapted for the India matrices because the sector definitions varied. This was done by a combination of interpolation and aggregation.

of imports in general had a more profound effect. When the countries were forced to produce everything domestically (autarky), the total number of transactions increased, as did all the measures of the average number of rounds.¹ For India, Italy, Japan, and Korea the effect was far greater than that due to changes in final demand. The averages R_1 and R_2 increased by 0.4–0.6 and R_3 by almost 0.1. The US was much less affected because it is the least dependent on imports. The ratios of the value of all imports to total domestic product are 5.4, 6.4, 5.6, and 1.7 per cent for India, Italy, Japan and the US respectively. For Korea, the ratios are 17.3 and 22.6 per cent for 1960 and 1966 respectively.

From Table II, it can be seen that variations in final demand affect production in the first round, but that by round two the differences are almost completely eliminated (except for Korea).² On the other hand, forcing autarky on the various economies causes a significant decrease in the share produced in each round through to the third round. This is reflected by the increases in R_3 .

Inter-country Comparisons

For all the number of rounds measures, there is a clear progression from the less to the more developed countries. Korea seems closer to Italy than to India, which is no real surprise. India seems to adjust about one round faster than Korea and Italy (R_1 or R_2) which in turn adjust about 0.5–1.0 round faster than the United States and Japan. India requires half as many transactions as Italy which requires about three-fourths as many as Korea, Japan, and the USA. Tables II and III indicate that the progression from less to more developed countries also holds when one examines the adjustment process round by round.

By R_1 and R_2 Japan requires about half a round more in the adjustment process than the US. They both require almost the same total number of transactions because the United States has a larger percentage of non-zero elements in its matrix. Thus Japan has a technology with fewer direct links, but it uses them more intensely than the United States.³ Korea has a very high share of non-zero elements and so has a total number of transactions comparable to Japan and the USA even though R_1 and R_2 are significantly lower.

Sectoral Comparisons

The sectoral shares of total orders (S_1) and total deliveries (S_2) are defined above. These two measures can be used to define the 'degree of linkage' of various sectors into the economic system. Sectors which are high both in S_1 and S_2 are clearly strongly linked into the system and, conversely, sectors low in both measures are weakly linked. Table IV gives the number of rounds each sector goes and also a measure of the degree of linkage of each sector. The linkage

¹ We conjecture that using DA and Df instead of A and f (where $0 < D_{ii} \leq 1$) cannot increase the number of rounds any sector—and thus the system—requires.

² This would explain the erratic behaviour of the Chenery W measure. Because we assume that final demand is produced in the first round, the Chenery W measure must equal one minus the share of total output produced in the first round.

³ One colleague suggested that the reason for this is obvious: Japan is a small country with shorter lines of communication!

measure indicates whether a sector is above (+) or below (–) the median for S_1^1 (first) or S_2 (second). Thus +– indicates a sector high in S_1 and low in S_2 .

It is interesting to compare the sectoral classifications in Table IV with those in Chenery and Clark (1959), Table 8.2, where the sectors are classified according to U and W. Table V lists those sectors for which there was substantial similarity in degree of linkage among the countries. For only five sectors was there complete agreement and for only sixteen was there agreement in three of the four countries. This contrasts with the substantial agreement found by Chenery and Clark for their classification by U and V.¹

It is also interesting to examine the sectors in which there is substantial disagreement in degree of linkage among the countries. Sectors such as processed food and other industry have a higher degree of linkage in Korea and Italy than in Japan and the US, while sectors such as transport, coal mining, coal products, agriculture, and electric power are more weakly linked in Korea. Agriculture is clearly a special case, but it is not surprising that sectors that are closely related to industrialization have a higher degree of linkage in the more developed of the countries.

Table VI gives Spearman rank correlations for various sectoral measures among and within the countries. It is clear from the table that sectoral rankings by S_1 , S_2 , and Chenery-W are quite different within the countries. There are also substantial differences between countries in the rankings by S_1 and S_2 , although in general, the US and Japan are more similar to one another than to the other countries.

VII. SUMMARY AND CONCLUSIONS

The empirical work discussed above indicates that the fable from which the various complexity measures arise is fairly robust. The measures do not seem highly sensitive to reasonable variations in the fable, although it would certainly be interesting to extend the fable to include things such as different cut-off points, and periods of production among sectors, and production bottlenecks. The measures were affected by changes in the structure of final demand and of imports. We were surprised to find that dramatic changes in final demand had relatively small effect, certainly less than forcing autarky on the countries. This result indicates that the measures are structural in that they seem to reflect properties of the input-output matrix itself instead of the structure of production.²

In general, the variations between countries were larger than the differences in the measures under different assumptions for the same country. It did not greatly matter for the inter-country comparison whether or not import structure or final demand were held constant, but in general it seemed better to use each country's measures with its own structure. The empirical work revealed a clear progression from the less to the more developed countries in the number of

¹ Chenery and Clark (1959), pp. 205–211. But see Kleiman (1970).

² It seems that the reason the measures are more sensitive to changes in imports is that the adjustment for imports is to change the elements of the A matrix ($\bar{A} = DA$). Of course, extreme changes in final demand (such as setting most of the vector to zero) must have a profound effect.

rounds and (except Korea) in the number of transactions required to adjust to changes in demand.

Earlier, we discussed the nature of adjustment costs in an economy and argued that they would be higher for more complex economies. During the process of development, countries have developed quite a sophisticated transactions technology (transportation and communications networks and various services) in response to the higher costs in a system becoming steadily more complex. It is often suggested that less developed countries should acquire the transactions technology of a developed country, but the results here suggest that this certainly need not be the case. Given that their systems are much less complex, then advanced communications, transportation, or financial systems are inefficient and unnecessary.

One empirical property of the development process that has often been observed is that it involves very rapid changes in the structure of production. It was argued earlier that the larger are such changes, then the larger are the costs of adjustment. It would seem, then, that to give India the Japanese input-output matrix would be a mixed blessing since India must undergo a period of rapid structural change that might well be far less costly with its own less complex matrix. We are implicitly assuming that the costs of adjustment to a given shock are an increasing function of complexity, independently of the technology of the transportation and communications network (which is imbedded in the input-output matrix we actually observe).

At first glance, the last two paragraphs seem very pessimistic—a less developed country should want neither the transacting technology nor the production technology of an advanced country. What we are really arguing is that increased complexity involves a new dimension of costs that should be considered when viewing an economy undergoing economic development. It is probably best to assume that any economy has evolved a transactions system consistent with its degree of complexity and rate of structural change. Any changes in the technology or in the rate of growth of the economy should not be considered independently of the economy's complexity, and furthermore there is no obvious reason to assume that the 'transactions sector' should (or should not be) a leading sector in the development process. Sorting out the interactions is certainly beyond the scope of this paper.¹ It is enough to point out the need for considering adjustment costs and to measure some of the factors that should enter any adjustment cost function.

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¹ Some thoughts on how one might include such interactions in a model of development are presented in Robinson (1972).

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TABLE I

Country	Aggregate Measures of Complexity and Adjustment		Average No. of Rounds			No. of Transactions, T^{**}	Chenery		Determinant of I-A
	Final Demand	Import Structure	R_1	R_2	R_3		W	U	
			India (11x11)	India (32x32)	Korea 1960				
India (11x11)	India	Autarky	4.86	5.06	1.36	323	0.259	0.259	0.467
India (11x11)	India	India	4.48	4.57	1.29	295	0.235	0.248	0.558
India (11x11)	USA	India	4.41	4.44	1.36	288	0.280	0.305	0.558
India (11x11)	USA	Autarky	5.05	5.22	1.47	338	0.328	0.328	0.467
India (32x32)	India	Autarky	5.01	5.46	1.38	1292	0.269	0.269	0.221
India (32x32)	India	India	4.54	4.95	1.30	1196	0.245	0.259	0.318
India (32x32)	USA	India	4.55	4.73	1.34	1216	0.270	0.293	0.318
Korea, 1960	Korea	Autarky	6.03	5.89	1.60	3685	0.365	0.368	0.087
Korea, 1960	Korea	Korea	5.70	5.54	1.53	3492	0.347	0.364	0.114
Korea, 1960	Japan	Korea	5.49	5.48	1.60	3348	0.391	0.415	0.114
Korea, 1960	USA	Korea	5.72	5.59	1.59	3484	0.377	0.403	0.114
Korea, 1960	Italy	Korea	5.68	5.64	1.60	3444	0.391	0.413	0.114
Korea, 1966	Korea	Autarky	6.18	6.04	1.71	4123	0.411	0.411	0.068
Korea, 1966	Korea	Korea	5.50	5.36	1.56	3670	0.370	0.398	0.120
Korea, 1966	Japan	Korea	5.54	5.47	1.58	3684	0.373	0.414	0.120
Korea, 1966	USA	Korea	5.57	5.37	1.59	3702	0.375	0.412	0.120
Korea, 1966	Italy	Korea	5.68	5.56	1.61	3734	0.394	0.426	0.120
Italy	Italy	Autarky	6.36	6.45	1.76	2463	0.340	0.430	0.025
Italy	Italy	Italy	5.90	6.06	1.65	2322	0.405	0.430	0.036
Italy	Japan	Italy	5.73	6.00	1.67	2278	0.417	0.444	0.036
Italy	USA	Italy	5.76	5.81	1.59	2281	0.366	0.391	0.036
Japan	Japan	Autarky	7.21	7.15	1.93	3704	0.478	0.478	0.016
Japan	Japan	Japan	6.80	6.72	1.85	3457	0.461	0.487	0.020
Japan	Italy	Japan	6.91	6.77	1.80	3480	0.435	0.460	0.020
Japan	USA	Japan	6.92	6.73	1.85	3512	0.442	0.461	0.020
USA	USA	Autarky	6.46	6.39	1.78	3609	0.429	0.429	0.013
USA	USA	USA	6.39	6.30	1.74	3565	0.420	0.427	0.015
USA	Italy	USA	6.67	6.61	1.83	3666	0.462	0.472	0.015
USA	Japan	USA	6.47	6.45	1.80	3593	0.449	0.458	0.015
Share of zero Elements of A Matrix			India (11x11)	India (32x32)	Korea 1960	Korea 1966	Italy	Japan	USA
			62.8%	28.2%	74.9%	81.8%	53.4%	65.2%	68.3%

R_1 = weighted by share of orders placed.

R_2 = weighted by share of deliveries made.

R_3 = weighted by share of value of total domestic production.

TABLE II

	Cumulative Percentage of Total Domestic Output Produced each Round						Average No. Rounds ^a
	1	2	3	4	5	6	
Own Final Demand							
Own Import Structure							
India (11x11)	76.4	94.6	98.8	99.8	100.0	100.0	1.30
India (32x32)	77.2	94.9	98.9	99.8	100.0	100.0	1.29
Korea, 1960	66.1	87.4	95.4	98.4	99.7	100.0	1.63
Korea, 1966	64.0	86.9	95.3	98.4	99.8	100.0	1.56
Italy	60.8	84.4	93.8	97.5	99.0	99.7	1.65
Japan	55.1	79.4	89.7	94.8	97.5	99.0	1.85
USA	58.2	81.6	91.8	96.4	98.6	99.6	1.74
Own Final Demand							
No Imports							
India (11x11)	74.1	92.9	97.9	99.4	99.8	100.0	1.36
India (32x32)	73.1	92.4	97.8	99.4	99.8	100.0	1.38
Korea, 1960	63.7	85.4	94.2	97.8	99.3	99.9	1.60
Korea, 1966	58.9	82.5	92.5	96.8	98.8	99.7	1.71
Italy	57.0	81.3	91.9	96.5	98.4	99.4	1.76
Japan	52.2	77.4	88.5	94.0	96.9	98.7	1.93
USA	57.1	80.6	91.1	96.0	98.3	99.5	1.78
USA FINAL Demand							
Own Import Structure							
India (11x11)	72.5	93.6	98.6	99.7	100.0	100.0	1.36
India (32x32)	73.9	93.9	98.7	99.7	100.0	100.0	1.34
Korea, 1960	62.7	85.7	94.6	98.1	99.5	100.0	1.59
Korea, 1966	62.7	85.9	94.8	98.2	99.6	100.0	1.59
Italy	64.9	85.5	93.9	97.6	99.0	99.7	1.59
Japan	56.7	78.9	89.4	94.5	97.4	98.7	1.85
USA	58.2	81.6	91.8	96.4	98.6	96.6	1.74

^a Weighted by share produced in each round.

TABLE III

Number of Sectors Completing more than 90 per cent of their Output in each Round

	1	2	3	4	5	6
Own Final Demand						
Own Import Structure						
India (32x32)	9	6	11	5	1	0
Korea, 1960	1	6	16	6	0	0
Korea, 1966	2	8	12	7	0	0
Italy	5	9	4	7	4	0
Japan	1	7	10	5	3	3
USA	0	6	10	9	4	0
Own Final Demand						
Autarky						
India (32x32)	8	6	7	10	1	0
Korea, 1960	1	5	15	7	1	0
Korea, 1966	1	3	16	8	1	0
Italy	4	5	9	5	5	1
Japan	1	7	7	6	4	4
USA	0	6	10	9	4	0
USA Final Demand						
Own Import Structure						
India (32x32)	7	6	17	2	0	0
Korea, 1960	1	7	11	10	0	0
Korea, 1966	1	6	14	8	0	0
Italy	5	7	6	10	1	0
Japan	0	8	8	7	3	3
USA	0	6	10	9	4	0

TABLE IV

Sector	Sectoral Measures No. of Rounds Gone				Degree of Linkage			
	Korea	Italy	Japan	USA	Korea	Italy	Japan	USA
1 Apparel	2	3	4	4	--	--	--	--
2 Shipbuilding	4	1	3	5	--	--	--	--
3 Leather and Prod.	5	5	6	5	--	+--	--	--
4 Processed Food	5	4	4	5	++	--	--	+--
5 Fishing	5	4	5	6	--	--	--	--
6 Grain Products	6	5	4	7	+--	--	--	+--
7 Transport	5	5	6	6	--	+--	++	++
8 Other Industry	5	4	4	5	++	++	--	--
9 Transport Equip.	5	3	5	5	--	--	--	+--
10 Rubber Products	5	5	7	6	--	+--	+--	++
11 Textiles	6	7	6	6	++	++	+--	++
12 Machinery	5	4	5	5	++	+--	+--	+--
13 Iron & Steel	7	8	9	7	++	+--	++	++
14 Non-Met. Min. Prod.	6	5	6	6	++	++	++	+--
15 Wood & Products	5	5	6	6	+--	+--	+--	++
16 Chemicals	6	7	7	7	++	++	++	++
17 Printing & Pub.	5	1	6	6	--	--	--	--
18 Agriculture	5	6	6	7	--	+--	+--	+--
19 Non-Met. Minerals	6	7	7	7	+--	+--	+--	+--
20 Petrol. Products	6	7	7	7	++	++	++	+--
21 Non-ferrous Metals	6	7	8	8	++	+--	++	+--
22 Metal Mining	6	8	10	8	+--	--	+--	--
23 Coal Products	5	7	10	7	--	++	+--	+--
24 Trade	5	4	5	5	--	+--	+--	+--
25 Paper & Products	7	6	8	8	+--	+--	+--	++
26 Electric Power	6	7	8	7	+--	+--	++	++
27 Coal Mining	6	8	9	7	--	++	++	+--
28 Services	5	6	6	6	+--	++	++	++

Notes:

- ++: Above median for share of total orders (S₁), above median for share of total Deliveries (S₂).
- + -: Above median for share of total orders (S₁), below median for share of total Deliveries (S₂).
- (S₂) - -: Below median for share of total orders (S₁), above median for share of total Deliveries (S₂).
- (S₂) -- -: Below median for share of total orders (S₁), below median for share of total Deliveries (S₂).

Each country has its own final demand and import structure. Sector 29 is omitted from the analysis because the sector definition was not compatible between Korea and the other countries.

TABLE V

Sectors whose Degree of Linkage is Similar among the Countries		
Strongly Linked (++)	Medium Linked (+-, -+)	Weakly Linked (---)
Textiles	Rubber Products	*Apparel
Iron and Steel	Non-metallic -Minerals	*Shipbuilding
Non-metallic Mineral Products	*Trade	Leather and Products
*Chemicals	Paper and Products	*Fishing
Petroleum Products		Transport Equipment
Services		Printing and Publishing

Source: Table IV.

Notes: 'Similarity' is defined as agreement in degree of linkage in at least three of the four countries. Those sectors marked with an * had the same degree of linkage in all four countries.

TABLE VI

	Spearman Rank Order Correlations				Between Selected Variables in each Country		
	Between Countries: for S ₁ above Diagonal for S ₂ below						
	Korea	Italy	Japan	USA	S ₁ -S ₂	W-S ₁	W-S ₂
Korea	—	.62	.52	.57	.28	.61	.29
Italy	.51	—	.52	.45	2.0	.51	.33
Japan	.64	.73	—	.63	.39	.79	.34
USA	.69	.68	.86	—	.40	.46	-.04