MACROECONOMIC POLICY UNDER CURRENCY INCONVERTIBILITY *

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Introduction

Currency inconvertibility is to be understood relative to particular items in the balance of recorded international transactions. Thus, restricting the ability to convert the domestic currency into foreign exchange may involve a restriction on international trade in goods. This would tend to promote factor mobility and trade in assets. Conversely, restricting convertibility for capital account transactions would tend to promote trade in goods.¹

The development of private international financial intermediation has led to an unprecedented growth of trade in assets taking place not only among industrialized countries but also spreading to semi-industrialized and even to some less developed countries, whose currencies are typically inconvertible. Exchange controls notwithstanding, channels have been created for individuals and organizations to build up foreign exchange balances, so that they can diversify their portfolios across assets denominated in different currencies.

The desire for currency diversification is consistent with the observed development of "black" market for foreign exchange in many countries with inconvertible currencies. Moreover, since the definition of convertibility in the Articles of Agreement of the International Monetary Fund only refers to current account transactions, "black" markets for foreign exchange exist even when the domestic currency is "convertible" by Fund standards.²

Analytically, portfolio diversification implies that domestic and foreign assets are not perfect substitutes and therefore that a conventional monetary approach to the inconvertible exchange rate would be particularly strained. The portfolio approach to the exchange rate, on the other hand, assumes a stock demand for foreign, as well as domestic currency. It can thus explain currency diversification and account for the macroeconomic implications of currency inconvertibility.
Section I of the paper contains a dynamic partial equilibrium model of the black market for foreign exchange using the portfolio view. Different mechanisms of expectations formation are contrasted, but emphasis is placed on the perfect foresight case.

The analysis in Section I ignores the special risks involved in illegal transactions. This neglect is justified in Section II by showing that the premium of the black market rate over the official rate is likely to be positively related to the probability of success in smuggling, along the lines of the "Beccaria formula".

Then, using a simple general equilibrium model, the black market for foreign exchange is analyzed in Section III, under the assumption of "stabilizing", or regressive, expectations about returns to domestic and foreign currencies. The portfolio view is contrasted with a monetary approach to the black market. The implications of perfect foresight are also taken up.

I. A partial equilibrium model

Consider a small country whose residents are not authorized to hold foreign assets, but who nevertheless allocate their financial wealth between domestic and foreign currency. The terms of trade between legal ("official") and illegal ("smuggled") imports and exports are given in the world market. Alternatively, the same good is traded through both channels, as in the Section II, and there is a non-traded good, as in Section III.

If foreigners do not hold the inconvertible domestic currency in their portfolios, the domestic currency price of foreign currency in the black market will be such that the existing stock is willingly held. Furthermore, the only way for domestic residents to acquire foreign currency is through the over invoicing of imports. Under a fixed official exchange rate and in the
absence of endogenous reported capital flows and errors and omissions, as well as interest payments on unrecorded capital flows, the two balances will give the change in the stock of foreign currency of the private sector and the central bank respectively.

As mentioned, the portfolio approach introduces the foreign exchange market in the determination of a perfectly floating exchange rate by recognizing that domestic and foreign assets are not perfect substitutes. The assumption of perfect substitutability is of course particularly strained if the domestic currency is inconvertible. The existence of a private stock demand for black market foreign exchange, as well as for domestic currency, is therefore easy to accept.

If the black market rate is determined so that the private stock of black market foreign exchange is willingly held, and demand α is proportional to wealth for simplicity, we have:

\[ e_F = \alpha (\psi) W \quad \alpha' > 0 \]

where \( e \) is the black market exchange rate

\( F \) is the stock of black market foreign exchange (in foreign currency)

\( W \) is private financial wealth (in domestic currency)

and \( \psi \) is the expected rate of change of the black market rate.

Given an official exchange rate, \( \hat{e} \), the requirement of portfolio balance can be written in terms of the black market premium and the financial wealth valued at the official rate:
(1') \quad pF = \alpha(\psi)\dot{\hat{W}}

where \quad p = e/\hat{e} \quad is the black market premium

and \quad \dot{\hat{W}} = \hat{W}/\hat{e} \quad is financial wealth valued at the official exchange rate.

The change in \( F \) is given by smuggling and other unreported current account transactions. The unreported trade surplus, \( B \), is zero for a particular value of the premium, taken to be one, and, if the elasticities condition holds, improves with an increase in the premium.\(^4\) Denoting rates of change by dots:

\[(2) \quad \dot{F} = B(p) \quad B(1) = 0 \quad B' > 0\]

Assuming that the opposite holds for the reported current account, \( \dot{R}(p) \), we show temporary equilibrium in Figure 1, a variant of the Kouri (1983) diagram. The right panel shows the instantaneous determination of the black market exchange rate, given demand for foreign currency and no expected change in relative returns, along a rectangular hyperbola PP. Under perfect foresight, demand for foreign currency will increase when the black market rate depreciates, constraining its initial value for a given stock of foreign currency \( F_o \) to \( e'_o \), on the saddle path \( S'_o \) (below the corresponding initial value under static expectations, \( e_o \)). For a given official exchange rate, \( \overline{e}_o \) (< \( e_o \) so that \( p_o > 1 \)), the left panel of Figure 1 plots the unreported trade surplus in foreign currency, \( \dot{\hat{B}} \) (which equals to the increase in \( F \)), the exogenous reported trade deficit, \( \dot{\hat{B}} \), and their sum, \( \ddot{B} \). Assuming a stronger effect of the black market rate on \( B \), the relative slopes will be as drawn. For simplicity, assume that the unreported trade balance equilibrium associated with the given official exchange rate is at \( e^* < e_o \).
Figure 1

Partial equilibrium analysis

of the black market rate
If the associated reported deficit $\hat{E}^*$ is corrected by a devaluation of the official exchange rate to its long-run equilibrium level, $\hat{e}_1$, the two schedules will shift up and intersect on the vertical axis at a point where, by choice of units, $e_1 = \hat{e}_1$ so that $p_1 = 1$. The implications of the official devaluation differ depending on the expectations formation mechanism. If expectations are static, the black market exchange rate will gradually depreciate from $e_0$ to $e_1$ as the stock of foreign assets decreases from $F_0$ to $F_1$. If expectations are "rational", the black market rate jumps from $e_0$ to $e_1$ and then gradually depreciates to the same long-run value. If the official devaluation generates an increase in the demand for foreign assets from PP to $P_2P_2'$, furthermore, the black market rate will overshoot to $e_2(e_2$ under perfect foresight) and gradually appreciate as the stock of foreign assets increases to $F_2^*$.

This analysis remains applicable when the official exchange rate, rather than being fixed, is set by the central bank according to Kouri's "acceleration hypothesis", so that it depreciates at a rate given by the rate of the reported trade balance over the stock of foreign assets of the central bank. But before we analyze the effect of an exogenously given rate of crawl and track the monetary consequences of the reported trade imbalances, we will turn to an overview of the special risks involved in black market trading.

II. The Beccaria formula revisited

The formal analysis on black markets can be traced back to the mid-eighteenth century, when - as Hume was writing on the specie-flow mechanism - Beccaria (1754-65) developed a break-even condition for smuggling activity, involving the probability of success and the tariff rate on imports.

\[(3) \quad zt = 1\]
where $z$ is the probability of success in smuggling
and $t$ is one plus the tariff rate.

A higher degree of enforcement of tariff regulations - to the extent
that it implies a lower $z$ - is associated with negative profits from smuggling
unless a higher tariff rate raises the domestic price and thus restores pro-
fitability. The objective of the analysis was precisely directing the atten-
tion of policy-makers to the trade-off between the revenue from higher
tariffs and the associated higher enforcement costs.  

The literature picked up about ten years ago. Bhagwati and Hansen (1973)
who refer to Beccaria's work - showed that tariffs do induce smuggling but,
together with Sheikh (1974), they claimed that a black market for foreign exchange
would not be induced by trade restrictions along. Sheikh (1976) also adapted
the traditional analysis of black markets by Boulding (1947), Brofenbrenner
(1947) and Michaely (1954) to the foreign exchange market.  

By introducing real costs to smuggling in the form of a "smuggling
function", Pitt (1981) and (1982) showed that legal and illegal trade in
goods and money coexist even in the absence of exchange restriction. If some
legal trade is necessary in order to engage in smuggling, the domestic price
may be less than the cost of legal imports, a phenomenon which he calls "price
disparity". Along the same lines, Martin and Panagaryia (1983) propose a
crime-theoretic approach to smuggling, which provides microeconomic foundations
to Pitt's smuggling function but they neglect the black market for foreign
exchange. Noting that this revival of interest in the supervision aspect would
have pleased Beccaria, a criminologist by training, we briefly show how the
Pitt-Martin-Panagaryia (PMP) analysis is consistent with his formula, modified
to allow for a black market for foreign exchange.
Consider the profit $\rho$ in domestic currency of a price-taking importer depending on whether smuggling is successful ($\rho_z$) or not ($\rho_{1-z}$). Setting the foreign currency price at unity, we get profits as a linear function of the amounts imported through legal and illegal channels:

\begin{align*}
(4) \quad \rho_z &= P(L + S) - \hat{\epsilon}tL - eS \\
(5) \quad \rho_{1-z} &= PL - \hat{\epsilon}tL - eS
\end{align*}

where $P$ is the domestic price

$L$ is the amount legally imported

$S$ is the amount smuggled

If the probability of success, $z$, is exogenous to the firm, and if there are no real costs of smuggling, expected profits will also be linear in the decision variables, $S$ and $L$, since they are a $z$-weighted average of (4) and (5):

\begin{equation}
(6) \quad \rho = z \rho_z + (1 - z) \rho_{1-z}
\end{equation}

Under perfect competition, profits will be zero at the optimum, which is independent of demand conditions. For the choice of the firm to determine the domestic price uniquely we will have the equivalent of Beccaria's formula when there is a black market for foreign exchange. Setting the partials of $\rho$ with respect to the decision variables to zero and solving we get:

\begin{equation}
(7) \quad zt = p
\end{equation}

Unless there is a unit probability of success, therefore, $t > p$ so that the premium is always less than the tariff. Put in another way, given the probability of success in smuggling, an increase in the premium lowers profits,
unless there is an increase in the tariff. The Beccaria trade-off is now between lowering \( z \) and lowering \( p/t \), the ratio of the premium to the tariff. This may involve a change in the tariff or in the official exchange rate.

On the other hand, a new trade-off arose between profits from the smuggling activity and profits in the black market for foreign exchange. Higher supervision of smuggling is now associated with a lower black market rate.

The basic idea behind the crime-theoretic approach is, however, to make the probability of success depend on the decision variables of the firm. If it is a decreasing function of the ratio of smuggled to legal imports, the first-order conditions for profit maximization can be solved for the optimal combination of price and import pattern, given the tariff and the premium:

\[
(8) \quad z = z(s); \quad \xi = -sz'/z
\]

where \( s = \frac{S}{z} \)

By adding a stock demand for black foreign exchange which is a positive function of the probability of success in smuggling, it is easy to incorporate the basic insight of the portfolio approach: Setting \( \psi = 0 \) in (1)', we get:

\[
(1'') \quad pF = \alpha(z)\hat{\gamma}; \quad \gamma = za'/\alpha
\]

Using (1'') and (8) in (6), we get from the first-order conditions:

\[
(9) \quad \hat{\nu}'(1 - z\xi s) = t + \gamma\xi sp
\]

\[
(10) \quad z\hat{\nu}'(1 - \xi) = p(1 - \gamma\xi)
\]

where \( \hat{\nu} = p/e' \)
Eliminating $\hat{y}$ between (9) and (10) we obtain what may be called the PMP version of the Beccaria formula:

(11) $zt = pA$

where $A = \frac{1-\xi[\gamma-zs(1-\gamma)]}{1-\xi}$

Clearly if $A = 1$, we get (7). This will be the case if $\xi = 0$ or if $\gamma = 1$. In the elastic case ($\gamma < 1$), the premium is less than the tariff weighted by the probability of success in smuggling, whereas if $\gamma > 1$, then $A < 1$ and $p > zt$ at the break-even point. In that case, if $\xi < 1$ an increase in $z$ will lower $A$ so that $iz$ will still be associated with an increase in $p$. If $\gamma < 1$ and $\xi < 1$, however, the opposite result may obtain.

The optimum combination of $\hat{p}$ and $s$ can be obtained by solving (9) and (10), which determine the optimum legal and illegal imports respectively. Using a diagram from Martin and Panagaryia (1983), the solution is presented graphically in Figure 2. If $\gamma < 1$, it can be shown that the locus of (zero profit from) legal imports (LL) is downward sloping and the locus of illegal imports (SS) is upward sloping, as drawn. In that case, an increase in the tariff raises $\hat{p}$ and $S$. A decrease in $z$ or in $F$ raises $\hat{p}$ but lowers $s$. 
Figure 2

The PMP model of smuggling

(inelastic case)
III. General equilibrium

While some basic features of the black market for foreign exchange can be derived in partial equilibrium models, as done in the previous sections, the interaction between the private and official valuation of foreign exchange, on the one hand, and the role of the black market premium as a relative price, on the other, call for a general equilibrium approach. In the model presented below, two relative prices are endogenously determined, the black market premium and the relative price of non-traded goods.

1. The Model

To sharpen the analysis, we will neglect the role of the black market premium emphasized in the previous section, namely determining whether a transaction will go through the official or the black foreign exchange market. Here there are exports and imports which either are always traded through the official market, or always smuggled. Their prices in foreign currency are, given by the small country assumption. It would, of course, be possible to reinterpret the results in terms of the Beccaria analysis by allowing for a tariff on the official import.

The portfolio balance condition is now written as:

\[
\hat{p}_F = c(\psi)\hat{W} = \frac{\alpha}{1 - \alpha} \hat{M} = h\hat{\hat{M}}
\]  

where \(\hat{\hat{M}} = M/\hat{e}\) is the domestic money stock valued at the official rate

\(h = eF/M\) is the currency substitution ratio

and \(\hat{\hat{W}} = \hat{\hat{M}} + pF\)
The assumption of regressive expectations implies, in turn, that:

(13) \( \psi = \psi(p) ; \dot{\psi} = -\psi \frac{p'}{\psi} \)

The measure of relative real return is the expected change in the black market rate because domestic residents are assumed to be identical in their consumption preferences. 9

If the money multiplier is given, changes in money can be derived from the balance sheet of the central bank, according to which (high-powered) money is made up of domestic credit and foreign exchange reserves, valued at the official exchange rate:

(14) \( M = C + e_F G \)

where \( M \) is the money stock

\( C \) is the domestic credit

and \( F^G \) is reserves in foreign currency

Since there is no explicit consideration of the government budget constraint, there is only one endogenous source of money creation, the change in reserves. As usually assumed, capital gains on the foreign currency value of reserves go into central bank net-worth forever, so that, denoting proportional changes by hats, the accumulation of real money balances can be written as:10

(15) \( \hat{M} = \delta + \frac{\hat{F}^G}{\hat{M}} (\hat{F}^G / \hat{M}) \)

where \( \delta = \frac{C}{M} - \hat{e} \)
We now define excess-supply functions for the officially traded good, the smuggled good and the non-traded good as functions of prices and wealth and express them in terms of the official exchange rate.\textsuperscript{11} Note that the neglect of the effect of relative asset returns on the demand for goods implies that an increase in $\psi$ increases $M/W$ and decreases $eF/w$ by the same amount, $\alpha_\psi$. We denote by $\varepsilon = \alpha_\psi \psi/\alpha$ the corresponding elasticity.

We also neglect the cross-price effect in the market for non-traded goods, which is always in equilibrium, so that excess supply is given by:

\begin{equation}
N(q, \overline{w}) = 0
\end{equation}

where $q = P_N/e$ \\
and $P_N$ is the price of the non-traded good.

Given asset stocks (and the terms of trade), (12) and (16) determine the relative prices $p$ and $q$ of the smuggled and non-traded good in terms of the officially traded good. To the extent that the official exchange rate is fixed, or that its rate of change is determined by the monetary authorities, furthermore, temporary equilibrium involves the adjustment of $e$ and $P_N$ to changes therein, so that $p$ and $q$ are constant.

Now excess supply for traded goods equal the accumulation of foreign currency by the central bank and the private sector through the reported and unreported trade balances respectively. Neglecting reported capital flows:

\begin{equation}
\dot{F} = B(p, \overline{q}, \overline{w})
\end{equation}

\begin{equation}
\dot{M} = B(p, q, \overline{w})
\end{equation}
In steady-state equilibrium, the excess-supply of traded goods is zero and asset stocks are constant.

2. A Comparison of stock and flow specifications

If $\delta \neq 0$ but the excess supply of traded goods is zero, we obtain from log differentiation of (12) and (16) the solution for the change in the relative prices as a function of $\delta$:

$$\hat{q} = \frac{\omega_N}{\nu} \delta$$  \hspace{1cm} (19)

$$\hat{p} = \frac{1}{1 + (\xi/1 - \alpha)} \delta$$ \hspace{1cm} (20)

where $\omega_N = -\frac{\partial \hat{N}}{\partial \hat{N}}$ is the positive semi-elasticity of the excess supply of non-traded goods with respect of real wealth; $\nu = -q\hat{N}/\hat{q}$ is the positive semi-elasticity of the excess supply of non-traded goods with respect to the relative price of non-traded goods, and $\xi = \xi \psi$ is the positive elasticity of $\alpha$ with respect to the premium.

We can contrast (19) and (20) with the equations implied by a flow specification of the black market rate. The basic reference is Blejer (1978a), who simplified his work on the monetary approach to the Mexican balance of payments (1977) to derive a model of the black market rate which collapses into the usual monetarist model of exchange rate determination when the real official exchange rate is fixed by a purchasing-power-parity reaction function.  \hspace{1cm} 12 We will ignore this last aspect in the analysis to follow.
Real money demand, the money multipliers and world inflation play no essential role in Blejer's model. They will be taken as constant and set to one by choice of units. Also, instead of using as deflator of nominal money balances a weighted average of the prices of traded and non-traded goods, the official exchange rate is again taken as the numeraire.

Arguing that excess demand for non-traded goods is a given fraction, \( \tau \), of excess demand in the goods market, and therefore of the ex-ante excess supply of money, Blejer posits that changes in the relative price of non-traded goods are proportional to \( \delta \):

\[(21) \quad \hat{q} = \tau \delta\]

By interpreting \( \tau \) as \( \omega N / \nu \), (21) is of course the same as (19). The difference comes from the specification of the black market for foreign exchange in flow terms. Supply per unit of time, \( S^B \), is a positive function of the black market premium:

\[(22) \quad \log S^B = c_{11} + c_{12} \log p\]

Demand per unit of time, \( D^B \), is a positive function of the expected rate of change of the black market rate (denoted above by \( 4 \)) and a negative function of expected inflation, as proxies for the real return to holding black market foreign exchange and holding the non-traded good. Since own and cross effects are the same, and expectations are assumed to be regressive, we get:

\[(23) \quad \log D^B = c_{21} - c_{22} \log \xi\]

where \( \xi = e / p_N \).
Note that (23) is not consistent with domestic residents having the same consumption preferences, as assumed in (12), where \( \psi \), rather measures than the expected change in \( z \), measures the real return differential.

Log differentiating (22) and (23) and equating yields a link between the two relative prices \( p \) and \( q \), represented in Figure 3 by point E:

\[
\hat{p} = c\hat{q}
\]

(24)

where \( c = c_{22}/(c_{12} + c_{22}) \)

Substituting (24) into (21) we obtain, instead of (20):

\[
\hat{p} = c\tau \delta
\]

(25)

According to (25), the effect on the premium is dampened by \( c\tau < 1 \). Clearly, if \( q \) is not responsive to \( \delta \) and \( \tau = 0 \), the black market premium is constant. The same happens when the flow supply of foreign exchange does not respond to the premium — say because expectations are static — and \( c = 0 \).

However in (20), if \( c = 0 \) we get \( \hat{p} = \delta = \hat{q}/\tau \), so that the premium moves by more than the relative price of non-traded goods, as seems plausible.

The equilibrium is now at \( E' \) in Figure 3, and the flow model understates the premium by \( E'E \).
Figure 3
Stock and Flow
Equilibrium Compared

\[ \delta \]
\[ \frac{1}{1+\varepsilon} \]
\[ c\tau \delta \]

\[ p \]
\[ q \]

E' and E represent the points of equilibrium.
We now analyze, in the case $\psi = 0$, the temporary equilibrium of the system, where excess supply for traded goods is non-zero, and steady-state equilibrium, where excess supply of traded goods is zero and, if $\delta = 0$, asset stocks are constant.

3. **Temporary equilibrium**

The neglect of goods prices in asset demands and of the cross-price effect in the non-traded goods market makes temporary equilibrium particularly simple. Portfolio balance is the definition of the currency substitution ratio, whereas equilibrium in the non-traded goods market relates the relative price to the stock of domestic money and the currency ratio:

\[
\hat{p} = \hat{M} - \hat{F} + \hat{h}
\]

\[
\hat{q} = \tau \hat{M} + \alpha_\omega \hat{h}
\]

Given asset stocks, and the currency ratio, from (26) there is a single value of $p$ consistent with portfolio balance. Similarly, from (27), an increase in $q$ must be matched by an increase in wealth (that is in $p$) to avoid an excess supply of non-traded goods.\(^\text{14}\) Note also that an exogenous increase in the return to foreign assets (captured by $\psi$) would increase $h$. Therefore $p$ would rise one-to-one and $q$ would rise by $\alpha_\omega N$ so that $\xi$ would be higher at the new equilibrium.

\[
\hat{\xi} = (1 - \tau)\hat{M} - \hat{F} + (1 - \alpha_\omega N)\hat{h}
\]

Domestic monetary expansion increases both relative prices while an increase in foreign money will decrease them. From (28), the net effect hinges on the size of $\tau$. 
If $\tau < 1$, the effect of domestic monetary expansion will be to increase the black market rate measured in terms of non-traded goods. To consider the effect of a once-and-for-all devaluation of the official rate, notice that it increases the demand for foreign currency one-to-one and increases excess demand for non-traded goods by $\nu - \omega_N$, so that the black market exchange rate and the price of non-traded goods rise. But, if $\tau < 1$, they will necessarily rise by less, so that $p$, $q$ and $\xi$ will fall.

4. Steady-state equilibrium

Still assuming static expectations about the change in the black market rate, $\tilde{\psi} = 0$, we investigate the properties of steady-state equilibrium, defined by constant asset stocks $\tilde{M} = \tilde{M}^*$ and $F = F^*$. These are such that, aside from portfolio balance and equilibrium in the non-traded goods market, there is zero excess-supply of the smuggled good, so that $B = 0$ and, therefore, zero excess-supply of the official traded good as well, $\tilde{B} = 0$. If asset accumulation is represented by (17) and (18), the linear approximation of the system around long-run equilibrium is:

$$
\begin{bmatrix}
\dot{\tilde{M}} \\
\dot{F}
\end{bmatrix}
= 
\begin{bmatrix}
-N & \tilde{N} \\
\Pi & -\Omega & \Pi
\end{bmatrix}
\begin{bmatrix}
\tilde{M} - \tilde{M}^* \\
\tilde{N}^* \\
F - F^*
\end{bmatrix}
$$

where

$\tilde{\omega} = \mu \tau + \tilde{\omega}$

$\Omega = \mu \tau + \omega$

$\tilde{\nu} = \tilde{\gamma} + \alpha \tilde{\omega}(\Pi = \pi - \alpha \omega)$ is the positive semi-elasticity of the reported (unreported) current account with respect to the premium $\tilde{\nu}(\mu)$ is the positive semi-elasticity of the reported (unreported) current account with respect to the relative price of non-traded goods;
and \( \hat{\sigma}(\omega) \) is the positive semi-elasticity of the reported (unreported) current accounts with respect to real wealth.

It is easy to see that, if \( \Omega = \hat{\sigma} \), the system in (29) has two negative eigenvalues:

\[
\begin{align*}
\lambda_1 &= -\Omega \\
\lambda_2 &= -(\Pi + \hat{\Omega})
\end{align*}
\]

It is therefore stable. The "own" effect of the domestic money stock on the reported trade balance is negative and the "cross" effect is positive so that the \( \hat{N} = 0 \) locus will be upward sloping in \( F, M \) space. The "own" and "cross" effects of the unreported trade balance are unambiguous because here the effect of the premium works in the opposite direction of the wealth effect. Since the system is stable, though, the slope of the \( \hat{P} = 0 \) locus will always be less than the slope of the \( \hat{N} = 0 \) locus.\(^{15}\) Then an increase in the demand for foreign assets will increase the currency substitution ratio, while a once-and-for-all devaluation of the official rate will leave it unchanged. To assess the effect of an increase in \( \delta \)-which can be interpreted either as an increase in domestic credit creation or as a decrease in the rate of crawl-take the total differential of the system:

\[
\begin{bmatrix}
1 & 0 & -1 & 1 \\
\alpha \tau & -1 & (1-\alpha) \tau & \alpha \tau \\
\hat{\Pi} & \hat{\mu} & (1-\alpha) \hat{\omega} & \alpha \hat{\omega} \\
-\Pi & \mu & (1-\alpha) \omega & \alpha \omega
\end{bmatrix}
\begin{bmatrix}
\hat{P} \\
\hat{q} \\
\hat{N} \\
\hat{\mu}_d \delta
\end{bmatrix}
= 
\begin{bmatrix}
0 \\
0 \\
0 \\
0
\end{bmatrix}
\]
An increase in $\delta$ increase $p$ and $F$ by the same amount it increase $\hat{N}$ so that there is no effect on the currency ratio: 16

\begin{align*}
(33) \quad \hat{p} &= \frac{\Omega}{\hat{\pi} + \hat{\pi}^*} \quad \hat{N} \delta \\
(34) \quad \hat{F} &= \frac{\pi - \Omega}{\hat{\pi} + \hat{\pi}^*} \quad \hat{N} \delta \\
(35) \quad \hat{\pi} &= \frac{\pi}{\hat{\pi} + \hat{\pi}^*} \quad \hat{N} \delta \\

\text{The increase in } q \text{ is given by} \\
(36) \quad \hat{q} &= \frac{\tau \pi}{\tau \hat{\pi} + \hat{\pi}^*} \quad \hat{N} \delta
\end{align*}

As a consequence, the effect on $\xi$ will be ambiguous. If $\Omega > \tau \pi$, there will be an increase in $\xi$, whereas if the own-price-effect is strong enough and $\tau \pi > \Omega$, there will be a real appreciation of the black market rate expressed in terms of non-traded goods. 17
5. The implications of perfect foresight

The well-known portfolio models of flexible exchange rates with non-traded goods of Kouri (1975) and Calvo and Rodriguez (1977) assume that expectations about relative returns are continuously realized:

\[ \psi = \hat{e} \tag{37} \]

Using (37) in (12) and inverting, we obtain a differential equation for the black market rate. Subtracting from a given rate of crawl, we get:

\[ \hat{p} = h^{-1} (p \hat{F}/\hat{M}) - \hat{e} \tag{38} \]

One method of solution in this case is to solve out for \( q \), since the rate of change of \( p_N \) is pegged to the rate of crawl by the assumption that the market for non-traded goods clears. To simplify, we assume that the official rate is also constant. Then we express the system in terms of \( p, \hat{M} \) and \( F \) by taking a linear approximation around steady-state equilibrium, where \( p = 1 \) by choice of units:

\[ \begin{bmatrix} \dot{p} \\ \dot{\hat{M}} \\ \dot{F} \end{bmatrix} = \begin{bmatrix} (1-\alpha)/\epsilon & -(1-\alpha)/\epsilon & (1-\alpha)/\epsilon \\ \gamma + \alpha \hat{\Omega} & -(1-\alpha)\hat{\Omega} & -\alpha \hat{\Omega} \\ \tau - \alpha \Omega & -(1-\alpha)\Omega & -\alpha \Omega \end{bmatrix} \begin{bmatrix} p - 1 \\ \frac{\hat{M} - \hat{M}^*}{\hat{M}} \\ \frac{F - F^*}{F} \end{bmatrix} \tag{39} \]

The determinant of the matrix in (39) is unambiguously positive and equal to \( 2(1 - \alpha)(\hat{\Omega} + \tau)/\epsilon \), so that, if \( (\hat{\Omega} + h\Omega)\epsilon > 1 \) (or, when \( \Omega = \hat{\Omega} \), if \( \hat{\Omega} > 1 - \alpha \) the system has one positive root associated with the black market premium and two negative roots, associated with the asset stocks. Equilibrium is thus a saddle-point. Suppose now that the rate of crawl increases from a steady-state situation where it is zero. In steady-state the premium is constant, so
that the rate of depreciation of the black market rate has to increase. This makes foreign currency more attractive and raises the currency substitution ratio. Note that the same steady-state result obtains if expectations are hyperopic.\textsuperscript{18}

\begin{equation}
\psi = \hat{\psi}
\end{equation}

To analyze the dynamics diagrammatically, assume that the domestic money stock is constant, $\hat{M} = 0$ and solve out for $\hat{M}$, to obtain a two-by-two dynamic system:

\begin{equation}
\begin{pmatrix}
\dot{P} \\ \dot{F}
\end{pmatrix} =
\begin{bmatrix}
\frac{1}{\varepsilon} (1 + \frac{q^p}{\pi^p}) & \frac{1}{\varepsilon} \\
\pi + \frac{q^p}{\pi^p} \Omega & 0
\end{bmatrix}
\begin{pmatrix}
P - 1 \\ \frac{F - F^*}{F^*}
\end{pmatrix}
\end{equation}

It is clear from (41) that the eigenvalues of the system are of opposite sign. To represent the linear approximation to the portfolio balance schedule, $p = 0$, note that it has slope less than one.

\begin{equation}
\frac{dp}{dF} \quad F = 0 = - \frac{q^p}{\pi^p + \Omega}
\end{equation}

On the other hand, since the wealth effects cancel, the locus where $F = 0$ is horizontal in $p, F$ space and defines the long run value of the premium.\textsuperscript{19} This can be represented in the right-hand panel of Figure 1 above, where the vertical axis would measure the premium, and the $F = 0$ locus is a straight line. Then, as stated in Section I, under perfect foresight, demand for foreign currency will increase when the black market rate depreciates, constraining its initial value for a given stock of foreign currency to a value below the corresponding initial value under static expectations. The effect of an increase
in the demand for foreign assets is also smaller than under static expectations. It is followed by a continuous decline of the black market premium along a higher perfect foresight path. Conversely, the effect of an exogenous increase in the foreign demand for the smuggled good is a downward shift in the $F = 0$ locus, which leads to a jump fall in $p$ and a continuous decline along a lower perfect foresight path to $F_2$. Under static expectations there would be no jump fall in $p$, but only a decline along the portfolio balance locus.
Conclusion

This paper has analyzed regimes of currency inconvertibility using the portfolio approach to exchange rate determination developed by Kouri (1983) and others. Reference was also made to a "crime-theoretic" approach pioneered by Beccaria (1764) and to the monetary approach of Blejer (1978a). In a simple three-good, two asset general equilibrium model, it was shown that continuous changes in the official rate (a crawling peg) are necessary to change the currency composition of private financial wealth. The direction of the effect hinges on the assumption about expected returns. If expectations are hyperopic or if there is perfect foresight in the "black" market, an increase in the rate of crawl increases the currency ratio whereas, if expectations are static or regressive, it decreases it.

The assumption of perfect foresight may seem particularly strained in a "black" market, but the contrast of the various expectations formation mechanisms brings out an important dilemma of currency inconvertibility. If the monetary authorities believe that, in the long-run, the private valuation of foreign currency adjusts to the official one, they also have to accept that the wrong exchange rate policy will set the black market for foreign exchange off the stable path where expectations are continuously realized, making domestic money worthless. Since instances of "destabilizing speculation" against inconvertible currencies come readily to mind, expectations can only be regressive to the extent that the official valuation also adjusts to the private valuation, as in a dual or even "gray" market for foreign exchange.
NOTES


2. In response to the greater mobility of capital, some governments have attempted to recapture foreign exchange by offering special advantages to certain types of transactions, namely the ones where evasion of exchange controls would be easier, like tourist services and migrant's remittances. See IMF, Annual Report, Table I.1 and Annual Report on Exchange Arrangements and Exchange Restrictions, Analytical Appendix. Only 33 out of 148 member countries have fully convertible currencies, while 67 (Western Europe, Middle-East and Latin America) have accepted convertibility for current account transactions only.

3. On the portfolio approach see Tobin (1969), Branson (1977) and Tobin and Macedo (1980), for example. A portfolio model of a dual exchange rate regime is in Macedo (1978). Flood (1978) and Cumby (1979) have also used the portfolio approach to flexible exchange rates in modelling dual foreign exchange markets but ignoring the linkage between the two markets. Taking the current account as given, the linkage was addressed by Fleming (1968) and (1947). Other useful references are Fleming and Mundell (1964), Lanyi (1975) and Swoboda (1974). Finally, see Macedo (1982b) and (1982c, Essay III) and Dornbusch et al. (1983) for evidence on black markets for foreign exchange in Egypt, Portugal and Brazil, respectively.

4. On the "elasticities condition" see Bickerdike (1920), Robinson (1937), Machlup (1939) and Metzler (1949).

5. The role of the current account is stressed in Kouri and Macedo (1978) and (1981) and Rodriguez (1980). Note that we assume that the stock of reserves is high enough so as to neglect the "crisis problem" of Krugman (1979).
6. As Cooper (1974) showed for the case of Indonesia, tariff collections as a percent of imports were lower than schedule rates at higher duties because of the increased incentive to smuggling activity. While higher collections are presumably achievable with greater law enforcement, net revenues might still be less than proportional to rates.

7. A brief survey of this literature is in Macedo (1979). On balance of payments effects see Bhagwati (1967) and (1964) and Winston (1974). Another strand of literature has concentrated on methods for the detection of smuggling by using partner country data. See Morgenstern (1950), and Bhagwati, Krueger and Wibulswaski (1974).

8. This case is also worked out in Macedo (1982a) where the shares depend on the premium. An alternative suggested by Al Fislow is to make shares a function of the expected change in the black market rate. Also, see Macedo (1983) for a model where exports and imports are not aggregated. Other two-price models are in Krueger (1984), Jones (174), Takayama (1984) and Dornbusch (1975) and (1980).

9. Strictly speaking, this also requires constant expenditure shares. See Kouri (1975).

10. See alternative assumptions in Macedo (1983) and the addendum to Johnson (1976).

11. The income effect could easily be added; see, for example, Buitier and Eaton (1981). Its irrelevance in this set-up has been noted by Kouri (1975).

12. Other references in the monetarist spirit are Culbertson (1975) who simply used annual data on black market rates in India, the Philippines and Turkey for purchasing power parity calculations; Giddy (1978) who attempted to
show the "efficiency" of black markets for foreign exchange in Colombia, Brazil and Israel by testing the randomness of the black market premiums and changes in the premium using weekly data; Blejer (1978b) who used annual data on a black market based on the real exchange rate for Brazil, Chile and Colombia to estimate money demand functions in these countries and Canto (1983), who performed time-series analysis on data from the Dominican Republic.

13. Under rational expectations the "real rate" would be a function of time only given by

\[ \xi_t = \exp \left[ \log \xi_0 \exp (-t) \right]. \]

14. If the portfolio balance schedule were not horizontal in p, q space, to the extent that relative price effects are stronger on the flow demand for goods than on the stock demand for assets, stability requires its slope to be less than the slope of the non-traded goods market equilibrium schedule. It is also less than the ray through the origin with slope \( \xi \). See the analysis in Macedo (1983).

15. See the analysis in Macedo (1982a).

16. See Macedo (1983) for the case where \( \psi \neq 0 \), so that the currency ratio changes.

17. This real appreciation is used to characterize the "Egyptian disease" in Macedo (1982b). See also Calvo (1979).


19. If, instead, we assumed that \( \hat{N} = \hat{N}^* \) at all times, the \( F = 0 \) locus would be upward sloping because of the wealth effect. None of the conclusions would be changed.
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