OPTIMAL PRICING OF PUBLICLY
SUPPLIED GOODS AND SERVICES

by

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INTRODUCTION

Many goods and services are publicly provided at prices below the costs of production. For example, most countries have public education systems where education services are sold at heavily subsidized prices (often equal to zero). Similarly, governments often operate subsidized health services. Is the below-cost pricing of a government supplied good or service in the consumers' interest?

If the only policy choice were the price of the publicly provided good, then clearly any price increase would make consumers worse off. But there are other policy variables. The government, in its role as a producer, typically cannot earn losses greater than some given level. Faced with a budget constraint, the government must make trade-offs between the good's price (i.e., the level of per-unit subsidy), its quality, and its total supply. Given the social budget constraint, an increase in the user fee allows the government to increase the level of output provided, or to raise the quality of the good. Thus, the existence of a government budget constraint gives rise to the possibility that consumers of the publicly provided good will benefit from an increase in its price; the increase in the amount or quality of the output supplied may more than compensate consumers of the good for the associated price rise.

The present chapter examines the socially optimal price for a good or service when aggregate subsidy for government supply is fixed. A simple, two-good general equilibrium model, presented in Section 2, provides the framework for the analysis. In Section 3, this model is used to examine
the efficiency and distributional consequences of marginally increasing the price and quality of the good while holding the aggregate subsidy constant. It is shown that such a price increase may redistribute income towards the poor and increase overall consumer welfare. The effects of a price increase and a concomitant expansion of output (holding the level of product quality fixed) are analyzed in Section 4. Again, conditions under which an increase in the user fee will raise consumer welfare are demonstrated. The relationship between the fixed and variable subsidy cases is discussed briefly in Section 5. The paper closes with a short concluding section.

2. A MODEL

Consider a two-good economy. Good 1 is taken as the numeraire, with the price equal to one. Each consumers is indexed by his or her initial endowment of good 1, \( n \), and by another characteristic, \( b \). \( n \) may thought of as income. \( b \) is some description of the agent, such as whether the agent lives in a rural or urban area. \( b \) and \( n \) are jointly distributed according to the frequency function \( f(b,n) \). That is, \( f(b,n) \) is the number of consumers who have both characteristic \( b \) and income \( n \).

All agents are assumed to have zero endowments of the second good. Production of \( X \) units of good 2 with quality \( q \) requires \( C[X,q] \) units of good 1 as input. Good 2 is sold at price \( p \) by the government, which is the sole producer of the good. As a producer, the government faces the following budget constraint:

\[
pX - C[X,q] + S \geq 0, \quad (2.1)
\]
where $X$ is the aggregate amount of good 2 supplied and $S$ is the aggregate subsidy for good 2. $S$ is assumed to be exogenously generated (e.g., it represents foreign aid). A completely general analysis of the problem would consider the simultaneous determination of the entire tax system, treating the subsidy of the publicly provided good as one of the parameters of the system. Although there clearly are important interactions between the pricing of publicly produced goods and the design of the overall commodity and income tax system, it may not be politically feasible to change the price of the publicly provided good and the overall tax structure simultaneously. Thus, finding the optimal price given a fixed aggregate subsidy is a case of practical interest. I assume that there are no other taxes or subsidies present in the economy.

The government chooses the price of the publicly provided good to maximize welfare, which is given by an individualistic welfare function. $w[x,y,q;b]$ is the social welfare that is generated by a household with characteristic $b$ consuming $y$ units of good 1 and $x$ units of good 2 with quality $q$. Aggregate social welfare is found by summing individual welfare over all households,

$$\int w[x(n,b),y(n,b),q;b]f[n,b]dn db,$$  \hspace{1cm} (2.2)

where $x(n,b)$ and $y(n,b)$ are the quantities of goods $x$ and $y$, respectively, consumed by an agent with income $n$ and characteristics $b$.

Possible motives for subsidization include: (a) income redistribution goals; (b) economies of scale in production (so that there is a conflict between marginal cost pricing and covering average costs); (c) externalities in the consumption of the subsidized good; (d) government paternalism.
(so-called merit goods); and (c) the presence of capital market imperfections (so that consumers cannot borrow against future income or there are incomplete markets for the risk associated with investment in education).2/

Under motives (a) and (b), the government does not care about consumption levels per se; it is concerned with the resulting utility levels. Assuming that the government respects individual preferences in its welfare function, a utility function, \( u \), can be found to represent consumer preferences such that

\[
w[x,y,q;b] = u[x,y,q;b].
\]

Objectives (c), (d), and (e) all are similar to one another in that, from the government's perspective, households will choose to consume too little of \( x \) relative to \( y \). The following function captures this type of relationship between social welfare and individual utility:

\[
w[x,y,q;b] = u[x,y,q;b] + E[x],
\]

where \( dE/dx \) is greater than zero. \( u \) and \( E \) are normalized so that \( E[0]=0 \). \( E[x] \) is the level of external (or perhaps one should say, "paternal") benefits that arise when a household consumes \( x \) units of the publicly provided good. \( E[x] \) is the amount by which the government values a household's consumption of good 2 by more than does the household itself.

The government does not choose the levels of \( x(n,b) \) and \( y(n,b) \) directly. Instead, households choose their consumption levels given the price and quality of the publicly supplied good (which are set by the government) and subject to any government-imposed rationing constraints.
In the absence of a rationing constraint, each household chooses its consumption levels to maximize utility subject to its budget constraint:

$$\begin{align*}
\text{maximize} & \quad u[x, y, q; b] \\
\text{subject to} & \quad px + y \leq n.
\end{align*} \tag{2.4}$$

Let $x^u(p; q, b, n)$ denote the consumer's unrationed demand for the publicly provided good.

The government may ration good 2. The ration allotments must depend only on signals or indices that the government can observe. I will assume that the government cannot observe $n$ directly, but that it is able to observe $b$ (b may be some imperfect, but observable measure of $n$). When a household is subject to a rationing constraint of the form $x \leq x^F(b)$, its consumption level, $x[p; q, x^F(b), b, n]$, will be equal to the smaller of $x^u(p; q, b, n)$ and $x^F(b)$.

3. PRICE AND QUALITY CHANGES

A. The Price-Quality Trade-off

Suppose that there is no rationing. For simplicity, assume that characteristic $b$ has no effect on utility or social welfare, so that we may suppress the index in this Section. For a given price of good 2, the only way to satisfy the social budget constraint is to adjust the quality of the good. Consider increasing the user fee charged for the publicly provided good by $dp$. Totally differentiating the social budget constraint, equation (2.1), and rearranging terms one obtains

$$\frac{dq}{dx} = \frac{(p - C_x)X_p + X}{C_x X_q + C_q} \tag{3.1}$$
The numerator is equal to marginal revenue minus marginal cost. As long as \( p \) is less than the monopoly price (given \( q \)), \( dq/dp \) will be positive; an increase in the user fee will raise social "profits" which then can be used to raise the quality of the publicly supplied good. Any price increase above the monopoly level would be unambiguously bad — the price would rise, and the quality would fall. There is little else that one can say in general; when price is below the monopoly level, an increase in the user fee and the concomitant quality change may raise or lower welfare.

B. Individual Welfare

Given the household budget constraint, a household with income \( m \) has utility given by \( u[x,n-px,q] \) when it consumes \( x \) units of good 2, the publicly supplied good. The change in the household's welfare from a price-quality shift is given by

\[
-u_x dp + u_q (dq/dp) dp. 
\]

The first term represents the loss in the household's utility due to the price increase, while the second term represents the gain from the increase in product quality. In general either effect may dominate. The larger is \( dq/dp \), the more likely it is that welfare will be increased by a price rise.

Suppose that utility is separable in the following way:

\[
u[x,y,q] = x[\delta(x,q),y],
\]

where \( \delta_{xx} < 0 \). \( \delta \) can be thought of as the consumption of good 2 measured in quality-adjusted units. Households choose their consumption levels to
maximize utility and set the marginal rate of substitution equal to the
price ratio, or \( z_2 / (z_1 \phi_x) = 1/p \). Hence, equation (3.2) may be written as

\[
\int \left[ - \frac{(x/p) \phi_x}{\phi_q} + \frac{\phi_q (dq/dp)}{\phi_q} \right] dp. \tag{3.4}
\]

From (3.4), one sees that the price and quality increases raise household
utility if and only if

\[
\left( \frac{p \phi_q}{\phi_x} \right) dq - x dp \tag{3.5}
\]

is positive. \( \phi_q / \phi_x \) is the household's marginal rate of substitution
between the quality and quantity of good 2, and \( (\phi_q / \phi_x) dq \) is the amount by
which \( x \) can be reduced while holding \( \phi(x, q) \) constant when quality rises.
Thus, the first term in equation (3.5) is the amount of money that the
household can save by reducing the number of physical units purchased while
holding quality-adjusted consumption constant. The second term represents
the effects of the price increase.

How do the effects on individual welfare vary across income groups?
The answer depends on the way in which the burden of the price increase and
the benefits of the quality increase are distributed across income classes.
For \( x \) a normal good, a household's consumption rises with income. From
equation (3.2), the burden of the price increase rises proportionally with
consumption. The valuation of the additional quality may rise proportion-
ately more or less than the level of consumption, however, and thus the net
benefits of the price and quality changes may rise or fall with household
income.
It follows from equation (3.5) that high income households are more (less) likely to benefit from increases in price and quality than are poor households when

\[
\frac{d}{dx} \left( \frac{xq}{\phi_q} \right) \quad (3.6)
\]

is negative (positive). Differentiating and rearranging terms, the sign of equation (3.6) is given by the sign of

\[
x_q \phi_{xx} + x \left\{ \frac{1}{x} \left( \int_0^x \phi_{q}^2 [s, q] ds - \phi_{q} [x, q] \right) \right\} \quad (3.7)
\]

\(z_q \phi_x\) is the marginal utility from consuming an additional unit of good 2. Thus, \(z_q \phi_q (x, q)\) is amount by which the quality increase raises the utility derived from the \(x\)th unit of good 2. The term in curly brackets is equal to \(1/z_q\) times the difference between the value of the extra quality averaged across all units and the value of the extra quality in the marginal unit.

It is instructive to examine several special cases. First, if \(\phi_q x > 0\), then an increase in quality raises the value of the marginal unit by increasing amounts as \(x\) rises (the value of the extra quality rises faster than does \(x\)) and the term in curly brackets is negative. The first term in equation (3.7) is nonpositive. Thus, when \(\phi_q x > 0\), the rich are more likely to benefit from the price and quality increase than are the poor.

Now, suppose instead that \(\phi(x, q) = xq\). The value of the additional quality rise proportionally with \(x\). When \(\phi\) takes this particular form, \(p/q\) can be thought of as the quality-adjusted (or hedonic) price of good 2 since \(x\) units of the good provide \(xq\) quality-adjusted units of service at a total cost (to the household) of \(xp\). All households will agree on whether
this hedonic price is increased by a given price and quality shift, and
the sign of the welfare effects will not vary across income classes. To
verify this intuition, note that equation (3.7) is equal to zero and
equation (3.5) becomes

\[ x(\frac{p}{q}dq - dp), \]

the sign of which is independent of \( x \).

There is, of course, a third possible relationship between household
income and the benefits from a price-quality shift. When \( \phi \) takes the form,

\[ \phi(x,q) = \begin{cases} 
0 & \text{if } x=0 \\
q+x & \text{if } x>0 
\end{cases}, \]

for example, the benefits of the extra quality rise more slowly than does
\( x \), and thus the benefits of the extra quality rise more slowly than does
the burden of the price increase. Equation (3.5) becomes \( pdq - xdp \). Low-
ingcome households are more likely to benefit from simultaneous increases in
price and quality than are high-income consumers.

C. Aggregate Welfare

When the social welfare function is of the form given in equation
(2.3), the change in welfare is given by the household utility effects and
external effects aggregated over all households:

\[ \int u_q(dq/dp)fdn - \int xu_yfdn + \int E \frac{dx(n)}{dp} \int fdn \]

(3.8)

where \( dx/dp = x_p^u + x_q^u(dq/dp) \) may be positive or negative. The first
integral represents the increase in social welfare that results from the
quality increase. The second term represents the social costs of the
additional quality for the original level of output, and the final integral is the change in the level of external benefits that results from the change in consumption of the publicly supplied good. Ceteris paribus, the price and quality changes will be most likely to increase social welfare when households with high levels of consumption (x) are ones for whom the marginal utility of income \( (u_y) \) is low, so that the social burden of the cost increase is low. If poor households highly value additional quality, but have low levels of consumption prior to the price-quality shift, then an increase in price and quality is likely to be socially desirable. A price increase and associated quality rise also are more likely to be socially beneficial when the external effects \( (E') \) are strong and the changes lead to increased consumption levels \( (dx/dp) \).

D. The Use of Consumption Data

Can one make inferences about the changes in aggregate and individual welfare by observing individual demand changes? Given the form of the externalities, clearly one can tell if they have risen or fallen. But can one determine the effects of a price-quality shift on the level of household welfare? More specifically, does the fact that a household raises its consumption of the publicly provided good in response to simultaneous changes in the good's price and quality imply that the household is better-off as a result of these changes? Unfortunately, often the answer is no.

To see the difficulty, suppose that there are no income effects, and that the household utility function is of the form

\[ u(x, y, q) = \delta(x, q) + y. \]
In this case, a household's willingness to pay for a marginal unit of the publicly provided good (the household demand curve) is given by \( u_x = \delta_x \), and \( u_{xq}(x,y,q) \) is the amount by which a quality increase raises the value of the \( x \)th unit of the publicly provided good (shifts the demand curve upwards). Absent income effects, the utility derived from consumption of the publicly provided good is equal to the area under the household's demand curve. As Figure 1.a illustrates, utility will rise only if the upwards shift in the demand curve due to the quality increase is on average greater than the price rise. (The welfare effects of the change in consumption induced by the price and quality increases will be second-order small, and for marginal changes they can be ignored when calculating the sign of the welfare effect.) While, utility changes depend on the relationship between the price rise and the average shift in the demand curve \((\frac{1}{x}\int_0^x u_{xq}(s,q)dsdq)\), the change in the level of consumption depends on the relationship between the price increase and the shift in the demand curve at the margin \((u_{xq}(x,y,q)dq)\). Thus, in Figure 1.b, consumption will rise as long as the price rise is less than \( p_2 - p_1 \). When the shift in the demand curve at the margin exceeds the average shift, consumption of the publicly supplied good may rise while utility falls. When the marginal shift is less than the average shift in the demand curve, however, a price-quality shift may raise a household's utility even though it induces the household to consume less of the publicly supplied good.

Somewhat more generally, one can allow for income effects by supposing that the utility function separable in the consumption of the two goods:

\[
 u[x,y,q] = \delta[x,q] + \Theta[y].
\]
vertical shading indicates gain from quality increase
horizontal shading indicates loss from price increase
solid triangle represents second-order-small gain from quantity adjustment
Solving for the comparative statics of the consumer’s utility maximization problem, equation (2.4), a household increases its consumption in response to the price and quality shifts if and only if

\[ p x u_{yy} - u_y + u_{xq}(dq/dp) > 0. \tag{3.9} \]

Dividing equation (3.2) by \( x \), one sees that household utility increases if and only if

\[ -u_y + (1/x)\int_0^x u_{xq}[s, y, q]ds (dq/dp) > 0. \tag{3.10} \]

The difference between equations (3.9) and (3.10) is

\[ p x u_{yy} + \left\{ u_{xq}[x, y, q] - (1/x)\int_0^x u_{xq}[s, y, q]ds \right\} (dq/dp). \]

The first term represents the income effect of a price change on the demand for \( x \). The term in curly brackets is equal to the difference between the value of the extra quality averaged across all units and the value of the extra quality in the marginal unit.

A government concerned with redistributing income to households at the bottom of the distribution would have a welfare function such that \( u_{yy} < 0 \). Hence, if the term in curly brackets is negative, then whenever equation (3.9) is satisfied so too is equation (3.10). When \( u_{xxq} \) is negative, the value of the extra quality is lower at the margin than on average, and any shift in price and quality that induces a household to increase its consumption of of the publicly provided good raises that household’s utility. Thus, if policymakers believe that a quality increase raises a consumers’ willingness to pay for the good by more for initial units of consumption than for later units, the government should continue to raise the price and
quality of the good as long as the net effect is to raise households' consumption levels.

4. PRICE AND RATIONING CHANGES

A. The Price-Quantity Trade-off

In the previous section, the government used the additional funds generated by a price increase to raise the quality of the publicly provided good. Now, suppose instead that the quality of the good is fixed and that the government satisfies the social budget constraint by adjusting the total quantity of the subsidized good that is supplied. Totally differentiating the social budget constraint, equation (2.1), and rearranging terms, one obtains

\[
\frac{dX}{dp} = \frac{-X}{p-C_x} \quad (4.1)
\]

From equation (4.1), it is clear that rationing will be optimal only in those cases where \( p-C_x \) is negative; if the price is greater than marginal cost, then relaxing the rationing constraint can only raise "social profits" and make it possible to increase the amount of good 2 supplied for a given level of aggregate subsidy.

As long as there is excess demand at the current price, consumers will be willing to purchase the additional output even when the price is increased to finance the additional production. Thus, when the price is below \( p^e \) in Figure 2, consumption is rationed and a price increase leads to movement along the social supply curve. When the price is above \( p^e \), there
is no rationing and a price increase leads to movement along the the demand curve.

The welfare effects of raising the price and aggregate consumption of the publicly provided good depend on the way in which the additional output is distributed to households. Parameterize the rationing scheme so that $x^*[b;X]$ denotes the rationing ceiling faced by a household with characteristic $b$ when total output of the publicly supplied good is $X$.

The rationing scheme must satisfy the constraint that rationed consumption (summed over all households) is less than or equal to production,

$$\int x[p;n,b,x^*[b;X]]f[n,b]db \leq X,$$  \hspace{1cm} (4.2)

where $x$ is the smaller of the household’s desired level of consumption and its ration allotment. It always is socially desirable to have all output that is produced consumed, and equation (4.2) will hold with equality at the optimum.

For all consumers for whom the ceiling is binding, consumption is equal to $x^*[b;X]$, $u_x - p_u y > 0$, and the change in consumption due to an increase in price and total output is $dx^*/dX$ (i.e., the household increases its consumption by the amount of the increase in its ration ceiling). For consumers for whom the ceiling is not binding, consumption is given by $x^u[p;n,b]$, $u_x - p_u y = 0$, and the change in consumption from a price and total output increase is $x^u_p < 0$.

B. Individual Welfare Changes

The essential property of the rationing scheme is the relationship between the increase in consumption and the increase in payments. It is useful to think of the policy change in terms of an increase in the level
of total output, \( X \), and to treat the concomitant changes in the price and ration levels as consequences. Suppose that the rationing constraints are binding for all consumers. The change in a household's welfare from a change in total output is

\[
(u_x - pu_y)(dx^F/dX) - x^F(dp/dX).
\]

(4.3)

The first term represents the increase in the household's utility from being allowed to shift some consumption from the privately produced good to the publicly produced good. The second term represents the loss in utility due to the increase in the price of the publicly produced good. A household will benefit from the price and quantity increase if consumption rises by more than does marginal-utility-weighted expenditure on the good.

How does the sign of equation (4.3) vary across income groups? Suppose that \( u_{xy} \geq 0 \), \( b \) does not enter into the utility function, and the ration allotment is the same for all households. Then as income rises, \( px^F \) rises, \( u_x \) rises, and \( u_y \) falls. The other variables in equation (4.3) are constant across income groups. Hence, the rich will favor an increase in the price and total output more than will low-income households. The intuition behind this result is the following. The publicly provided good is a normal one, and thus when all households are forced to have the same level of consumption, high-income households have their consumption more distorted than do low-income households. High-income households have the most to gain from a marginal relaxation of the rationing ceiling. At the same time, all income groups bear equal shares of the additional costs of production.
Now, consider a rationing scheme where each household faces either a ceiling of zero or no ceiling, depending on whether its value of \( b \) is above or below some standard. Letting \( h \) denote the cutoff value,

\[
x^r[b; X] = \begin{cases} 
0 & \text{for all } b < h \\
\infty & \text{for all } b \geq h.
\end{cases}
\]

The use of tests for admission to college or secondary school is an example of this type of scheme.\(^7\) Raising the price and the total amount supplied will make it possible for the government to lower the cutoff value; \( \frac{db}{dp} < 0 \).

A household with a given income level may have any one of several values of \( b \). Thus, one must consider the expected effects of a price and output shift on a household with income \( n \), taking the expectation over the possible values of \( b \). The change in the expected utility of a household with income \( n \) is

\[
\left\{ u[x^n, n-x^n, h] - u[0, n, h] \right\} f(n, b) \left\{ -\frac{db}{dp} \right\} - x^n u_y \int_b^\infty f(n, b) db.
\]

The first term represents the additional utility that the household derives from being allowed to purchase the publicly supplied good multiplied by the change in the household's chance of being at the admissions margin times the change in the cutoff level. \( x^n \) in the second term is the quantity purchased by a household who is not subject to the zero consumption ceiling. \( \int_b^\infty f(n, b) db \) is the probability that a household with income \( n \) will be above the cutoff level. Thus, the second term represents the welfare effects of the price rise.
As before, the burden of the price increase (given by the integral) is distributed according to pre-shift consumption levels. The benefits are given by the increase in the chance that the household is allowed to purchase the publicly supplied good. An income group is most likely to benefit from the price and output increases if its members were unlikely to have been allowed to purchase the service before the changes, but had values of \( b \) near the cutoff level. For example, if \( b \) is highly correlated with income, then households currently allowed to purchase the good will tend to have higher incomes than those households who are at the margin of being allowed to purchase. Hence, a price increase and accompanying reduction in the cutoff level will be a progressive policy move.

C. Aggregate Welfare

Under the first type of rationing scheme considered in Part B, the change in social welfare from a marginal increase in the level of total output and the accompanying price increase is

\[
\int \{u_x - pu_y\} (dx^* / dX) - xu_y(dp/dX) \} f \, dbdn + \int E'(dx^* / dX)f \, dbdn
\]

\[
= \int \{u_x + E'\} \{dx^* / dX\} f \, dbdn - \int u_y\{p(dx^* / dX) + x(dp/dX)\} f \, dbdn
\]  

(4.4)

The first integral represents the private and public (external) increases in gross consumption benefits. The additional revenues raised are equal to the additional costs of production;

\[
\int \{p(dx/dX) + x(dp/dX)\} f \, dbdn = C_x.
\]

Thus, the second integral represents the social cost of the additional output, weighted by the social marginal utility of income of those con-
sumers who bear the additional cost. For a given level of production costs, the socially-weighted costs will be lower when \( u_y \) and \( x \) are negatively correlated than when they are positively correlated.

Under the admissions cutoff type of rationing scheme, the change in welfare is given by

\[
-(dh/dp) \int_0^\infty \left\{ u[x, m-x, b] - u[0, n, b] + E(x) - E[0] \right\} f[n, b] \, dn
\]

\[
- \int_0^\infty x u_y f[n, b] \, dnb,
\]

(4.5)

More generally, the essential features of a rationing scheme are whether marginal output is distributed in a socially efficient way and whether the marginal revenues are raised in a socially efficient way. The marginal social benefits of the additional output depend on the covariance of \( u_x \) and \( dx^x/dx \) across households. When those consumers receiving the additional output are the ones for whom the social marginal utility of consumption is high, the price and output increases will be more likely to raise welfare. The social costs depend largely on the relationship between \( u_y \) and \( x^x \). When those consumers with low social marginal utilities of income are the ones who pay the largest share of the increased costs (i.e., the ones who consume large amounts of the publicly provided good prior to the price increase), the socially weighted costs will be low. Thus, the price increase will be more likely to be socially beneficial when \( u_y \) and \( x^x \) are negatively correlated. Finally, note that a price and output increase is more likely to lead to a welfare increase when there are strong externalities so that the increase in consumption has a high marginal social value.
D. The Use of Consumption Data

Given that households are rationed, changes in consumption levels yield very little information about changes in either individual or aggregate welfare levels. First consider the relationship between consumption and utility changes at the household level. Assuming-away income effects for simplicity, household welfare is given by the area under the demand curve as shown in Figure 3. The level of utility will increase only if the increase in consumer surplus due to the extra consumption (area abde) is greater than the loss in surplus due to the price increase (area cdfg). The level of consumption will increase in response to a relaxation of the rationing constraint as long as the price is less than marginal utility. As illustrated in Figure 3, even though consumption rises, area abde is less than cdfg, and household utility declines. Of course, if consumption falls when price rises, then consumer surplus falls.

There is one important case in which a rise in household utility can be inferred from simultaneous increases in the price and a household's consumption. Suppose that the household faces a ration ceiling of zero under the old price and has a positive consumption level under the new, higher price. Prior to the price increase, consumer surplus is zero. Under the new price and relaxed rationing constraint, surplus is positive. In terms of Figure 3, area abde is larger than area cdfg because area cdfgh is equal to zero. Thus, if a price rise allows the government to expand its output, those low-income households that choose to purchase the service and who previously were excluded from receiving the service definitely benefit from the price increase. Moreover, households with near-zero initial consumption levels are very likely to gain from any policy that raises their consumption.
FIGURE 3

The diagram illustrates a demand curve with a new price and an old price. The new price is marked as 'g', and the old price is marked as 'f'. The demand curve intersects the quantity axis at 'old ceiling' and the price axis at 'new ceiling'. The diagram shows the change in quantity demanded when the price changes from old to new.
What about aggregate welfare changes? Continue the assumption of no income effects and, further, assume that all households are identical, so that it is valid to consider a representative household. Clearly, this case is the one for which there is the greatest chance of being able to make correct inferences. However, as illustrated in Figure 4, even in this case an increase in consumption does not indicate an increase in aggregate welfare when price is set below marginal cost (recall from equation (4.1) that rationing never is optimal when price equals or exceeds marginal cost). For households with a binding rationing constraint, the gain in utility from a marginal unit of the good is \( m \), which is greater than \( p \). There is excess demand, and an increase in the price leads to an increase in the total amount of the good consumed as output moves along the social supply curve generated by the budget constraint. The increase in social benefits is the sum of the increases in the household and external benefits, \( m + e \). For sufficiently weak externalities, \( m + e \) is less than \( C_X \) even though \( m \) is greater than \( p \). Thus, the increase in social benefits is less than the increase in costs in social costs. A small price rise may lead to inefficiently high consumption.

5. THE PRESENCE OF OTHER TAXES AND SUBSIDIES

The analysis above assumes that there are no other taxes or subsidies present in the economy. Clearly this condition will not be satisfied in actual cases. Thus, it useful to consider the effects that the presence of other taxes would have on the results derived above. Essentially, there are three types of interaction between the price of the publicly provided good and the overall tax system.
First, if the other tax rates can be adjusted, there is no need to hold the aggregate subsidy, S, at a fixed level. With a fixed aggregate subsidy, the additional revenues necessary to finance an increase in the quality or total quantity of the publicly supplied good have to be raised by increasing the price of the good. When the aggregate subsidy is variable, some or all of the additional funds can be raised through changes in other tax rates. Instead of a fixed level of subsidy, the budget constraint would contain a shadow price for public funds, reflecting the social costs of raising additional revenues through optimal (feasible) tax changes. This shadow price could be greater or less than the shadow price of funds used in the analysis above, which reflected the social costs of raising additional revenues through an increase in the price of the publicly produced good for a given level of contribution by other taxes, S. The relationship between the two shadow prices depends on whether the fixed level of aggregate subsidy considered above is greater or less than the socially optimal level of aggregate subsidy.

Second, the presence of a tax system will affect the benefits of changes in the consumption of and expenditures on the publicly supplied good. The presence of redistributive taxes may reduce the need to use the price of the publicly produced good as a means of income redistribution. Further, by distorting relative prices and, hence, consumption decisions, other taxes will alter the marginal benefits associated with an increase in the consumption of the publicly produced good.

Finally, changes in the price, quality, and quantity of the publicly provided good will affect the revenues raised by other taxes. For example, if there are commodity taxes on goods other than the government supplied
one, an increase in expenditures for the publicly produced good will reduce expenditures on other goods and will lower the revenues collected from fixed taxes on these goods. Similarly, a quality change may induce shifts in consumption patterns that alter the revenues collected by other taxes.

6. CONCLUSION

When there is a social budget constraint, a price increase may allow the government to increase the quality or the quantity of a publicly provided good or service. As a result, a price increase and the concomitant quality and output changes may lead to an increase in the aggregate welfare of the consumers of the good. Moreover, a price increase may raise the welfare of low-income households, particularly if prior to the price change these households consumed little of the good or service. Consequently, even when the government has strong redistributational objectives favoring low-income households, it may be socially desirable to raise the prices of publicly supplied goods and services.

FOOTNOTES

1/ When the aggregate subsidy can be varied, there are additional tradeoffs between the subsidy to this good and the tax rates on other goods and income.


3/ The terms involving dx all cancel; the household chose x optimally given the original price, so that to, a first-order approximation, changes in x have no effect on utility.

4/ In writing this equation, I have made use of the fact that a function is equal to the integral of its derivative.
5/ In a series of papers, Birdsall has investigated the difficulties with, and the methodology for, estimating the welfare effects of price changes. See, for example, Birdsall (1982), "Strategies for Analyzing Effects of User Charges in Social Sectors," Country Policy Department, The World Bank.

6/ Again, I have made use of the fact that a function is equal to the integral of its derivative.


100. "Environmental Pollution and Optimum Taxation Under Market Distortions in Developing Countries" by Gill C. Lim and Joon Koo Lee, October 1981, pp. 18.


102. "Environmental Policies in Developing Countries: A Case of International Movements of Polluting Industries" by Joon Koo Lee and Gi-Chin Lim, March 1982, pp. 19.


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