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CAPITAL ACCUMULATION AND CAPITAL INTENSITY
OF PRODUCTION TECHNIQUES

Charles R. Frank, Jr.

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One of the most persistent notions in the theory of economic development is that a less developed country should use more labor intensive techniques. To put the same thing in another way, the more developed a country becomes, the optimal technique requires less labour intensity. Intuitively, the proposition makes sense -- in countries where labour is plentiful and capital scarce, more labour should be used relative to capital. The proposition can be demonstrated simply as follows:

Let p_L be the price ratio of labor relative to capital. The amount of capital is given. The amount of labor employed with the given capital stock equates the marginal product of labor with its price.

$$(1) \quad f_L(K, L) = p_L$$

where $f_L(K, L)$ is the partial of the production function $f(K, L)$ with respect to labor. As capital is accumulated, suppose the price of labor rises relative to capital ($\dot{p}_L > 0$)¹. A new equilibrium is reached in which

$$(2) \quad f_{LK} \cdot \dot{K} + f_{LL} \cdot \dot{L} = \dot{p}_L \quad \text{or}$$

$$(3) \quad \dot{L} = \frac{1}{f_{LL}} \dot{p}_L - \frac{f_{LK}}{f_{LL}} \dot{K}$$

The change in the capital labor ratio is

$$(4) \quad \left(\frac{\dot{K}}{L} \right) = \frac{1}{L} \dot{K} - \frac{K}{L^2} \dot{L}$$

Substituting (3) into (4)

$$(5) \quad \left(\frac{\dot{K}}{L} \right) = \frac{1}{L} \dot{K} - \frac{K}{L^2} \left(\frac{1}{f_{LL}} \dot{p}_L - \frac{f_{LK}}{f_{LL}} \dot{K} \right)$$

Assuming constant returns to scale, Euler's Theorem states that

$$(6) \quad f = f_L \cdot L + f_K \cdot K$$

Differentiating (6) with respect to K, we get

$$(7) \quad \frac{f_{LK}}{f_{KK}} = - \frac{L}{K}$$

Substitute (7) into (5).

$$(8) \quad \left(\frac{\dot{K}}{L} \right) = - \frac{K}{L^2} \cdot \frac{1}{f_{LL}} \dot{p}_L$$

Since we assume diminishing returns to labor ($f_{LL} < 0$), the expression in (8) is positive and the capital/labor ratio increases as capital accumulation takes place, and the price of labor rises relative to capital.

There are numerous dissenters to this proposition. The arguments usually take one of three forms. First, a more labor intensive technique may be less efficient than a capital intensive technique. To say that capital scarce or labor surplus countries should use labor intensive techniques may result in inefficient techniques which use both more labor and more capital to produce less output, although the ratio of capital to labor is lower.² Secondly, labor intensive techniques may result in less reinvestment because savings out of profits are higher than out of wages.³ Thirdly, labor intensive techniques may not be profitable if a) skilled labor is needed to

supervise less skilled workers, b) the necessary ratio between skilled and unskilled workers is relatively fixed, and c) skilled labor is limited in supply.⁴ All of these arguments are well taken, but they assume either fixed technical coefficients or that the relationship between savings and consumption is completely determined by the distribution of income between wages and profits. It is generally taken for granted that if one assumes a neoclassical production function with continuous substitutability and that the reinvestment problem can be solved either through income redistribution or investment incentives of various sorts, then the original labor intensity proposition holds.

This is not true. A neoclassical production function may result in an inverse relationship between the optimal capital/labor ratio and the relative scarcity of labor if a) there are non-constant returns to scale, b) there are capital saving innovations, or c) there is a high degree of complementarity between skilled and unskilled labor.

A. Non-constant Returns to Scale

The expression (5) for the change in the capital/labor ratio is not necessarily positive unless the assumption of constant returns to scale is made. In particular, the term $\frac{K}{L^2} \frac{f_{LK}}{f_{LL}} \dot{K}$ is negative if we assume that f_{LK} is positive, i.e., an increase in the capital stock raises the marginal productivity of labor. If the marginal product of labor is increased enough by addition to the capital stock, the capital/labor ratio may fall.

Another way of putting equation (5) is as follows:

$$(5^*) \quad \left(\frac{\dot{K}}{L} \right) = \left(\frac{1}{L} + \frac{K}{L^2} \frac{f_{LK}}{f_{LL}} \right) \dot{K} - \frac{K}{L^2} \frac{1}{f_{LL}} \dot{p}_L$$

The first term may be called a capital accumulation or scale effect. It may be positive or negative depending on whether $\frac{K}{L} \frac{f_{LK}}{f_{LL}}$ is greater or less than

minus one. The second term is the price or substitution effect which is always positive.⁵ If the scale effect is negative and outweighs the substitution effect, capital intensity decreases with capital accumulation.

B. Innovations

Next, let us assume that there are constant returns to scale but that both capital augmenting and labor augmenting innovations take place. The production function is

$$(9) \quad Y = f(Ke^{\beta t}, Le^{\alpha t}) = f(K^*, L^*)$$

where K^* and L^* are "effective" capital and labor, and K and L are actual capital and labor magnitudes. Then equilibrium requires

$$(10) \quad f_L(K^*, L^*) = p_L$$

As the capital stock is augmented and the price of labor relative to capital rises, the new equilibrium requires

$$(11) \quad f_{LL^*} \dot{L}^* + f_{LK^*} \dot{K}^* = \dot{p}_L$$

Using the chain rule, however,

$$(12) \quad f_{LL^*} = f_{LL} \cdot e^{-\alpha t}$$

$$(12) \quad f_{LK^*} = f_{LK} e^{-\beta t}$$

Furthermore

$$\dot{L}^* = e^{\alpha t} \dot{L} + \alpha e^{\alpha t} L$$

$$(13) \quad \dot{K}^* = e^{\beta t} \dot{K} + \beta e^{\beta t} K$$

Substitute (12), (13), and (7) into (11) and solve for \dot{L} .

$$(14) \quad \dot{L} = -(\alpha - \beta) L + \frac{1}{f_{LL}} \dot{p}_L + \frac{L}{K} \dot{K}$$

Substitute (14) into (4) to obtain the change in the capital/labor ratio

$$(15) \quad \left(\frac{\dot{K}}{L} \right) = \frac{K}{L} (\alpha - \beta) - \frac{K}{L^2} \frac{1}{f_{LL}} \dot{p}_L$$

The second term on the right hand side (call it the price effect) of (15) is always positive. The first term (call it the innovation effect) may be negative or positive. If the rate of capital saving innovation exceeds the rate of labor saving innovation ($\beta > \alpha$), then the net effect of innovation is a tendency to increase the labor requirements faster than the increase in capital stock. If the innovation effect (the first term) outweighs the price effect (second term), the result of development may be to reduce the capital intensity of production.

C. The Skilled Labor Factor

Next let us consider the case in which skilled labor (denoted by S) is included in the production function.

$$(16) \quad Y = f(K, L, S)$$

Labor and skilled labor are hired up to the point where price equals marginal revenue productivity.

$$(17) \quad \begin{aligned} f_L(K, L, S) &= p_L \\ f_S(K, L, S) &= p_S \end{aligned}$$

As capital is accumulated and factor prices change, the equilibrium is disturbed

$$f_{LK} \dot{K} + f_{LL} \dot{L} + f_{LS} \dot{S} = \dot{p}_L$$

(18)

$$f_{SK} \dot{K} + f_{SL} \dot{L} + f_{SS} \dot{S} = \dot{p}_S$$

Solve for \dot{L} in terms of \dot{K} , \dot{p}_L and \dot{p}_S

$$\dot{L} = \frac{f_{SS} \dot{p}_L - f_{LS} \dot{p}_S + (f_{LS} f_{SK} - f_{SS} f_{LS}) \dot{K}}{f_{LL} f_{SS} - f_{LS}^2}$$

(19)

For simplicity, let us assume constant returns to scale. Euler's Theorem holds

$$f = f_K \cdot K + f_S \cdot S + f_L \cdot L$$

(20)

Differentiate (20) with respect to L and S to obtain

$$f_{KL} \dot{K} + f_{LL} \dot{L} + f_{SL} \dot{S} = 0$$

(21)

$$f_{KS} \dot{K} + f_{LS} \dot{L} + f_{SS} \dot{S} = 0$$

Solve (21) for L in terms of K.

$$L = \frac{(f_{LS} \cdot f_{KS} - f_{SS} f_{KL}) K}{f_{LL} f_{SS} - f_{LS}^2}$$

(22)

Solve (22) for the expression in the denominator on the right hand side and substitute into (19)

$$(23) \quad \dot{L} = \frac{(f_{SS} \dot{p}_L - f_{LS} \dot{p}_S)}{(f_{SL} f_{KS} - f_{SS} f_{KL})} \frac{L}{K} + \frac{L}{K} \cdot \dot{K}$$

Substitute (23) into (4)

$$(24) \quad \left(\frac{\dot{K}}{L} \right) = \frac{1}{L} \frac{(f_{LS} \dot{p}_S - f_{SS} \dot{p}_L)}{(f_{SL} f_{KS} - f_{SS} f_{KL})}$$

Under the usual assumptions of diminishing returns ($f_{SS} < 0$) and that ^{increases} in the amount of one factor raises the marginal productivity of other factors ($f_{SL} = f_{LS} > 0$, $f_{KS} > 0$, $f_{KL} > 0$), the denominator of (24) is positive and the second term in the numerator is positive for increases in the price of unskilled labor ($\dot{p}_L > 0$). If the price of skilled labor falls, however, ($\dot{p}_S < 0$), the second term in the numerator is negative. This negative effect of a decreasing scarcity of skilled labor may outweigh the positive effect of increasing scarcity of unskilled labor and cause the capital/labor ratio to fall as development proceeds. In other words, the less developed country may have a higher capital/labor ratio because of a scarcity of skilled labor. This is more likely, the higher is the degree of complementarity of skilled and unskilled labor (f_{LS} positive and large) and the less the strength of decreasing returns to skilled labor (the smaller is f_{SS} in absolute value).

D. Conclusions

The analysis here presupposes that labor supply is not completely inelastic. Obviously, if this were the case and the marginal product of labor were not zero, a country with more capital relative to labor would always employ a more capital intensive technique (merely because maximum output would be attained by employing all capital and all labor). If, however, the labor supply is more than zero elastic, then capital intensity of technique is not a monotonically increasing function of the price of labor relative to capital⁶ because of a) scale effects, b) innovational effects, and c) the presence of a scarce third factor.

¹A dot over a variable represents the time rate of change of that variable, i.e., $\dot{p}_L = \frac{dp_L}{dt}$.

²Examples of this type of argument can be found in Hans Singer, International Development: Growth and Change, New York, McGraw-Hill, 1964, pp. 59-60; Walter Galenson and Harvey Leibenstein, "Investment Criteria, Productivity and Economic Development," Quarterly Journal of Economics, Vol. LXIX (August 1955), p. 348 and International Labor Office, Employment Objectives in Economic Development, Report of a Meeting of Experts (Geneva 1961), p. 67.

³See Galenson and Leibenstein, op. cit., A.K. Sen, "Some Notes on the Choice of Capital Intensity in Development Planning," Quarterly Journal of Economics, Vol. LXXI (November 1957), p. ; and M. Dobb, "Second Thoughts on Capital Intensity of Investment," Review of Economic Studies, Vol. XXIV (1956-57), p. .

⁴See A. O. Hirschman, The Strategy of Economic Development, New Haven, Yale University Press, p. 145; Hla Myint, The Economics of Developing Countries, London, Hutchinson and Co., 1964, p. 137 and Werner Baer and Michel E.A. Hervé, "Employment and Industrialization in Developing Countries," Quarterly Journal of Economics, Vol. LXXX (February 1966), pp. 97-102.

⁵The terms substitution effect and scale effect are not exactly but somewhat analogous to the substitution and income effects in the theory of consumer equilibrium.

⁶As the labor supply curve becomes more and more inelastic, the effect of capital accumulation is unambiguous in the direction of more capital intensity. Thus a "reversal" of the direction of change of the capital/labor ratio is possible. This is not to be confused with the factor reversal phenomenon noted by Sraffa, Robinson, Pasinetti and others. (See "Paradoxes in Capital Theory: A Symposium" in the Quarterly Journal of Economics, Vol. LXXX (November 1966), pp. 503-583.) If capital goods are not homogeneous, then there is not necessarily a monotonically increasing relation between the rate of profit and the capital intensity of production regardless of what method of capital aggregation is used. We assume this problem away by assuming homogeneous capital goods but switches in capital intensity come about for other reasons.