PLANNING MODELS AND DEVELOPMENT POLICY: COMPUTABLE GENERAL EQUILIBRIUM MODELS

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Tentative Chapter Outline

PLANNING MODELS AND DEVELOPMENT POLICY

Chapter 1: Introduction

1.1 The Goals of Planning. This section will discuss the objectives and nature of the planning process.

1.2 The Policy Environment. Here we will briefly discuss the nature of economic institutions, policy tools and strategic policy choices in developing countries.

1.3 Planning Models and Economic Policy. This section will review the kinds of models that have been used and their relation to policy analysis. We will discuss in some detail the nature of policy instruments included in different kinds of models.

1.4 The Accounting Framework. This section will discuss the range and types of data required to implement planning exercises. We will develop the Social Accounting Matrix (SAM) as an organizing framework.

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2.2 Dynamic Input-Output Models and Consistent Growth Paths.

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2.4 Linear Programming Models.

2.5 Dual Solutions and Shadow Prices.

2.6 Foreign Trade and the "Make-or-Buy" Choice.

2.7 Terminal Conditions and Long-run Equilibrium Concepts.

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Chapter 3: Computable General Equilibrium Models

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3.3 The General Equilibrium Solution.

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Introductory Statement

This book is concerned with economic planning at the central government level in developing countries. It deals with the mathematical models that have been and are being constructed to help central planners in their attempts to hasten and harmonize the pace of development.

While the exact meaning of "planning" may vary depending on context and circumstances, the process of planning always implies a desire to increase control over events, to reduce the realm of chance, to steer the economy onto a desired path. Planning does imply normative choices and centralized action. But the process of planning need not be viewed as incompatible and antagonistic to the market process. We feel that much of the "planning or the market" debate has been misguided. It is quite clear that like many biological or technical systems, economic and social systems are characterized by important and inherent self-regulating mechanisms. But it seems equally obvious that these self-regulating mechanisms are subject to serious disturbances, that even when they function normally their outcomes are not always desirable and that the direction they give to the economy may not be optimal.

We therefore place ourselves resolutely within the context of the mixed-economy throughout this book. Planning is not seen and analyzed as a process that replaces the market but as an endeavor to complement it and help achieve more desirable outcomes. The focus is on development planning and therefore on long-term problems of growth, structural change,
trade strategy and income distribution. These problems are more micro-economic than macroeconomic in nature and the planning models we use and discuss have to build on general equilibrium theory, trade theory and growth theory. An elaborate analysis of more short-term macroeconomic and monetary problems is outside the scope of this book. The emphasis is on long-run trends and development policy, not on short-run cycles or stabilization policy.

The finished 3 chapters and the as yet unfinished chapter on linear models are designed to set the stage for the applications and policy experiments to be undertaken later. It is hoped that they will clarify the basic nature of economy-wide development planning models, their relationship to theoretical reasoning and their usefulness in generating a dialogue between the model builder, the theorist and the practical policy maker. The second half of the book will then turn to applications focusing on a detailed analysis of the interrelationships between government policy and trade, growth and income distribution in developing economies.
Chapter 6: Economic Planning and the Distribution of Income

6.1 Introduction.


6.3 Generating the Distribution of Income in a CGE Model.

6.4 Measures of the Distribution of Income.

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Chapter 7: Applications: The Analysis of Distributional Issues

7.1 Analyzing the Sources of Inequality: An Application of the Decomposition Methodology to Turkey. This section will use the methodology presented in chapter 6, section 6.3, to analyze the nature and sources of income inequality in Turkey without using a formal planning model.

7.2 Trade Policy and Income Distribution in Colombia: An Analysis with an Eight-Sector Dynamic Model.

Chapter 8: Conclusion

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Computable General Equilibrium Models

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CHAPTER IV

Computable General Equilibrium Models

4.1 Introduction

Multi-sector, economy-wide mathematical models have now been used for over two decades as development planning models. Although the more sophisticated dynamic programming models have not yet significantly crossed academic boundaries, most developing countries have extended their national accounting effort to include input-output transaction tables. Indeed, many countries had by the early seventies adopted input-output analysis as the basic framework for their formal planning exercises. There is no doubt that the techniques of development planning have acquired a wide field of potential real-world application.

The word "potential" should however be stressed. In spite of a generally very favorable atmosphere at the national policy making level, it is very hard to claim that the by now traditional linear models have had much impact on development strategy or policy making. Paradoxically, even in the hands of official central planners, these models have largely remained academic demonstration exercises.

Why has this been so? The answer to this question may not be a simple one and certain institutional and political factors play a more important role than is sometimes realized. When considering the origin and development of this branch of economics, a few important facts
relating to the nature of the models proposed as planning models will help explain why they have remained as largely academic constructs.

Both input-output analysis and linear programming were first designed as planning and decision tools in a setting where a central decision-maker fully in control of the various quantity variables in the system has to make consistent or optimal decisions. W. Leontief's work in input-output analysis was initially motivated by the problems of consistent plan formulation in the non-market setting of the Soviet economy. Indeed input-output analysis is often primarily regarded as the solution to the famous problem of material balancing in the productive sphere of a centrally planned economy. Kantorovich perceived in a programming approach a clear link between centralized planning and the scarcity price concept of neo-classical economic theory, while Dantzig developed linear programming as a tool for optimal central decision making for primarily military purposes. It is quite clear that the standard linear programming formulation is best suited to problems where a single decision maker optimizes a central welfare function subject to technological and physical constraints. The standard formulation does not appear so well suited to modelling situations where many agents independently maximize their own welfare functions and jointly but inadvertently determine an outcome that can only be affected indirectly by the planner or policy maker.

Once these basic features of the traditional linear techniques are recalled, it is perhaps not surprising that the resulting development planning models have been constructs that fit much better the setting of
a pure command economy than the real-world of the developing economies, almost all characterized by mixed economic systems where a great deal of economic behavior is not under the direct control of a central planner and where autonomous decision making by sub-units and market-price formation mechanisms have a determining impact on resource allocation.

The situation is reflected in the fact that, given the institutional set-up of most LDC's, neither linear programming models nor input-output models usually contain variables that can be considered to be instrument variables controlled and determined by the policy maker or planner. Although policy makers are often impressed by the economy-wide coverage and overall consistent picture provided by the mathematical models, they cannot easily relate the computed variables to any actual policy decision. Most often what emerges is only a rather vague notion of "desired sectoral structure" that may influence investment licensing and credit rationing practices.

In order to achieve greater policy relevance, it is clear that the fiction of a central command economy must be abandoned in the very specification of the model and be replaced by a framework in which endogenous price and quantity variables are allowed to interact so as to simulate the workings of at least partly decentralized markets and autonomous economic decision-makers. Such price endogeneity and general equilibrium interaction cannot be achieved using the standard linear programming formulation. The crucial difficulty lies in the fact that
economic behavior and relations such as budget constraints, consumption functions and saving functions must be expressed in current endogenous-factor and commodity prices. But the standard primal constraint equations of a linear program cannot include the "shadow" prices that result as a by-product of the maximization. Or, to put it differently, one cannot in general expect that the resource allocation and production structure determined by the solution of a linear program is consistent with the incomes and budgets that result from its dual solution. Indeed, if factor prices have any impact on the structure of demand, the quantities supplied that are the outcome of the primal solution will in general not equal the quantities demanded that are implied by the dual solution.

It is convenient to discuss the problem using the basic material balance equations that form the core of any planning model. Let us for simplicity consider the case of a closed economy. In equilibrium we must always have:

\[(4.1.1) \quad X_i = \sum_{j=1}^{n} a_{ij} X_j + C_i \quad i=1, \ldots, n \quad (4.1)\]

where \(X_i\) denotes gross sectoral output, \(C_i\) represents final consumption and the \(a_{ij}\) are the familiar input-output coefficients determining intermediate demand. In a mixed market economy the relative prices of commodities and factors will have a determining influence on the vector of outputs supplied \(X=[X_1, \ldots, X_n]\), and final demands \(C=[C_1, \ldots, C_n]\). Now let us assume for the moment that the production sphere of the economy is characterized by linear-coefficients activities and let us denote by
the various resources, fixed in supply during the period considered, that limit productive capacity. The production activities are characterized by the usual commodity input coefficients, \( a_{ij} \), supplemented by resource input coefficients, \( r_{ij} \). Given an arbitrary commodity price system \( P_i \), \( i = 1, \ldots, n \) it is clear that the solution of the following linear program can be considered as a market solution for the productive sphere of the economy:

\[
\text{Max: } \sum_i V_i X_i
\]

Subject to:

\[
\begin{align*}
\sum_j r_{sj} X_j &\leq \bar{R}_s & s = 1, \ldots, m \\
X_j &\geq 0 & j = 1, \ldots, n
\end{align*}
\]

where \( V_i \) is the net price or per-unit value added defined as:

\[
V_i = P_i - \sum_j a_{ij} P_j
\]

Associating a "producer" or "firm" with each productive activity, the solution of this program is consistent with what would be achieved by a market process during which the autonomous "firms" would attempt to maximize profits bidding for the scarce factors that themselves must be considered as mobile between activities and offering their services to the highest bidder. The shadow prices that the dual solution of the programming problem assigns to the scarce resources would in fact correspond
to the values the market would assign to them. If we denote these resource
prices by \( W_i \), the duality theorem also guarantees that total factor in-
come equals total net product value, i.e:

\[
\sum_{s} \bar{W}_s = \sum_{i} X_i
\]

Summarizing, we see that to each price vector \((P_1, \ldots, P_n)\)
corresponds a linear programming solution that associates outputs \((X_1, \ldots, X_n)\)
and factor prices \((W_1, \ldots, W_m)\) with the price vector. We can regard
the relationship between the price vector and the elements of the output
vector as \( n \) supply functions of the form:

\[
X_i^s = g_i (P_1, \ldots, P_n)
\]

Note that since the feasible set defined by the input-use and
non-negativity constraints is bounded, any price vector will lead to a
solution of the linear program.\(^1\) It is however true that an arbitrary
positive price vector may lead to a value-added vector with negative com-
ponents. We must therefore assume that the only "allowable" price vectors
will be those such that \( P_i - \sum_j a_{ij} P_j \geq 0 \) for all \( i \) and return to the
problem later.

Another problem relates to the uniqueness of the supply response.
When the price hyperplane is tangent to a facet of the feasible set, the

\( ^1 \) We can of course restrict our search for an appropriate price vector
to the economically meaningful non-negative orthant.
linear program will have a non-unique solution and more than one vector of output supplies will be consistent with a given price vector. This does not create any fundamental problem: one simply has to redefine the supply functions as supply correspondences. We shall in fact use the term function more loosely to include point-to-set mappings.

But let us now consider the demand side of this simple market economy. We shall assume that all resources and factors \( R_{oi} \) are owned by households that use their income derived from letting firms use their resources to demand the commodities produced for final consumption. Each category of households, \( q \), will have a system of expenditure equations that relate quantities demanded, \( C_{iq}^d \), to household incomes and commodity prices. Let us denote the income of household category or socioeconomic group \( q \) by \( Y_q \). It is clear that no matter how resource ownership is distributed, \( Y_q \) will be a linear homogenous function of resource rental prices, \( (W_1, \ldots, W_m) \). The consumer demand functions are of the form:

\[
(4.1.6) \quad C_{iq}^D = h_{iq}(P_1, \ldots, P_n, Y_q)
\]

Where \( C_{iq}^D \) is the amount of commodity \( i \) demanded by household category \( q \), \( h_{iq} \) expresses the form of the demand functions and \( Y_q \) is determined by exogenous ownership shares and resource rentals. Aggregating over groups we obtain market demand functions of the form:

\[
(4.1.7) \quad C_i^D = \sum_{q} C_{iq}^D = \sum_{q} h_{iq}(P_1, \ldots, P_n, Y_q) \quad i=1, \ldots, n
\]

To consumer demand, we must add intermediate demand by firms:
(4.1.8) \[ N_i^D = \sum_{j=1}^{n} a_{ij} y_j^S \]

\[ i=1, \ldots, n \]

Together, consumer and intermediate demand constitute total demand for the various products produced in the system:

(4.1.9) \[ x_i^D = c_i^D + n_i^D \]

But resource rentals (i.e., factor prices) and therefore incomes were themselves determined by the solution of the linear program which of course depends on the initial choice of commodity prices. The price vector \((P_1, \ldots, P_n)\) is thus seen to lead to a supply solution \((x_1^S, \ldots, x_n^S)\) via the linear program and to a demand solution \((x_1^D, \ldots, x_n^D)\) via the demand functions derived from the preferences and budget constraints of the various household categories. We have:

\[ (P_1, \ldots, P_n) \xrightarrow{g_i} (x_1^S, \ldots, x_n^S) \quad i=1, \ldots, n \]

\[ \quad \quad \quad (P_1, \ldots, P_n) \xrightarrow{d_i} (x_1^D, \ldots, x_n^D) \quad i=1, \ldots, n \]

When the \(g_i\) and the \(d_i\) are the aggregate market supply and demand functions, the former determined by the linear programs and the latter representing the sum of consumption and intermediate demands, taking account of the fact that consumer incomes are themselves functions of the initial price vector.

Our basic material balance equation requires that \(x_i^S = x_i^D\) for all \(i\). But clearly it would be pure coincidence if this equality were satisfied
for an arbitrary price vector \((P_1, \ldots, P_n)\). In general we will have \(X^S_1 \neq X^D_1\). Rewriting the equilibrium requirement in a form exactly analogous to input-output analysis, we have:

\[(4.1.10) \quad X^S_1 = C^D_1 + N^D_1 = X^D_1\]

where \(X^S_1\) can be interpreted as gross supply functions associating production levels with commodity prices. \(N^D_1\) are the intermediate demand functions derived from the gross supply functions and \(C^D_1\) are the final demand functions. We now see how the solution problem reduces to that of finding a set of commodity prices (weights in the objective function of the linear program) such that the supply decisions made by firms exactly match the demand decisions made by households whose incomes are determined by the shadow prices emerging from the linear program.

This general equilibrium or fixed-point problem is clearly a much more subtle one than that faced by input-output analysis or standard economy-wide programming models. In the former the solution is very simple and we simply have the familiar input-output matrix equation:

\[X = (I-A)^{-1}C\]

For linear programming models the solution is a little more difficult to obtain. Given certain exogenous preference weights in the objective function, \(X\) is obtained by "solving" the productive sphere of the economy. But the final demand vector is not linked to the factor incomes implicit in the solution and there is therefore no feedback
mechanism that would require an adjustment in prices. The simple model outlined above did, however, include such a feedback.

Generalizing, we shall define a computable general equilibrium model (CGE model) to be a model that includes this basic feedback. These models are also sometimes called price-endogenous models because commodity prices must adjust until the decisions made in the productive sphere of the economy are consistent with the final demand decisions made by households and other autonomous decision makers. General equilibrium feedback mechanisms and autonomous decision making are the concepts that must be stressed. This does not imply that the CGE framework insists on perfectly competitive models and instantaneous market clearing or that it can handle only very limited government intervention. On the contrary, imperfectly competitive behavior, quantity or price adjustment lags and widespread government intervention are compatible with the CGE framework. But the CGE approach does stress horizontal interaction between economic agents, suboptimizing autonomous behavior and the workings of market clearing processes. It aims to be applied general equilibrium analysis. As such, the approach is much better suited to planning and policy analysis in mixed market economies than to planning in highly centralized systems, be they whole economies or individual corporations.

The CGE approach is worthwhile when analyzing problems for which we believe general equilibrium interactions to be important. In the context of planning models for policy analysis, investment allocation, foreign trade strategies and policies directed towards income distribution issues
provide examples where complex interactions among various agents in the economy affect the outcome of policies. These various interactions and their quantitative importance will emerge in the course of the discussion of specific numerical applications in later chapters.

There are two remaining tasks for this chapter. The first is to provide a more detailed presentation of a streamlined CGE model, emphasizing the core equations and establishing the framework which will form the basis of discussions of foreign trade and income distribution in chapters 5 and 6. This presentation will abstract from specific functional forms, with the exception of the assumption of a Leontief technology for intermediate and capital goods which has been a basic assumption of the linear models of chapters 2 and 3 and which we have chosen to maintain to provide continuity in the presentation. The resulting model is quite neoclassical in spirit. The next three sections will mostly leave aside consideration of market structures, imperfections and institutional characteristics of developing countries. Consideration of these issues is the second task for this chapter and will be covered in sections 4.6 and 4.7 where the problems of equilibrium and intertemporal linkages will be reviewed in some detail. There we will consider some important non-neoclassical characteristics of developing countries that must be incorporated into CGE planning models if they are to be useful tools for policy analysis.

4.2 The Basic Structure of a CGE Model

In the introductory section above we presented the fundamental
"feedback" problem faced as soon as one models the decentralized interaction of autonomous producers and consumers. A simple linear program was used to describe the supply side of the economy and we ignored government activity and investment. The problem of overall equilibrium was posed but not solved. In this section we shall discuss the basic structure and equations of a somewhat more complete closed-economy CGE model that can be thought of as a core model underlying the whole family of models that have been built using the same basic approach. A discussion of the general equilibrium solution follows in section 4.3 while alternative solution strategies and model specifications are briefly discussed in section 4.4.

A few notational rules will be generally observed in this and the following chapters. Roman capital letters without bars are endogenous variables. All other characters denote predetermined variables. There are \( n \) sectors, \( m \) labor categories and capital; the corresponding subscripts are \( i, j \) for sectors and \( s \) for labor categories. Superscripts \( D \) and \( S \) are used to distinguish quantities demanded from quantities supplied.

**The Supply of Commodities**

In the typical CGE model each commodity distinguished in the economy is associated with a production sector, very much in the tradition of input-output analysis. Indeed the sectors are usually sectors in an input-output table. In the linear models presented in chapter two, a fixed coefficients technology was specified for intermediate inputs,
capital composition, and for non-produced primary factors. Although it is by no means necessary, existing CGE models have often retained the assumptions of fixed-coefficients for intermediate technology and the composition of capital goods. In contrast, the production technology for primary factors is described by a neoclassical production function that allows smooth substitution between several factor inputs. The degree of substitutability is governed by the elasticities of substitution specified. It would be quite feasible and certainly compatible with the basic modelling approach to specify sets of alternative linear production activities instead of neoclassical production functions. But production functions have usually been preferred to activity analysis representations of the technology because they require a much more moderate data collection effort. The really important question in this context relates to the degree of substitutability one believes ought to be specified. For most purposes in economy-wide modelling it can reasonably be argued that the use of CES production functions with realistic substitution elasticities will capture most of the interactions one wants to analyze. Given the necessity of aggregating sectors in any applied economy-wide model into a relatively small number, it should be made quite clear that the production functions are only very rough representations of actual technical production processes. But they constitute an extremely flexible and convenient tool and their careful use at the sectoral level is unlikely to distort seriously the representation of the underlying technology. Rather than using a linear programming specification, we shall therefore use neoclassical production
functions throughout the various applications in this book.

Sectoral gross outputs are related to inputs according to a
two-level production function which can be written generally as:

\[(4.2.1) \quad X_i = f(\bar{A}_i, \bar{K}_i, L_i^a, N_i^a) \quad i=1, \ldots, n\]

where:

- $X_i$ is sectoral output,
- $\bar{A}_i$ is a shift parameter which dynamically will reflect disembodied technical progress,
- $\bar{K}_i$ is the stock of the aggregate capital good assumed to be fixed by sectors,
- $L_i^a$ is an aggregation of labor inputs,
- $N_i^a$ is an aggregation index of intermediate inputs.

The parameter $\bar{A}_i$ is constant within a period and depends on the units in which output and inputs are measured. The sectoral capital stock, $\bar{K}_i$, is assumed fixed within each period. A unit of sectoral capital stock is assumed to consist of fixed proportions of different investment goods (construction, machinery, etc.), with the proportions varying among sectors, exactly as in dynamic input-output models.

Since we choose to assume a Leontief input-output technology for intermediate inputs, we need not specify a separate aggregation function for an intermediate goods aggregate. Given that both the shares among different intermediate inputs in a sector and
the ratios of intermediate inputs to output are fixed, we can write the

demands for intermediate inputs directly:

\[
(4.2.2) \quad N_{ij} = a_{ij}^\varepsilon j \quad i = 1, \ldots, n \quad j = 1, \ldots, n
\]

where \(a_{ij}\) are the input-output coefficients. We can then aggregate inter-
mediate demands to get total intermediate demand by sector of origin:

\[
(4.2.3) \quad N_i = \sum_{j=1}^{n} N_{ij} = \sum_{j=1}^{n} a_{ij}^\varepsilon j \quad i = 1, \ldots, n
\]

The use of fixed coefficients obviates the need for a separate aggregation
function to define \(N_i^a\) and, in this case, the variable is not required.
If we had wished to specify substitution possibilities among intermediate
goods or among aggregate intermediate goods and labor and capital, then a
separate aggregation function for intermediate goods would be required.

The sectoral labor input is assumed to be an aggregation of labor
of different skill categories. The function will be represented by the
equation:

\[
(4.2.4) \quad L_i^a = L_i^a(L_{i1}, \ldots, L_{im}) \quad i = 1, \ldots, n
\]

In all applications, this aggregation is given by either a CES or Cobb-
Douglas function with a single elasticity of substitution among all labor
categories.

To summarize, the production technology exhibits a number of
special characteristics. It is a CES or Cobb-Douglas function of aggregate capital and aggregate labor. Capital is a fixed coefficient aggregation of investment goods. Labor is a CES or Cobb-Douglas aggregation of labor of different skill categories. The production function is thus a two-level function. Intermediate goods are required according to fixed coefficients and so can be treated separately.

In terms of modern neoclassical production functions, this specification of production technology seems cumbersome and even raises some problems in proving the existence of an equilibrium solution that are considered in the next section. Mathematically, specifying equation (1) as a generalized or multi-level CES function would have been more consistent. There are, however, a number of advantages in this specification. First, retaining a number of the assumptions underlying linear models facilitates exposition and comparisons between the CGE and earlier models. We have extended the earlier specification in those areas where it is necessary for an adequate specification of a model designed to capture price-sensitive general-equilibrium market interactions. Second, specifying more elaborate production functions leads to a proliferation of parameters that must be estimated. Parameter estimation has always been a difficult problem in economy-wide planning models, and one must always consider the trade-offs between adequate econometric estimation and adequate structural specification. This specification requires a minimum of additional parameter estimation beyond that already required for linear planning models. Finally, the
particular simplifications used here seem to be empirically reason-
able for the levels of aggregation, the types of issues, and time horizons
usually considered.

The judgments underlying the model choices are all subject to
revision as more information and experience are acquired. Given more data
and a higher level of disaggregation, one might well wish to use different
specifications of technology. One certainly need not be restricted to CES
functions. Given the data and the appropriate focus on problems, one
might specify activity analysis, translog functions, or directly estimate
cost functions. It is clear from the presentation of the core model above that
the CGE framework is flexible enough to incorporate a wide variety of
specifications of production technology.

In addition to the specification of technology, the basic model
also incorporates important assumptions about factor mobility. Usually,
it is reasonable to assume that the amount of capital in each sector is
fixed at the beginning of the period being modelled. This implies that
current investment will add to capacity only in future periods: a realistic
assumption. For some purposes, it may however be better to leave sectoral
capital stocks as endogenous variables. Assume, for instance, that one is
simply interested in finding an equilibrium configuration for some future
year without actually modelling the path leading to this configuration.
Then it is sensible to allow the model to determine sectoral capital stocks
endogenously. It might be possible to find, with a separate "transition
path" model, a sequence of investment allocations that would lead to the
terminal capital configuration.

Most often, however, CGE models are run forward in time from given initial conditions and the most important of these initial conditions is given by the capacity installed in the various sectors in the base year. We shall therefore, at this stage of the discussion, assume fixed sectoral capital stocks.

The specification of the production set of the economy is incomplete without a set of factor availability constraints. They usually take the simple form:

\begin{equation}
\sum_{i} L_{is} = L_{s} \quad \text{s=1, \ldots, m}
\end{equation}

Where \( L_{s} \) denotes the fixed supplies of the various categories of labor. In principle one could generalize to let the \( L_{s} \) include natural resources, although this is seldom done.

To distinguish the net production possibility set from the gross production possibility set \( \mathbf{X} = (X_1, \ldots, X_n) \), denote the net production of sector \( i \) by \( X_i^N = X_i - \sum_{j} a_{ij} X_j \). The resulting net production possibility set corresponding to the gross production possibility set is strictly convex if the gross production possibility set is strictly convex and the Hawkins-Simons conditions are satisfied. Convexity is assumed if either capital stocks are fixed or capital stocks are mobile and none of the sectors have the same factor intensity. The net production possibility set will be "flatter" than the gross production possibility set and completely
contained within it, except at the boundaries. The degree of convexity is of course increased when the number of sectorally fixed factors increases since this assumption amounts to assuming decreasing returns to scale to the variable factors of production.

The specification of the production possibility set is the first step in specifying the supply side of the economy. It should be noted that the "production possibility set" is a strictly technical description of attainable combinations of outputs. It should be distinguished from the "transformation set" which includes, in addition, various assumptions about market behavior. The nature of these additional assumptions will be considered below and in the next chapter.

If we were interested in specifying and maximizing a central objective function ignoring market behavior and the feedback from factor prices to commodity demands, then we could treat the system as a non-linear program and maximize the objective function subject to the production possibility set and the appropriate non-negativity restrictions. The problem is a non-linear generalization of the linear programming maximization problem presented in section 4.1 above. Exactly as in the linear programming case, there will correspond to each net price vector (the weights in the objective function) a solution on the boundary of the net production set. Conversely, at each point on the boundary of the production set, there exists a net price vector (i.e., a separating hyperplane) that will make the point the solution to the maximization problem.
But rather than viewing the problem as a programming problem, one can specify the behavior of firms in the economy and add an explicit description of the workings of factor markets. Under the assumption of perfect competition in both factor and commodity markets, the supply response will be the same whether we treat the production side of the economy as a programming problem or as a decentralized market. If we drop the assumption of perfect competition, the supply functions will no longer be the same and we must replace the production possibility frontier with a transformation set.

Each sector in the economy is treated as made up of many similar firms maximising profits and bidding for the scarce factors. The assumption of perfect competition in product markets amounts to assuming that firms take commodity prices as given. Under these circumstances, one can treat each sector as one large price-taking firm. The aggregate sectoral profit functions are given by:

\[(4.2.6) \quad \Pi_i = P_i (1 - t_i^X) X_i - \sum_j a_{ji} X_j - \sum_s W_s L_s \quad i=1, \ldots, n\]

where \(W_s\) is the wage of labor type \(s\) and \(t_i^X\) is the indirect tax rate.

Although the primary concern in this simple model is not with policy formulation, the inclusion of a government sector leads naturally to the inclusion of indirect and direct taxes in the system.

The profit equation can be rewritten as:

\[\Pi_i = v_i X_i - \sum_s W_s L_s \quad i=1, \ldots , n\]
where \( V_i = P_i (1 - t_i) - \sum_{i=1}^{n} P_j a_{j_i} \), is the net price or value-added coefficient, this time net of indirect taxes.

To maximize the profit function, set its derivatives with respect to variable factors of production equal to zero:

\[
(4.2.7) \quad V_i \frac{\partial X_i}{\partial L_i^a} = \frac{\partial L_i^a}{\partial L_is} = W_s \quad i=1, \ldots, n \\
\quad s=1, \ldots, m
\]

These are the familiar conditions that wages equal the value of the marginal products of different labor types. These equations give the demand functions for labor by sectors and may be written as:

\[
(4.2.8) \quad L_{is}^D = F(K_i, V_i, W_s)
\]

Depending on the production function used, these \( n \cdot m \) equations (one for each sector and each labor category) may be solved directly or by numerical methods. Should a neoclassical specification requiring full employment of all categories of labor be adopted, then the resource constraints will be binding and the wage for each labor category will adjust until the sum of sectoral demands for each category equals the fixed supply of that category.

However, in most of our applications, a specification where the supply of labor of certain categories is a function of the wage rate will be preferred. In the extreme case, labor is in infinitely elastic supply at a fixed real wage given by:
(4.2.9) \( \bar{W}_s = \overline{W}_s \prod_{i=1}^{l} \Omega_i \)

where the \( \Omega_i \) are weights in the price index that define the real wage \( \bar{W}_s \).

This formulation creates no complication since with fixed capital stocks the transformation set will still be strictly convex.\(^{1/}\) If this formulation is adopted for certain categories of labor, the resource contraints become side equations giving the total possible employment of labor in the economy, and the wages in the labor demand equation become a fixed argument along with capital stocks.

Whether a neoclassical or fixed wage specification is adopted creates no difficulty since capital - assumed to be fixed during the period considered - plays the role of a constant, sector-specific factor. Given the ensuing diminishing returns to labor and given the general continuity properties of neoclassical production functions, solving for a given set of positive net prices and wages will never constitute a theoretical problem. The payments to capital in each sector are defined residually after payments for labor and intermediate inputs. Total factor payments will therefore, by definition, equal total value-added generated.

Given an arbitrary vector of allowable commodity prices leading to a non-negative vector of net prices, each sector will minimize profits subject to its capital stock, its technology and the wages of the various types of labor. For neoclassical production functions, the marginal

\(^{1/}\) If capital stock were mobile across sectors, the transformation curve would contain linear segments. For further discussion, see Chapter 5.
productivity curves are strictly concave and the solution will in fact be unique. Substituting the solution values of \( L_{is} \), \( i=1, \ldots, n \), \( s=1, \ldots, m \) into the labor aggregation and production functions will yield a unique vector of outputs, \((X^S_1, \ldots, X^S_n)\) that constitutes the supply vector associated with a given price vector \((P_1, \ldots, P_n)\).

In the case where wages are not fixed, we simply add the \( m \) factor exhaustion equations to the system defining the supply response of the economy. We then have \( n \cdot m + m \) equations to be solved for the \( n \cdot m + m \) variables, \( L_{is} \) and \( W_s \). Again the strict convexity guarantees a unique solution so that we obtain a set of well behaved numerical supply functions of the form:

\[
\text{(4.2.10)} \quad X^S_i = \varphi_i (P_1, \ldots, P_n)
\]

that associate a vector of output supplies with each allowable price vector. It will usually not be possible to write them out explicitly, but they can be obtained numerically.

As a by-product of computing sectoral supplies associated with a given price vector by solving factor markets we obtain factor incomes, including the residual value added accruing to capital. We are thus ready to move on to the demand side of the system.

**Income Generation and the Demand for Commodities**

The decision-making units that determine the demand for commodities are the various categories of households which demand consumer goods, the
government which also demands consumer goods, and the firms themselves which demand intermediate goods and capital goods. For simplicity we shall assume in this chapter that each household category is characterized by a single type of factor that it owns and supplies. Thus there will be \( m+1 \) categories of households, the first \( m \) categories supplying the different kinds of labor distinguished in the discussion of factor markets and the last category being constituted by the owners of capital who receive the non-contractual residual amount of value-added. Again for simplicity assume that the government does not own any capital and receives its income only through direct and indirect taxes. Given these simplifying assumptions, one can write:

\[
(4.2.11) \quad Y_s = \sum_i \sum_{L_i} s (1 - t_s^Y) \quad s=1, \ldots, m
\]

\[
(4.2.12) \quad Y_k = [\sum_i \sum_{L_i} s (1 - t_k^Y)] (1 - t_k^Y)
\]

\[
(4.2.13) \quad Y = \sum_s \frac{t_s^Y}{1 - t_s^Y} Y_s + \frac{t_k^Y}{1 - t_k^Y} Y_k + \sum_{i,i} \sum_{L_i} X_i^P Y_i
\]

where \( Y_s, s=1, \ldots, m, Y_k \) and \( Y \) represent the net incomes of the \( m+1 \) household categories and the government, and \( t_s^Y, s=1, \ldots, m \) and \( t_k^Y \) are the direct average tax rates applying to the different groups and are assumed to be independent of the level of income. Note that by definition we always have:
(4.2.14) \[ \sum_{s} Y_s + Y_k + Y_g = \sum_{j} \hat{a}_{ij} X_j - \sum_{j} \hat{a}_{ij} X_j \]

so that total income generated in the system always equals total national product at market prices.

The government and the capitalist and labor households must now decide how to spend their incomes. Following a convenient if not theoretically very satisfying practice, assume that prior to any consumption decisions they make, the various household groups and the government decide on the proportion of their income that will be saved. Total saving, denoted by TS, is withdrawn from the system and may be written as follows:

(4.2.15) \[ TS = \sum_{s} \hat{S}_s Y_s + \hat{S}_k Y_k + \hat{S}_g Y_g \]

This leaves each spending group with a reduced amount of income to be spent on consumer goods. We shall have:

(4.2.16) \[ C_{is}^D = h_{is} (P_1, \ldots, P_n, (1 - \hat{S}_s) Y_s) \quad i=1, \ldots, n \quad s=1, \ldots, m \]

(4.2.17) \[ C_{ik}^D = h_{ik} (P_1, \ldots, P_n, (1 - \hat{S}_k) Y_k) \quad i=1, \ldots, n \]

(4.2.18) \[ C_{ig}^D = h_{ig} (P_1, \ldots, P_n, (1 - \hat{S}_g) Y_g) \quad i=1, \ldots, n \]

where \( C_{is} \), \( C_{ik} \) and \( C_{ig} \) are the amounts of consumer good \( i \) demanded by labor of type \( s \), capitalists and the government respectively.

This leads to the aggregate demand functions,
(4.2.19) \[ c^D_i = \sum_s h_{is}(p_1, \ldots, p_n, (1 - \hat{s}_s)Y_s) + h_{ik}(p_1, \ldots, p_n, (1 - \hat{s}_k)Y_k) + h_{ig}(p_1, \ldots, p_n, (1 - \hat{s}_g)Y_g) \quad i=1, \ldots, n \]

Note that one need not in practical applications insist that these demand functions be derived from explicit utility functions. They are, however, required to be homogeneous of degree zero in prices and incomes. In most empirical applications, the estimated demand equations are usually derivable from additively separable utility functions. This assumption simplifies the problem of parameter estimation, but is limiting. For example, there can be no specific substitution effects and no inferior goods.\(^1\) At this point, we shall simply require that the demand functions be continuous and "well-behaved" so that to each set of prices and associated incomes, we can associate a unique vector of consumption demands. What cannot be ruled out when there is more than one group of consumers is that the same vector of consumption demands recurs for different sets of prices.

Since factor incomes are fully determined by the set of commodity prices one initially gives to the system, one can write the consumption function more simply as:

\[ \Phi (2.20) \quad c^D_i = h_i(p_1, \ldots, p_n) \quad i=1, \ldots, n \]

\(^1\) For a recent survey of empirical demand analysis, see Brown and Denton (1973).
It is understood that behind the equation lies the solution of factor markets as well as the various equations defining disposable incomes. But fundamentally there is a simple chain of causality leading from the price vector to the vector of consumption demands and it is this chain that is represented by (4.2.20).

To close the model it remains to discuss what happens to the total savings withdrawn from the flow-of-funds. Assume that all savings are spent on investment goods. Denoting by $\bar{H}_i$ the share of investment going to sector $i$,

\[(4.2.21) \quad \bar{H}_i = U_i \Delta K_i / TS\]

and therefore

\[(4.2.22) \quad \Delta K_i = \bar{H}_i \cdot TS / U_i\]

where $U_i$ denotes the price of capital (defined below) and $\Delta K_i$ is real investment in sector $i$. The shares of $\bar{H}_i$ will for the moment be assumed to be predetermined. We shall extensively discuss the problem of their determination in section (4.4) below.

Since capital in each sector is simply a fixed-proportions composite commodity, the price of capital is the weighted average of its components:

\[(4.2.23) \quad U_i = \sum_{j \in J} P_j i = 1, \ldots, n\]
where \( b_{ij} \) are the shares in the capital composition matrix. The sectoral capital accumulations, \( \Delta K_i \), are therefore uniquely determined by the price system which uniquely determines the capital prices, \( U_i \), and total savings \( TS \). It remains to translate the sectoral pattern of capital accumulation into demands for investment goods by sector of origin. This is achieved by using the elements of the capital composition matrix. Letting \( Z_i \) denote total investment demand by sector of origin, we have, just as in a dynamic input-output model,

\[
(4.2.24) \quad Z_i = \sum_{j=1}^{n} b_{ij} \Delta K_j \quad i=1, \ldots, n
\]

It is again possible to write investment demands \( Z_i \) as a function of the initial price vector only, since a unique causal chain leads us from the \( P_i \) to the \( Z_i \). One can therefore write,

\[
(4.2.25) \quad Z_i = Z_i(P_1, \ldots, P_n)
\]

in similar fashion to the consumer demand equations.

4.3 The General Equilibrium Solution

We thus have the following situation:

\[
\begin{align*}
X_i^S &= g_i(P_1, \ldots, P_n) \\
X_i^D &= d_i(P_1, \ldots, P_n)
\end{align*}
\]

where
\[(4.3.1) \quad d_1(P_1, \ldots, P_n) = h_1(P_1, \ldots, P_n) + Z_1 + N_1 \]

The problem faced is almost exactly analogous to the simpler one already discussed in the introductory section. A solution to the general equilibrium model is constituted by a price vector \((P_1, \ldots, P_n)\) such that excess demands equal zero in all sectors:

\[(4.3.2) \quad EX_1 = d_1(P_1, \ldots, P_n) - g_1(P_1, \ldots, P_n) = 0 \quad i=1, \ldots, n \]

Before discussing the problem of existence of the equilibrium price vector, it is worth emphasizing certain important properties of the excess demand functions.

The first important property is that they are homogeneous of degree zero in all prices. To see the homogeneity properties, first consider the factor market equations. Doubling all prices implies doubling all net prices. Thus doubling all wages \(W_{s1}\) in equations \((4.12)\) and leaving the labor allocations \(L_{s1}\) unchanged will not affect the equalities and thus the same factor allocation and factor prices will remain a solution to the marginal productivity equations. Since value-added has doubled and wage incomes have doubled, the residual capital incomes must also double. While incomes have doubled in nominal terms, the sectoral supplies \(x_i^S\) have remained constant after the proportional increase in all prices; \(g_1(P_1, \ldots, P_n)\) is not affected by such proportional price changes.

Turning to the demand side, \(d_1(P_1, \ldots, P_n)\), it also will remain unaffected by a proportional change in all prices. As we have already noted,
all incomes will change proportionately with prices. Relative
prices and real incomes have remained constant and therefore consumption
demands will remain unchanged. The price of composite capital goods —
equation (4.2.23) — also doubles and hence the demand for real composite
capital goods by sector of destination — equation (4.2.22) — does not
change since both the numerator and the denominator
have doubled. Thus, in equation (4.2.24), the demand for investment
goods by sector of origin will also remain unchanged. Finally, gross
output remaining constant, the intermediate demands by producers will also
remain unchanged. Hence neither \( g_i(P_1, \ldots, P_n) \) nor \( d_i(P_1, \ldots, P_n) \)
is affected by proportional price changes and the excess demand equations
(4.3.2) are indeed homogeneous of degree zero in all prices and wages.
This means that if a vector \( (P_1, \ldots, P_n) \) constitutes a solution to the
system of \( n \) excess demand equations, any vector \( \lambda(P_1, \ldots, P_n) \) pro-
portional to it \( (\lambda > 0) \) will also constitute a solution. There seems
to be an infinite number of solutions to a system of \( n \) equations in \( n \) unknowns.

In fact, the second important property of the excess demand equations
is that they are not independent. For any allowable price vector \( (P_1, \ldots, P_n) \),
the following identity, known as Walras' Law, holds:

\[
(4.3.3) \quad \sum_{i=1}^{n} \left( d_i(P_1, \ldots, P_n) - g_i(P_1, \ldots, P_n) \right) = 0
\]

To see that Walras' Law always holds it is sufficient to
remember that:
\[(4.3.4) \quad \sum_{i} p_i x_i (P_1, \ldots, P_n) = \sum_{i} p_i x_i, \text{ the total value of output}\]

and

\[(4.3.5) \quad \sum_{i} d_i (P_1, \ldots, P_n) = \sum_{i} p_i c_i + \sum_{i} p_i z_i + \sum_{i} p_i n_i, \text{ the total value of expenditures.}\]

Noting that \[n_i = \sum_{j=1}^{n} a_{ij} x_j\] and subtracting \[\sum_{i=1}^{n} p_i n_i\] from both \[(4.3.1)\] and \[(4.3.2)\], Walras' Law reduces to the requirement:

\[(4.3.6) \quad \sum_{i} p_i x_i + \sum_{i} \frac{x_i}{p_i} x_i = \sum_{i} p_i c_i + \sum_{i} p_i z_i\]

or that

\[(4.3.7) \quad \sum_{s} y_s + y_k + y_g = \sum_{i} p_i c_i + \sum_{i} p_i z_i\]

Since each spending unit's demands are subject to a budget constraint which says that outlays must equal income, it is clear that such a budget constraint also holds in the aggregate and \[(4.3.7)\] will hold not only at equilibrium, but for all allowable price vectors. There are thus only \((n-1)\) independent excess demand equations to determine \((n-1)\) relative price ratios.

In pure general equilibrium theory, one can leave matters at that point. In practice a CGE model must usually arrive at some way of determining an absolute price level as well as relative prices. A wide variety of price normalization equations have been used. Johansen, in his path-breaking (1960) study, fixed the wage of labor and thus expressed all prices in terms of wages. One could alternatively fix the price of
any one commodity and express all prices in terms of this numeraire commodity. Indeed, one could normalize around virtually any nominal magnitude in the model economy.

For development planning models to be used as tools of analysis and policy formulation, it seems best to use a price normalization rule that provides a "no inflation" benchmark against which all price-changes are relative price changes. The equation used will be of the form:

\[(4.3.8) \quad \sum_{i} \Omega_i = \bar{p}\]

where the \(\Omega_i\) are weights defining the index \(\bar{p}\) that one wants to hold constant. The normalization equation does not have any impact on the real structure of the model. But it may become a link between the CGE model and a macro-monetary model that would determine the value of the price-index.

The extent to which one may want to introduce money and money-holding behavior into a CGE model is a difficult question which we shall not discuss at this stage. We only note that unless monetary mechanisms are explicitly introduced and modelled, we feel that it is best not to use formulations that have implicit macroeconomic implications.

Such formulations may be confusing unless a genuine effort is made to couple the very Wairasian CGE structure with a macroeconomic monetary model.

At this point, the presentation of the model equations is complete. In the next section we will provide a summary listing of the model equations and discuss how they incorporate the "circular flow" in an economy. We will
then discuss various approaches to solving the model to determine all the endogenous variables.

Before considering how to solve the model as a set of non-linear equations, it is reasonable to ask if, in fact, a solution exists. Most model builders have not however worried too much about the general existence problems. After all, a solution is numerically computed and an existence proof may appear unnecessary. The models are always quite well-behaved and given that very general existence proofs have been established for theoretical models of which CGE models form a rather well-behaved subset, it is reasonable to expect that non-existence problems will not arise in practice. Nevertheless it is worthwhile to examine briefly the existence problem for the core model and sketch the basic reasoning of an existence proof. Note that we will use some set notation and matrix notation that are unique to this section and which deviate slightly from our standard notation conventions.

Consider the set of "allowable" price vectors such that net prices are non-negative. Define this set as:

\[(4.3.9) \quad P^s = \{ P / P-PA \geq 0, \ P \geq 0 \} \]

Provided that the input-output matrix is productive and satisfies the Hawkins-Simon conditions, \( P^s \) is non-empty and it is clear that it is a convex cone. For if \( P \in P^s \), \( \lambda P \in P^s \) for all \( \lambda \geq 0 \) and if \( P^1 \in P^s \) and \( P^2 \in P^s \), \( \lambda P^1 + (1-\lambda)P^2 \in P^s \). Figure 4.1 describes the set \( P^s \) in a two-sector model:
Figure 4.1: The Cone of Allowable Prices in a Two-Sector Model

The two rays through the origin, R₁ and R₂, reflect the condition of non-negative net prices (value added) in sectors 1 and 2. The Hawkins-Simon (or productiveness conditions) guarantee that the slope of R₁ is greater than the slope of R₂ and that both slopes are positive:

\[(1 - a_{11})(1 - a_{22}) - a_{12}a_{21} > 0 \Rightarrow \frac{1 - a_{11}}{a_{21}} > \frac{a_{12}}{1 - a_{22}}\]

and

\[(1 - a_{11}) > 0 \Rightarrow \frac{1 - a_{11}}{a_{21}} > 0\]

\[(1 - a_{22}) > 0 \Rightarrow \frac{a_{12}}{1 - a_{22}} > 0\]
Each ray passes through the origin and therefore defines two half-spaces that are convex cones. The intersection of the two half-spaces defined by the inequalities is itself the convex cone of allowable prices. Note finally that the boundaries of the allowable set (i.e., the two rays $R_1$ and $R_2$) are constituted by points for which either $V_1$ or $V_2$ equals zero. For any non-negative net price vector $V$, there exists a vector $P = V(I-A)^{-1}$ in the allowable set.

We must now add the restriction that prices satisfy the normalization rule so that we define a new "normalized" allowable set $^nP$ such that:

$$^nP = \{ P/P - PA \geq 0, P \geq \sum_{i=1}^{n} \Omega_i \} = \bar{P}$$

Clearly the set $^nP$ is a closed interval of the form NN shown in figure (4.1). The equilibrium price vector must be contained in that closed interval that itself constitutes a closed, bounded and convex set.

Let us now consider the following mapping from $^nP$ into itself. Take any price vector $P \in ^nP$. Use it to solve for the production side of the economy and obtain factor prices and household and government incomes. Feeding these incomes into the demand side, obtain a net demand vector $X^D = (X^D_1, \ldots, X^D_n)$ by subtracting intermediate demands from total gross demands. Scale this vector up or down by a scale factor $\lambda$ until it is situated on the boundary of the net production set $X^N$. There will exist a separating hyperplane at that point which defines a non-negative vector of net prices $V = (V_1, \ldots, V_n)$ that would lead profit maximizing firms
(or a central planner) to the scaled point $\lambda_1 \cdot \mathbf{X}^P$. This vector of net prices is arbitrary up to a multiplicative constant, so it is possible to scale $V$ with a second scaling factor $\lambda_2$ such that $P = V(I-A)^{-1} \in \mathbb{P}^S$.

The above steps define a mapping from $\mathbb{P}^S$ into itself. Using a theorem on the continuity properties of optimal solutions, one can establish that the mapping satisfies the continuity properties required for fixed point theorems. In this case, characterized by strict convexity of the production set, every step of the mapping constitutes a point-to-point mapping and the entire mapping is therefore a continuous point-to-point mapping of a compact convex set into itself. One can therefore appeal to Brouwer's Fixed Point theorem and claim that there exists at least one vector $P^e$ such that

$$P^e = M(P^e)$$

where $M(P)$ defines the mapping of $\mathbb{P}^S$ into itself. By using Walras' Law one can then show that the two scaling factors used in the mapping must equal unity at the fixed point because the total value of net output must equal the sum of all incomes. At that point, when $P = P^e$, $\lambda_1 = \lambda_2 = 1$ and the supply decisions of firms will be consistent with the demand decisions of households and of the government. This argument establishes the general existence of an overall equilibrium solution to the basic CGE model.

1/ See for instance Debreu (1961), Section 1.8k.
We conclude this section with a note on uniqueness. The fixed-point discussed above cannot in the general case be shown to be unique. If one were to assume a single decision-maker on the demand side, the convexity of preferences leading to the weak axiom of revealed preference would imply uniqueness of equilibrium. But the weak axiom need not hold in the general case when there is more than one category of demanders because as prices change, the distribution of income changes and the weak axiom no longer follows from the assumption of well-behaved individual utility functions: equilibrium may therefore not be unique. In practice, none of the CGE models so far built seems to suffer from non-uniqueness. It appears that the changes in the community indifference map due to shifts in income distribution are never enough to create a uniqueness problem. Whether this result will continue to hold for all kinds of CGE models remains an open question.

4.4 CGE Models, Social Accounts, and Solution Strategies

In this section, we will collect the basic CGE model equations together and discuss them in terms of the kind of statistical "picture" they provide of the workings of an economy. As an organizing framework, we will use the device of a social accounting matrix which presents in one unified set of accounts a picture of the "circular flow" of a market economy. We will then discuss algorithmic strategies for solving a CGE model.

Figure 1 presents a simplified social accounting matrix (SAM) that corresponds to the basic CGE model. A social accounting matrix is
essentially an expanded input-output table that traces out all the monetary flow-of-funds in an economy. 1/ In the simplified table, one can distinguish four types of accounts: production activities, factors of production, institutions, and the capital account. There are also three kinds of institutions: labor households, capitalists, and government. Figure 1 indicates in the cells of the matrix the types of transaction that occurs. For example, in the upper left-hand corner there is a cell marked "intermediate goods." This is the standard Leontief input-output flows table giving the flow of intermediate goods from a row "production activity" to a column "production activity." The flow of funds, however, is in the opposite direction: it is an expenditure by a column activity and a receipt for the row activity. Continuing down the "production activities" column, one can trace out all the expenditures by production activities. They buy intermediate goods from producers and factors of production from the factor markets and also pay indirect taxes to the government. The other columns trace out the expenditures for each type of account. Accounting consistency requires that each column sum must equal the corresponding row sum. Essentially, each row and column represents a set of double-entry bookkeeping accounts which must balance.

The simplified SAM deviates in one respect from standard conventions. In the definition of institutions, "capitalists" are

1/ For a discussion of social accounting, see Stone ( ), Pyatt and Thorbecke (1976), and United Nations ( ).
## Expenditures

<table>
<thead>
<tr>
<th>Receipts</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Production Activities</td>
<td></td>
<td></td>
<td>Factors of Production</td>
<td></td>
<td>Labor Households</td>
<td></td>
</tr>
<tr>
<td>1. Production Activities</td>
<td>Intermediate goods</td>
<td></td>
<td>consumption</td>
<td></td>
<td>consumption</td>
<td></td>
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<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>government consumption</td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td>investment</td>
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</tr>
<tr>
<td>2. Factors of Production</td>
<td>factor payments</td>
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<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Institutions:</td>
<td></td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3. Labor Households</td>
<td></td>
<td></td>
<td>distribution of labor income</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4. Capitalists</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>capital income</td>
<td></td>
</tr>
<tr>
<td>5. Government</td>
<td></td>
<td></td>
<td>indirect taxes</td>
<td></td>
<td>direct taxes</td>
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<td></td>
<td></td>
<td></td>
<td>direct taxes</td>
<td></td>
</tr>
<tr>
<td>6. Capital Account</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>labor saving</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>capitalist saving</td>
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<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>government saving</td>
<td></td>
</tr>
</tbody>
</table>
a separate institution. They receive all capital income (cell 4,2 of
the matrix) from which they pay direct taxes, save and consume. This
treatment follows the model formulation presented in this chapter.
However, standard accounting practice is to define an institution called
"firms" which is considered as a separate entity. In this treatment,
"capitalists" would be classified as a type of household. Then cell 3,2
would give the distribution of factor income to all households, including
capitalists. Cell 4,2 ("capital income") would then give "retained earn-
ings" by firms which would, in turn, presumably be spent on investment
goods.

The treatment of capitalists as a separate institution has important
economic implications. We are choosing not to treat firms as separate
actors in the economy, distinct from their owners and with different
behavioral rules. Instead, capitalists are treated as receiving all of capital
income which is then taxed and allocated between investment and consumption.
In this framework, for example, one cannot easily handle a separate corporate
income tax or include a behavioral rule which determines the level of
distributed profits. For a less developed country, this treatment seems
justifiable, although for many purposes one might want to treat firms as
separate behavioral units.1/

The complete set of equations for the model are collected to-
gether in Tables 1 and 2. Table 1 gives the equations for the factor

1/ See, for example, Adelman and Robinson (1978) who explicitly model the
retained earnings and investment decisions by firms. In principle,
there is no great difficulty in doing so.
markets which essentially underlie the aggregate supply or production transformation set considered earlier. The endogenous variables in these equations describe all the flows in the first column of the social accounting matrix. In real terms, they give employment, intermediate demand, and product supply. In terms of monetary flows, they give payments for intermediate goods, labor, capital and indirect taxes. Table 2 gives the equations for the product markets in the model economy. The endogenous variables describe the flows in the last five columns of the social accounting matrix. They give the distribution of factor income to "institutions" (labor households, capitalists, and government); its allocation among taxes, consumption and saving; and finally the resulting demand for products.

In the treatment of the capital account, the simple social accounting matrix does not capture the full treatment of investment in the model. In the model, we distinguish investment both by sector of origin and by sector of destination. That is, we considered both how much real capital each sector would receive and what would be the demands for investment goods by sector of origin. The SAM, however, only treats investment by sector of origin. All savings are gathered into a single account and allocated (in cell 1,6) for the purchase of investment goods. In this treatment, the capital account acts as a kind of bank which receives all savings in the economy and spends the funds on investment goods.

There are two modelling issues related to investment that are mixed in the social accounting framework: first, the determination of the
Table 1

Factor Market and Product Supply Equations

Production functions:

\[ (4.4.1) \quad X_i^S = f(K_i, L_i^a; N_{il}, \ldots, N_{ni}) \quad i = 1, \ldots, n \quad (4.2.1) \]

Intermediate goods demand:

\[ (4.4.2) \quad N_{ij} = a_{ij}X_j \quad i = 1, \ldots, n \quad (4.2.2) \]

\[ (4.4.3) \quad N_i = \Sigma j N_{ij} \quad i = 1, \ldots, n \quad (4.2.3) \]

Labor aggregation:

\[ (4.4.4) \quad L_i^a = L_i^a(L_{il}, \ldots, L_{im}) \quad i = 1, \ldots, n \quad (4.2.4) \]

Net prices:

\[ (4.4.5) \quad V_i = P_i - \Sigma a_{ij}P_j - \Sigma_i^X P_i \quad i = 1, \ldots, n \quad (4.2.6) \]

Labor demand equations:

\[ (4.4.6) \quad V_i \frac{\partial X_i}{\partial L_i^a} \frac{\partial L_i^a}{\partial L_{is}} = W_s \quad i = 1, \ldots, n \quad (4.2.7) \]

Aggregate labor demands:

\[ (4.4.7) \quad L_s^D = \Sigma L_{is} \quad s = 1, \ldots, m \quad (4.2.9) \]

Aggregate labor supply:

\[ (4.4.8) \quad L_s^S = \Sigma_i L_{is} \quad s = 1, \ldots, m \quad (4.2.5) \]
Table 1 (cont.)

Excess demand for labor equations:

\[(4.4.9) \quad L^F_s - L^S_s = 0 \quad s=1, \ldots, m\]

**Endogenous variables**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$X^S_i$</td>
<td>sectoral production</td>
</tr>
<tr>
<td>$N_{ij}$</td>
<td>intermediate goods demand</td>
</tr>
<tr>
<td>$N_i$</td>
<td>aggregate intermediate goods demand</td>
</tr>
<tr>
<td>$L^a_i$</td>
<td>aggregate labor by sector</td>
</tr>
<tr>
<td>$V_i$</td>
<td>net prices</td>
</tr>
<tr>
<td>$L_{is}$</td>
<td>labor demand by sector and type</td>
</tr>
<tr>
<td>$L^D_s$</td>
<td>aggregate labor demand by type</td>
</tr>
<tr>
<td>$L^S_s$</td>
<td>aggregate labor supply by type</td>
</tr>
<tr>
<td>$W_s$</td>
<td>wage of labor by type</td>
</tr>
</tbody>
</table>
Table 2
Product Market Equations

Household, capitalist and government income:

(4.4.11) \[ Y_s = \sum_{i} W_i s L_{is} (1 - t_i^Y) \]
\[ s=1, \ldots, m \] (4.2.12)

(4.4.12) \[ Y_k = \left[ \sum_{i} X_{i1} - \sum_{i} W_i s L_{is} \right] (1 - t_k^Y) \] one
(4.2.13)

\[ Y_s = \sum_{i} \frac{t_i^Y}{1-t_i^Y} Y_s + \frac{t_k^Y}{1-t_k^Y} \sum_{i} \hat{P}_i X_i \] one
(4.2.14)

Total saving:

(4.4.14) \[ TS = \sum_{s} S_s Y_s + \hat{S}_k Y_k + \hat{S}_g Y_g \] one
(4.2.16)

Consumer demand:

(4.4.15) \[ C_{is} = h_{is} (P_1, \ldots, P_n, (1-S_s)Y_s) \]
\[ i=1, \ldots, n \]
\[ s=1, \ldots, m \] (4.2.17)

(4.4.16) \[ C_{ik} = h_{ik} (P_1, \ldots, P_n, (1-S_k)Y_k) \]
\[ i=1, \ldots, n \] (4.2.18)

(4.4.17) \[ C_{ig} = h_{ig} (P_1, \ldots, P_n, (1-S_g)Y_g) \]
\[ i=1, \ldots, n \] (4.2.19)

(4.4.18) \[ C^D_i = \sum_{s} C^D_{is} + C^D_{ik} + C^D_{ig} \]
\[ i=1, \ldots, n \] (4.2.20)

Investment demand:

(4.4.19) \[ U_i = \sum_{j} b_{ji} P_j \]
\[ i=1, \ldots, n \] (4.2.24)

(4.4.20) \[ \Delta K_i = \bar{H}_i TS / U_i \]
\[ i=1, \ldots, n \] (4.2.23)
Table 2 (cont.)

Investment demand cont.:

\[(4.4.21) \quad z_i = \sum_{j} b_{ij} \Delta k_j \quad \text{i=1, \ldots, n} \quad (4.2.25)\]

Aggregate excess demands:

\[(4.4.22) \quad x_i^D = c_i^D + z_i + n_i \quad \text{i=1, \ldots, n} \quad (4.3.1)\]

\[(4.4.23) \quad x_i^D - x_i^S = 0 \quad \text{i=1, \ldots, n} \quad (4.3.2)\]

Price normalization:

\[(4.4.24) \quad \sum_{i} \rho_i \Omega_i = \bar{p} \quad \text{one} \quad (4.3.8)\]

Endogenous variables

\[Y_s \quad \text{labor income by type}\]

\[Y_k \quad \text{capital income}\]

\[Y_g \quad \text{government revenue}\]

\[TS \quad \text{total saving}\]

\[C_{is}^D \quad \text{consumer demands by labor households}\]

\[C_{ik}^D \quad \text{consumer demands by capitalists}\]

\[C_{ig}^D \quad \text{government consumption}\]

\[C_{i}^D \quad \text{aggregate consumption demand}\]

\[U_i \quad \text{capital goods price}\]

\[\Delta K_i \quad \text{real investment by sector of destination}\]
Table 2 (cont.)

Endogenous variables (cont.)

\( Z_1 \)  
investment goods demands by sector of origin

\( X^D_1 \)  
aggregate demands by sector

\( P_i \)  
prices
volume of investment and, second, its sectoral allocation. We will treat the second issue in our discussion of intertemporal linkages. In the model equations presented in Table 2, the sectoral share parameters for investment ($\bar{H}_i$) are simply assumed to be fixed. The level of total investment, however, is determined endogenously. In the capital account row in the SAM, total savings (and hence investment) are determined by applying exogenous savings rates to the income of each institution in the economy. Total investment is thus determined by savings behavior and is thus also a function of the distribution of income among the different institutions (assuming that their savings rates differ). This model of total savings and investment is very classical in spirit, except that we have added government to the usual capitalist and labor classes.

The determination of aggregate investment is clearly a very important part of any dynamic planning model and the classical approach presented above is not the only reasonable way to treat investment in such a model. There is no shortage of theories of investment in economics -- the problem is more that there is no wide-spread agreement on which theories are best. In a planning model such as our CGE model, a number of different choices seems reasonable. We will give two alternative examples. First, in a model intended for policy planning, one might wish to specify the level of real investment exogenously. A planner might well wish to trace out the impact of alternative investment scenarios on the model economy and
simply assume that policies can be chosen which will achieve the desired level of investment. The model can be adapted to embody this approach. Instead of being endogenous, real capital stock growth (usually measured in base year prices) would be set exogenously. Some other parameters such as the savings rates in equation 14 would then have to be adjusted endogenously so that sufficient savings are generated to finance the purchase of the capital goods.

Second, a different savings determined approach has some appeal. Instead of having savings be partly a function of the distribution of income among institutions, one might assume that the society determines aggregate savings by setting aside a fixed portion of total national income. The precise mechanism by which this fixed rate is determined and applied is left purposely vague. One might appeal to a complex interaction among government credit policy, the workings of the banking system, and inflation -- all of which are not explicitly included in the planning model. The essential view is that society as a whole determines the savings rate -- a view that might well be labelled neoclassical. Again, some parameters will have to adjust endogenously for the flow-of-funds to remain consistent.

The three investment theories we have discussed -- the one actually modelled and the two alternatives discussed above -- certainly do not represent an exhaustive list of possible approaches. However, they do represent an interesting range and all have some appeal for inclusion in a
planning model intended to be used for policy analysis in the medium to long-run. In the next chapters, we will discuss the implications of using these approaches to investment when the model is extended to include international trade and the possibility of foreign capital inflows.

In the last section, we argued that the model economy will have at least one general equilibrium solution with non-negative values for all the endogenous variables. While not sufficient, it is also reassuring to count equations and endogenous variables in Tables 1 and 2. Assuming that prices are given (in Table 2), there are \(4n + 3m + mn + n^2\) factor market equations in Table 1 in as many unknowns. In Table 2, there are \(8n + m + mn + 4\) equations, one more than the number of endogenous variables. However, as discussed earlier, Walras' Law implies that the equations are not independent and hence can only determine relative prices. The last equation defines price normalization and sets the absolute level of prices.

The case where we assume that for some labor categories the wage is fixed instead of the supply of labor does not, as discussed above, add any real problems. We simply add an equation for each fixed wage and drop the corresponding labor supply equation. In this case, the corresponding excess demand for labor equation serves only to determine the supply of labor rather than as an equilibrium condition determining the wage.

A number of different approaches has been used to solve CGE models such as the one presented above. In discussing different approaches, it is useful to distinguish between a solution strategy and a solution
algorithm. A solution strategy refers to the way in which the equations of the model are substituted and rearranged so as to reduce the solution problem as much as possible before solving it on a computer. A solution algorithm refers to the actual numerical algorithm used to solve the reduced set of equations on the computer. In devising a solution strategy for CGE models, it is important to take advantage of our knowledge of the economic properties of the system of equations. In this section, we describe our basic solution strategy and use it as a framework for clarifying further the workings of the model.

Our basic solution strategy is, in fact, evident from the way in which the model equations were presented. The entire model can be reduced by substitution -- either analytical or numerical -- to sets of excess demand equations for the factor and product markets. The solution problem thus is reduced to that of finding a set of equilibrium wages and prices - a fixed point problem.

In choosing a solution strategy, two criteria are most important. First, how hard is it to solve the reduced equations numerically? Second, how hard is it to reduce the model equations to the chosen set especially when, as is often the case, one wants to be able to experiment with a variety of analytic formulations? Reducing a CGE model to sets of market excess demand equations is usually a straightforward procedure requiring, with one exception, simply the evaluation of the equations of the model in order. The only place where numerical techniques may have to be used is in the derivation of labor demands by firms given wages - the
marginal revenue product equation. The resulting excess demand equations are, of course, extremely non-linear and it is virtually impossible even to write out their analytic representation. Numerically, however they can be handled.

Given our solution strategy, all solution algorithms will follow the same general procedure. They will all start with some initial set of wages and prices (which satisfy the normalization rule), calculate the excess demands in both factor and product markets, and then revise wages and prices iteratively based on calculated excess demands. The iterations stop when equilibrium is reached; that is, a set of wages and prices is found such that all excess demands are sufficiently close to zero. Different algorithms use quite different techniques for revising wages and prices given the excess demands from the last iteration, and there are a variety of such algorithms from which to choose. A brief survey plus a detailed description of our particular choice is given in an appendix. ¹ Note, however, that for models using more-or-less "standard" production functions and demand systems, the excess demand equations are quite well-behaved (after some scaling transformations) and are solvable by the imaginative application of standard numerical algorithms for solving systems of non-linear simultaneous equations.

Inherent in the general solution strategy described here is a "price adjustment" rather than a "quantity adjustment" specification. The problem is seen as that of adjusting prices and wages until equilibrium is reached. One might instead use a quantity

¹ Scarf and Eansan (197 ), Ginsburgh and Waelbroeck (197 ), and Adelman and Robinson (1978) are examples of the three major approaches: fixed-point algorithms, sequences of mathematical programs, and direct solution algorithms, based on the idea of Walrasian tâtonnement.
adjustment approach. For example, in the labor markets one might find it difficult or inconvenient to invert the marginal revenue product equations to solve for labor demands given fixed wages. One can use a quantity adjustment solution strategy that avoids the problem. Instead of assuming fixed wages and solving for labor demands, one can start with a labor allocation and solve for marginal revenue products by sectors. The equilibrium condition for the labor markets is restated. Instead of being "demand equals supply for each category of labor," it becomes "marginal revenue products are equal across all sectors for the same labor category." The solution problem is then to find a physical allocation of labor such that all labor is employed and marginal revenue products are equal across sectors for each category of labor.\(^1\)

In discussing solution strategies, one must be careful not to confuse properties of an algorithm with properties of an economy. For example, the choice between a "price adjustment" and a "quantity adjustment" solution strategy has nothing to do with how one views the behavior of an actual economy out of equilibrium. In the literature on decomposition algorithms for solving linear programming models, there has been some tendency to interpret the behavior of different algorithms as reflecting differences in actual economies. Kornai (19) has warned against making such interpretations and his warning is equally valid in interpreting

\(^1\) This technique has in fact been used for some models, [See Dervis (1975), De Melo (197), and De Melo and Dervis (1977)].
solution strategies for CGE models. The CGE model is formulated as an
equilibrium model and one should place no significance on the actual path
the model economy follows during iterations designed to find the equilibrium.

In terms of solution strategy, it is convenient to separate
the factor and product markets and solve them seriatim. For example, for
a given set of product prices, it is possible to solve for a set of wages
which will clear the factor markets. This solution will yield product
supplies and factor incomes which are then used to determine excess demands
in the product markets and hence to a new guess at prices. Solving the two
markets separately reduces the number of endogenous variables which has to
be solved at any one time, although it means that one set of markets must
be solved more often (in this case, the factor market must be solved for
each price iteration). In general, reducing the number of endogenous
variables per iteration yields algorithmic improvements that outweigh the
disadvantages of having to solve the markets more often.

Separating the factor and product market equations in fact
underlies the discussion of the model in earlier sections. Given a set
of fixed prices, we assumed that the factor market equations could be solved
to give the supply of products. The set of such factor market solutions
given different price vectors defines the production transformation act.
Assuming, as was done, that there is perfect competition in the factor
markets, the solution of the factor market equations will yield a vector
of product supplies that is on the technical production possibility frontier.
4.5 Equilibrium, Time and Market-Clearing

The CGE model presented so far is purely static. Time did not enter into its specification and all endogenous variables are therefore viewed as simultaneously determined. The solution constitutes a simultaneous intersection of the sectoral supply and demand functions. In a very Walrasian fashion, prices play a parametric role in the sense that decision makers react passively to prices and it is these reactions that determine the demand and supply schedules. But, assuming uniqueness conditions hold, there is only one vector of prices at which all demand and supply decisions of the various agents in the economy are mutually compatible, and it is that price vector that defines the general equilibrium solution.

Such a description is extremely neoclassical in spirit and so close to the Walrasian ideal that one can rightly question its relevance for the formulation of policy in a developing country where institutional features and structuralist characteristics result in an economy far removed from the idealized description given so far.

First there may be elements of imperfect competition that need to be included in the model. This will not fundamentally alter the character of the solution: prices will remain market-clearing prices though some of
the supply functions will now reflect monopoly power. One could also extend the model to incorporate monopoly power in the factor or product markets. In either case, one needs to model the relevant perceived demand elasticities in the factor and/or product demand equations. Monopoly-pricing rules could be replaced by mark-up pricing rules, in which case the supply schedules will include profits as arguments. It would, of course, be far more difficult to model imperfect competition properly since certain indeterminacies may occur. However, aside from such problems of indeterminancy, these model extensions do not change the fundamental market-clearing nature of the model solution.

Second, as was discussed in connection with labor markets, there may be some markets which will not be allowed to clear. The solution of such an applied model can be characterized as a "constrained" general equilibrium, close in spirit to the Walrasian construct but incorporating quantity constraints. The question of which markets should be constrained in a CGE model is a difficult one. It is certainly far from evident that prices observed in the real world of developing countries are in fact market-clearing prices. We shall discuss further below how the solution prices should be interpreted, but it should be noted that the problem is present not only in factor markets but also in product markets. This aspect of market adjustment has been recently emphasized in the theoretical work on the micro-foundations of macro-economics. This work emphasizes non-rééquilibre processes and mixed quantity-cum-price adjustment mechanisms that may have consequences more important than allowed for or
foreseen by traditional Walrasian theory.

One would of course like to address these issues properly, incorporating disequilibrium and partial adjustment in the actual model formulation. But given the medium to long-term planning purposes served by CCE models, it would in our view be premature at the present time to give up the basic notion of Walrasian equilibrium for a generalized disequilibrium model. The reason general equilibrium theory (and, indeed, partial equilibrium theory) is of any use at all is that the equilibrium solution does exert a certain "pull" on the economic system: while the economy may never actually be at equilibrium, it should not be too far away from it. This is clearly our fundamental premise. It does mean that one gives up short-term "tracking" as well as short-term prediction. But this should not be upsetting. It is the equilibrium shifts over time due to fundamental trends in technology, tastes, policies and other exogenous factors that one can analyze with an equilibrium model. For these purposes, the assumption of market-clearing prices is a valuable one to retain as the basic organizing principle in models focusing on medium to long-run change.

It remains true, however, that certain constraints on prices -- particularly factor prices -- are of a permanent and structural kind. In a sense, they form part of the notion of equilibrium. Take, for instance, the Lewis labor surplus model with its roots in classical political economy. The fact that the real-wage is fixed in the modern urban sector and that the "labor market" is not allowed to clear does not reflect "disequilibrium" in the usual sense. Instead, it reflects a long-run structural feature of the
economy that should be incorporated in a planning model. In fact, taking real-wage determination outside of the equilibrium model goes back to the Marxian-Ricardian foundations of general equilibrium theory as recently revived in England, most notably by Morishima.\footnote{See Morishima ( )}. Taking a more classical view, there is no real conflict between letting product prices clear product markets (thus regulating the supply and demand of commodities) and, at the same time, regarding the real wage (and hence the average rate of profit) as more exogenous, strongly influenced by factors that are not explicitly modelled.

A second area in which the concept of equilibrium and the notion of market clearing is not always clearly defined relates to investment and the determination of capital asset prices. When capital is treated as a stock of produced means of production that differs in composition among sectors, the assumption of perfect capital mobility cannot very well be maintained in a dynamic analysis. It is much better to assume that, once installed, capital cannot move from one sector to another within a given time period. As we saw earlier, this assumption leads to upward sloping short-run supply curves. The assumption of heterogeneous and sector-specific capital greatly increases the importance of today's decisions on tomorrow's alternatives. Considering the question of how sectoral supply curves shift over time, one realizes that they shift essentially due to technical progress which is usually considered exogenous and due to sectoral capital accumulation which is usually endogenously
modelled. If labor supply constraints are binding, demographic change will also effect the supply schedules. But capital accumulation is the most important endogenous cause of change, and traditionally constitutes one of the most important areas of concern for the development planner.

What does one mean by an equilibrium allocation of investment? Much depends on how one views the workings of capital markets. In perfect neo-classical intertemporal equilibrium, the market price and production price of capital will always be the same, the profit-rate will be the same at any given moment of time in all sectors, and the economy will be traveling along an intertemporally efficient path where prices are correctly foreseen and no mistakes are being made. It is perfect foresight or the intertemporal clearing of capital markets that is here the crucial assumption about linkage. A giant intertemporal tâtonnement process that determines capital prices and equalizes profit rates is assumed so that if ever production costs were to exceed market prices, investment would be cut back and conversely. This would be the mechanism that regulates capital accumulation.1/

1/ Note that fixed sectoral capital stocks need not prevent the equalization of profit-rates. They are equalized by an appropriate adjustment in the market prices of capital, i.e., share prices on the stock market.
While it is perhaps possible to argue that intertemporally efficient paths exert a certain pull on the economy, just as static equilibrium is assumed to do, one is clearly here on much more slippery ground. First, it is clear that even in the most developed economies, futures markets operate only for a few commodities and to assume some kind of giant intertemporal tâtonnement to reflect the actual workings of a market economy requires more than the usual abstraction from reality. Second, even if one were willing to model such an intertemporal tâtonnement, the question of boundary conditions remains open. An intertemporal equilibrium system can be reduced to a system of $2n$ difference equations in capital stocks and prices. While one set of boundary conditions is provided by the $n$ initial capital stocks, the remaining set of $n$ boundary conditions constitutes a problem. If we were to give the system the base period set of prices as $n$ initial conditions, the resulting path can be shown to be highly unstable: the system, trying to validate the initial conditions, would soon attempt to achieve negative output values.\footnote{See Hahn \textit{( )} for a discussion of this problem.} One therefore has
to give it either a terminal capital stock vector or a terminal price vector to achieve a more reasonable solution. Still another alternative may be to assume capital mobility in the initial year so as to free the system from the necessity of validating the historical capital stock and combine an initial price vector with terminal growth requirement on the capital stocks. Whichever reasonable alternative one chooses, however, the system becomes indecomposable over time whenever boundary conditions are split and one ends up with a very large system of intertemporal simultaneous equations. Even such systems can however be solved by taking advantage of their sparse structure.  

1/ Most often the concept of intertemporal equilibrium is in fact abandoned, and profit-rates, defined as the return to capital valued at production cost prices, remain unequal. This does not necessarily mean that claims on existing capital stocks do not get revalued, but that the stock valuation process is not being modelled. Given the extremely thin nature of equity markets in all developing countries, there is good reason for this neglect.

There remains, however, the problem of determining the allocation of investment by sector of destination. In an intertemporal equilibrium model, this allocation is determined by the requirement that production costs must equal the discounted sum of future returns. But in a model that abandons the concept of intertemporal equilibrium, the allocation

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1/ See Dervis (1975) for a model that achieves intertemporal efficiency and must therefore be solved simultaneously for all time periods. The alternative chosen there is to let capital be mobile in the initial period.
of investment remains open and must be governed by an alternative mechanism.

One way of solving this problem is to assume that the allocation of investment by sector of destination in any given period is determined by the prices, production costs and profit-rates of the previous period. The model becomes a recursive model that can be run forward in time given initial conditions consisting of the historically given capital stocks and prices. Because we no longer insist on validating the initial configuration by equalizing profit-rates, the system is not necessarily unstable. How stable it will in fact be will of course depend on how the mechanism determining investment allocation is formulated dynamically. One gives up the notion of an intertemporal equilibrium and settles for a recursive model where the only input needed to solve any one period's model are the exogenous parameters and the past history of the economy. It may be that investment is governed by expectations about the future, but one assumes that these expectations are formed purely on the basis of past experience, not on the basis of some kind of tâtonnement into the future where decision-makers could test the consistency of their expectations.

What is really proposed is a two-stage specification of the overall dynamic model: in the first stage, all markets are assumed to clear subject to a number of restrictions on the ability of certain markets, such
as both capital and labor markets, to adjust. In a second stage, the
dynamic adjustment of certain variables whose values were fixed in the
first stage is explicitly modelled. The overall dynamic model is thus
partitioned into a static within-period equilibrium model and a separate
between-period model which provides the necessary intertemporal linkages and
shifts the sectoral supply and demand functions.1/ Both the problem of
structural disequilibrium in the labor market and the pattern of investment
determination are thus "taken out" of the core equilibrium model and put into
a stage 2 dynamic adjustment model which links the sequence of years considered
during a given planning exercise into a consistent whole. This solution to the
dynamic linkage problem, a solution that gives up the notion of intertemporal
equilibrium, has been chosen by most general equilibrium model builders. While
it constitutes a practical solution, long adopted by model builders in the
advanced economies, it does represent a retreat from the ambitious attempts of
earlier planning models at determining intertemporally efficient or optimal
growth paths. Because the "terminal conditions" problem has never really been
satisfactorily solved (and probably cannot be given the inherent open-endedness
of the future), the paths proposed by optimizing models were never fully con-
vincing.2/ Nevertheless, the next generation of non-linear computable general
equilibrium models will probably return to the problems of optimization. A
fully recursive structure where the past governs the future may assign too

1/ See S. Robinson (AER, 1976) for a more detailed discussion of stages in a
similar approach to the formulation of linkages in a dynamic modelling framework.

2/ See the discussion in chapter 2 on linear models.
great a role to that part of the economic system which is regarded as self-regulating. We shall return to the difficult problems involved in extending these models in an optimizing direction later, in Chapter . In the next section we turn to a brief description of the practical dynamic formulation that has so far been adopted by most model-builders.

4.6 A General Two-Stage Dynamic Formulation

In a general, two-stage approach to dynamic formulation, the role of the stage 2 dynamic adjustment model is to update all the exogenous variables entering the static stage 1 model which will then be solved for the next period. In turn, when various variables are updated for the following period, the dynamic adjustment model will take the solution of past period stage 1 models as given.

Not all of the updating equations will be of a behavioral kind. Some demographic and technological variables will be updated following some separately calculated or projected trends. Others will simply remain the same, as for instance is usually the case for input-output coefficients. Only a subset of the dynamic linkage equations will attempt to model economic behavior or government policies. Table 3 summarizes a typical set of variables likely to be updated by the dynamic linkage equations of a between-period model.
While the classification given in Table 3 encompasses a wide range of variables, it is not meant to apply to all models. It provides examples of variables which have typically been included in dynamic CGE models, but other models have used different approaches. There are, for instance, models in which demographic variables such as population growth and dependency ratios are treated as functions of economic variables.\(^1\) Other models may endogenize technological change (making it, for instance, depend on accumulated past investment) or attempt to model government behavior as endogenously reacting to a set of economic indicators determined by the equilibrium model. These more ambitious models remain exceptions however. In most cases investment allocation and rural-urban migration are the only important economic mechanisms modelled in stage 2.

One could of course take a lot "out" of stage 1 and put it "into" stage 2. Indeed, price formation could be modelled dynamically as

\(^1\) For example the ILO, Bachue models.
Table 3

The Endogenous Variables of a Stage 2 Dynamic Model

A. Variables and Parameters That Most Often Remain Unchanged
   - Share parameters in the sectoral production functions
   - Input-output coefficients
   - Capital composition coefficients

B. Variables and Parameters That Are Usually Updated According to Simple Trends
   - Technological shift parameters in sectoral production functions
   - Total labor supply, sometimes by skill categories
   - Some variables determined in the "rest-of-the-world" in open-economy models
   - Engel elasticities in the consumer demand functions

C. Variables That Are Seen as Determined Largely by Government Policy or Political Mechanisms
   - Money supply or the overall price index
   - Tax rates and government expenditure shares
   - Wages for some labor categories
   - Tariffs, quotas and export subsidies in open economy models

D. Variables That Are Updated By Modelling Economy Behavior
   - Sector capital stocks
   - Labor supplies to subsets of sectors: in particular the rural-urban composition of the labor force
   - Investment shares by sector of destination
a disequilibrium adjustment process in the between-period model. This would dramatically reduce the degree of simultaneity in the stage 1 model! But we have already argued that for development planning purposes an attempt to set up a general disequilibrium model that would be focused on short-term adjustments and disequilibrium phenomena in the product markets might well be counter-productive. For our purposes, and for most empirical applications, what needs most to be included in the stage 2 between-period model are submodels of the sectoral allocation of investment and of rural-urban migration.

We will therefore discuss a simple set of dynamic equations that can be used to describe investment allocation and rural-urban migration. They form the core of the between-period model and together with trend equations and the updating of government policy variables, they allow the model to be run forward in time.

**Investment Allocation**

After determining the volume of investment, a multi-sector model must specify the structure of investment by sector of destination. In the presentation of the static model, we have so far treated these shares as predetermined. For some purposes such as testing the feasibility, consisting and/or desirability of otherwise formulated investment plans and growth targets, it may be best to treat them as predetermined and analyze the consequences of a specified set of investment shares. But in most cases one will want to model the determination of investment by sector
of destination endogenously and thereby attempt to capture the effects of government policy on the structure of investment.

Theoretically, probably the most satisfying way to model sectoral investment allocation would be to split the process into two parts. First, determine the desired investment by each sector and, second, model the allocation process by which the supply of investable funds is reconciled with the demand. This explicit distinction of the demand and the supply side underlies much of the recent work on investment theory in macro-models.\(^1\) In the multi-sector context, each sector would determine its desired capital stock according to various rules - such as accelerators, expected profitability, expected sales. This demand for capital stock must then be translated into a demand for a flow of investment. Finally this flow demand must be compared to the supply of funds available for investment to the particular sector. If the two do not match, the adjustment mechanism must be modelled.

This approach necessitates a realistic specification of both the demand and supply relationships including the explicit modelling of the loanable funds market and the banking system. Adelman and Robinson have used this general approach with a simple model of expectations and investment demand linked to a fairly elaborate model of the supply of loanable funds that distinguished among retained earnings, bank credit and unorganized money market loans.

\(^1\) See Jorgenson ( ).
The data base and effort required for such a full specification makes it a very difficult one to achieve. For many purposes, a less ambitious approach will be sufficient and we shall describe a simple formulation that can be regarded as a minimum core specification to be qualified and expanded in planning applications.

An extreme specification would be to assume that money markets do not exist and allocate the investable funds in proportion to each sector's share in aggregate capital income (or profits). More realistically, one can adjust the proportions as a function of the relative profit rate of each sector compared to the average profit rate for the economy as a whole. Sectors with a higher than average profit rate would get a larger share of investable funds than their share in aggregate profits. We will use this formulation for our simple dynamic model. The shares are given by:

\[ H_{it} = SP_{i,t-1} + \mu SP_{i,t-1} \left( \frac{R_{i,t-1} - AR_{t-1}}{AR_{t-1}} \right) \]

where

- \( SP_{it} \) is sectoral share in aggregate profits
- \( \mu_{Rit} \) is "mobility" of investable funds parameter
- \( R_{it} \) is sectoral profit rate
- \( AR_t \) is average profit rate

1/ Note that since \( \sum_{i} SP_{it} = 1 \), then it is true that \( \sum_{i} H_{it} = 1 \) for any value of \( \mu \) since the average profit rate is equal to the sum of the sector profit rates weighted by the shares in total profits.
The profit rates are defined as returns to capital when the entire capital stock is valued in current prices and also includes capital gains. The equation is:

\[
R_{it} = \frac{V_{it} - \mathbb{E}W_{st}}{U_{i,t-1}K_{i,t-1}} + \frac{U_{it} - (1-d_{i})U_{i,t-1}}{U_{i,t-1}}
\]

where

- \(K_{i,t-1}\) is the capital stock at the end of the last period (and which is used in production in this period),
- \(d_{i}\) is fixed sectoral depreciation rates, and
- \(U_{i}\) is the capital goods price in equation (4.2.23).

When the investment mobility parameter \(\mu\) is zero in equation (4.6.1), there is no intersectoral mobility of investment funds. In essence, all investment is financed by retained profits (ignoring savings from government and labor income). When \(\mu\) is positive, the sectoral allocation of investment will respond to profit rate differentials and high profit sectors will attract funds from low profit rate sectors. Thus \(\mu\) measures the intersectoral mobility of investment funds. It is not, however, an index of the degree of perfection of capital markets. Even if \(\mu\) is zero, the system may move towards equalizing profit rates over time, and, if \(\mu\) is too large, it is easy to make sectoral profit rates oscillate. The parameter \(\mu\) is rather an indicator of the responsiveness of capital markets to static market signals namely, current profit rates in the various sectors.

While we have presented a lagged version of the simple investment
model, we could have let current profit shares and profit rates determine the investment shares. Not much would change, again on the assumption that there are no serious oscillations in the underlying technological and taste parameters. Thus, once we abandon the concept of an inter-temporal tâtonnement, we could in fact incorporate the determination of investment in the within-period model.

It is not necessary to define the $R_{it}$ variables as sectoral shares in aggregate profits. They could represent any measure of the "normal" allocation of investable funds. For example, one might argue that in the absence of a money market response to profit rate differentials, investable funds would be allocated according to current sectoral shares in the capital stock. In practice, given the important role played by government investment, the simple investment theory described above may in any case have to be seriously qualified in any specific application.

Where the government controls a substantial proportion of total investment funds, the allocation of funds should reflect government policy objectives. Although we would argue that the simple theory outlined here is a good point of departure, it is clear that the realistic specification of investment allocation even in a purely descriptive model and quite apart from problems of optimization still represents a major challenge for modelers.

Rural-Urban Migration

Capturing the essence of structural dualism characterizing many developing economies constitutes another major problem for the model builder. Rapid urbanization, even in the face of urban unemployment
and poverty has accompanied growth in most developing countries. Efforts 
have been made to reconcile the observed simultaneous urban unemployment 
and migration from rural to urban areas. The popular model developed by 
Harris and Todaro (1971) explains migration from rural areas to cities 
in terms of the relation between the wage in rural areas, $w_1$, and the 
expected urban wage, $w_2^e$, which is the urban wage weighted by the rate of 
employment in the urban sector.

Migration models have been included in CGE models and have 
empirically turned out to be very important.\footnote{See Ahmed (1975), Adelman and Robinson (1977) and De Melo and Dervis (1977).}
Ahmed treated migration endogenously in the CGE model (Stage 1) 
and determined the distribution of the labor force such that the rural wage 
equalled the expected urban wage. Migration was thus treated exactly as 
in the Harris-Todaro model as the movement of labor required to bring 
about the equality between the rural wage and the expected urban wage.

It is also possible to treat migration as being a function of 
the differential between the rural and urban wages without assuming that 
equilibrium is reached; i.e., that the rural wage equals the expected 
urban wage. In this approach, migration is seen as a disequilibrium ad-
justment mechanism and is modelled as part of stage 2.

Assume that labor category 1 is rural labor and that labor
category 2 is unskilled urban labor, the category which migrants will join. Alternatively, it is possible to spread the migrants among the different urban labor categories, but this will not be done here. Let the two labor categories have natural rates of growth, $\hat{G}_1$ and $\hat{G}_2$. Thus, the labor supply equations for the two categories of labor become:

\begin{align*}
(4.6.3) & \quad \overline{L}_1^s(t+1) = (1 + \hat{G}_1) \overline{L}_1^s(t) - \text{MIG}(t) \\
(4.6.4) & \quad \overline{L}_2^s(t+1) = (1 + \hat{G}_2) \overline{L}_2^s(t) + \text{MIG}(t) \\
(4.6.5) & \quad \text{MIG}(t) = e \left[ \frac{w_2^e}{w_1^e} - 1 \right] \overline{L}_1^s(t) \\
(4.6.6) & \quad w_2^e = w_s \left( \frac{L_2^D}{L_2^s} \right)
\end{align*}

This formulation is exactly the Harris-Todaro formulation except that there is no assumption that equilibrium is reached (i.e., $w_2^e = w_1^e$). As in the Harris-Todaro formulation, the definition of the expected wage in equation (4.6.6) only makes sense if there is urban unemployment. This implies that we are using the fixed real wage version for urban unskilled labor since otherwise $L_2^D = L_2^s$ and $w_2^e = w_2$. If at the fixed wage, $L_1^D > L_1^s$, then one should switch to a full employment model in which the urban wage is determined endogenously and $w_2^e = w_2$.

Since the migration equations are part of the Stage 2 model, migration is seen as a quantity adjustment process that need not achieve full equilibrium. Harris and Todaro require that the urban wage, $w_2$, be
fixed and define migration by comparative statics, imposing the condition that \( \bar{w}_2 = \bar{w}_1 \). In the Stage 2 model, however, it is not assumed that \( \bar{w}_2 = \bar{w}_1 \) and, indeed, the model dynamics may be such that equality is never achieved. In this disequilibrium adjustment model, the definition of the expected wage is less important than in the Harris-Todaro comparative statics formulation. However, the response elasticity parameter \( (\epsilon) \) in equation (4.6.5) is correspondingly more important since it determines the magnitude of the migration response this period to a gap between the rural and expected urban wages.

Whether one wishes to model migration in Stage 1 or Stage 2 is partly a question of empirical realism. If it is modelled as part of Stage 1, then there is assumed to be no constraints on the amount of migration that can take place within the period. If it is modelled as part of Stage 2, one must specify a parameter such as \( \epsilon \) which gives the degree of responsiveness of the quantity adjustment model. While the Harris-Todaro model might be useful for exploring comparative statics questions of "equilibrium" migration, an explicitly dynamic model must deal with the adjustment process.

It should be clear from this discussion that there is considerable scope for flexibility in the choice of which variables are to be determined in each stage. Naturally the outcome of this selection will determine how close the overall CGE model remains to the core Walrasian model discussed above. The specification of intertemporal
linkages is a difficult but crucial element in the formulation of any planning model and we shall see in later chapters how the outcome of specific policy experiments is profoundly affected by alternative specifications of dynamic linkages.

4.7 Conclusion

We have seen at the outset how traditional linear models do not incorporate some of the important general equilibrium interactions present in any economic system. We then proceeded to build step-by-step the general equilibrium demand and supply curves of a simple core CGE model. The presentation emphasized the links and points of departure from the linear models of earlier chapters. We saw that the solution of the model resulted in the selection of a price vector equating all product and factor market demands and supplies. We also examined briefly how one might solve such a model numerically.

The simple model developed in the early sections of the chapter was extremely Walrasian and neoclassical in spirit. With the exception of taxes on factor incomes and a brief allusion to fixed wages for certain labor categories, there was essentially no difference between the technologically feasible production possibility set and the resulting transformation set reflecting market behavior and institutional characteristics of the economy. All markets cleared. Applied models however, although close in spirit to the Walrasian construct, can be characterized as reflecting a "constrained" general equilibrium. The discussion of fragmented capital and labor markets
in the two-stage formulation of the dynamic model indicated that the extent of factor mobility between time periods is affected by the degree of structural rigidities in the economy.

It should be clear, however, that an applied CGE model cannot be viewed as a short-run projections model. It is not intended for that purpose. It should instead be used to explain medium-to-long term trends and structural responses to changes in development policy. It is in terms of the light thrown on structural problems and the help they provide in perspective planning and strategic policy formulation that the models should be evaluated. The analysis of short-run cyclical variations around basic trends is clearly beyond their scope and requires a quite different approach. This is not to say that one is not entitled to expect good predictive performance from CGE models. On the contrary, they should be able to predict, conditional upon policy variables, trends in sectoral structure, intersectoral terms-of-trade, income distribution, trade performance and even government revenue. If they cannot do this better and more consistently than linear models or more informal methods, it is not worth building general equilibrium models. But the models as they have so far been developed are not intended for tracking quarterly or even annual growth rates, predicting short-run movements in price indices and handling speculative behavior. The problems facing development policy are inherently longer-run problems and the models have focussed on these problems. They naturally reflect an emphasis on the long-run rather than the short-run, on growth rather than stabilization, on trends rather than on cyclical variations.
Bibliography


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