PORTFOLIO DIVERSIFICATION
ACROSS CURRENCIES

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Discussion Paper # 90

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November 1980

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Proposals for replacing the doomed Bretton-Woods system by a "world money" standard were made long before the actual demise, particularly by Robert Triffin, the Cassandra of the dollar standard.\textsuperscript{1/} Ten years after, there is no agreement in sight about implementing a world monetary system based on international liquidity like the Fund's Special Drawing Rights. It turns out, however, that the cosmopolitan spirit underlying proposals for a SDR standard is useful in the investigation of portfolio rules for currency diversification by risk averse investors with an international consumption and investment horizon.

In fact, more and more economic agents, from multinational firms and banks to monetary authorities, consume (or are affected by the consumption of) bundles of goods and services produced in different countries and hold portfolios of assets denominated in different currencies. This internationalization has occurred despite the fact that real returns on financial assets denominated in the major currencies have been highly volatile. The variability of observed real returns can be attributed to the divergent rates of inflation across countries and to the changes in the relative prices of different currencies that have prevailed in the floating rate period.

In such an environment, an index of value appropriate for consumers and investors with an international horizon can be defined as the purchasing power of different currencies over goods and services produced in different countries. In Section I this index, introduced in earlier work with Pentti Kouri, is exposted and used to interpret recent trends in the relative attractiveness of major currencies.

Armed with an index of value to find the relevant real return for the international investor, we proceed in Section II to derive an optimal portfolio rule. Assuming perfectly integrated world financial markets and costless trading, real asset returns can be described as Brownian motion, serially independent continuous functions of Wiener processes with constant instantaneous mean and variance. In addition it is assumed that the consumption preferences of the international investor can be described by a separable utility function with constant relative degree of risk aversion, so that the portfolio rule is independent of the consumption rule. These assumptions are of course restrictive, but they allow an intuitive and computationally convenient derivation of the portfolio rule.1

As shown by Kouri (1975) in a center-periphery monetary world that is a stylized representation of the Bretton-Woods system, the optimal portfolio derived under these assumptions is the sum of a minimum variance portfolio and of a zero net worth speculative portfolio dependent on real returns and scaled by risk aversion. A two-country two-asset model is also used in Section II, where a further decomposition of the minimum variance portfolio into the expenditure shares and a zero net worth hedging portfolio depending on the covariance between exchange rate changes and inflation and on consumption preferences is emphasized. It is also shown that, because of Jensen's inequality, expected mean real return differentials depend on preferences. As a consequence, the degree of relative risk aversion determines if increased demand for goods produced in one country will imply increased demand for the currency of that country, as is the case in the popular flow model of the foreign exchange market.

* The consumption and portfolio rules for the general N country non-Brownian motion case are derived in Macedo (1980).
Optimal portfolios for the eight major currencies discussed in Section I are computed in Section III, using actual quarterly changes in their purchasing powers. The time-invariant parameters that transform real return differentials into portfolio proportions are reported for three alternative weighting schemes and three different assumptions about risk aversion.

The findings are relevant for the problem of optimal reserve diversification by central banks, insofar as they can be assumed to pursue a risk averse strategy of maximizing real returns with consumption preferences given by foreign trade patterns.\(^1\) For example, recent estimates of the share of dollars in world official reserves being somewhere between 60 and 70\% depending on coverage\(^2\) would not be much higher than the optimal dollar share if the expenditure share on US goods were about 40\% and the investor had a degree of relative risk aversion close to unity (the Bernouilli case).

It is not surprising that in this case the optimal portfolio should be lower than the actual. Aside from political considerations that may enter in the central banks reserve management policies\(^3\), this framework entirely abandons the presumption that transactions costs and regulations imply a "preferred monetary habitat" for the domestic currency. It would, therefore, take a very high degree of risk aversion for substantial further

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\(^1\) For an application of this framework to the central bank of a less developed country, when real returns are equalized, see Healy (1980b) who estimates the optimal reserve portfolios of four LDCs and contrast them with popular alternatives like pure dollar holdings, expenditure shares and SDR weights.


\(^3\) As argued by Heller-Knight (1978) and Ben-Bassat (1980) particularly for central banks of developed countries.
diversification out of dollars to have been optimal, given the real return structure of April 1978.

Other implications as well as suggestions for future research are taken up in the conclusion of the paper.
I. Indices of purchasing power described

1. We begin by deriving an optimal price index for the international consumer in terms of the standard maximization problem of the household. The utility function is assumed to be homothetic and loglinear so that relative risk aversion is unity. While the assumption of constant expenditure shares is an important drawback for the application of such indices over long periods of time\(^1\), it nevertheless provides a suggestive benchmark. It is also convenient for the purpose of establishing the optimizing foundations of international currency diversification in a well-known framework.\(^2\)

Consider the standard consumption problem in an environment where goods produced in different countries are not perfect substitutes for each other, so that each good \(j\) is also indexed by its country of origin \(i\), where country \(i\) produces \(M_i\) goods and there are \(N\) countries. If the utility function of the 'representative' international consumer based in country \(k\) is as assumed, an indirect utility function, \(v^k\), separable between expenditure, \(E_k\), and prices, \(Q_k\), can be written as

\[
v^k = E_k Q_k .
\]

The budget constraint in terms of the currency of country \(k\) is given by

\[
E_k = \sum_{i=1}^{N} \sum_{j=1}^{M_i} P_{ij} S_{ki} X_{ij} .
\]

where \(P_{ij}\) is the price of good \(j, j=1,\ldots,M_i\), in terms of the currency of country \(i, i=1,\ldots,N\);

\(^1\) McKinnon (1979, pp. 130-131) uses such indices computed by Kawai for the period 1953-1977.

\(^2\) Other specifications of the utility function could provide exact or at least "superlative" indices. See Diewert (1976) and further discussion in Macedo (1979a).
$S_{ki}$ is the price of the currency of country $i$ in terms of the currency of country $k$;

and $X_{ij}$ is the quantity of good $ij$, that is to say good $j$ produced in country $i$.

The purchasing power of currency $k$ for the representative international consumer of country $k$, denoted above by $Q_k$, is then the measure of the utility of an extra unit of currency $k$, or

$$Q_k = \frac{\partial V^k}{\partial E^k} = \prod_{i=1}^{N} \prod_{j=1}^{N_i} (P_{ij} S_{ki})^{-\alpha_i \beta_j^i}$$

where $\sum_{j=1}^{N_i} \beta_j^i = 1$,

and $\sum_{i=1}^{N} \alpha_i = 1$.

The purchasing power of money, applied to any particular good, is of course the inverse of the price of that good. Here the purchasing power of currency $k$ is the inverse of the geometric average of prices expressed in currency $k$ using the expenditure shares as weights.

2. Using the fact that exchange rates do not depend on $j$, the purchasing power of currency $k$ can be decomposed into an index of the purchasing power of 'world money' and a multilateral effective exchange rate for currency $k$. The latter is defined as:

$$S_{ki}^e = \prod_{i=1}^{N} \frac{1}{-\alpha_i} S_{ki}$$

so that, whereas $S_{ki}$ was defined as the price of currency $i$ in terms of currency $k$, $S_{ki}^e$ is defined as the average price of currency $k$ in terms
of the other currencies. An increase in $S_k^e$ therefore represents an effective appreciation of currency $k$.

Using currency $N$ as the numeraire, we can express this multilateral effective exchange rate as

$$S_k^e = \prod_{i=1}^{N} (S_k/S_i)^{-\alpha_i} = S_k^{1-\alpha_k} \prod_{i \neq k}^{N} S_i^{\alpha_i}$$

where $S_i = S_i/S_N, \ i=1,...,N-1$

and $S_N = 1$.

Thus $S_k^e$ can be seen as the ratio between the average price of foreign currencies $\prod_{i \neq k}^{N} S_i^{\alpha_i}$ and the price of currency $k$ in terms of currency $N$, raised to the power of $\sum_{i \neq k}^{N} \alpha_i = 1 - \alpha_k$. Note that, if the expenditure share of goods from country $k$ for the international consumer is negligible, or used as the standard of reference, we obtain the usual index of an effective exchange rate where bilateral exchange rates are weighted by their importance in the foreign trade of country $k$.\(^{1/}\)

We now define the world price level as a weighted average of domestic currency prices

$$P = \prod_{i=1}^{N} \prod_{j=1}^{M_i} P_i^{\alpha_i b_i}$$

Its inverse, $Q$, which can be denoted as 'world money',\(^{2/}\), is the purchasing power of a basket of currencies using the country weights $\alpha_i$. The reason for this is that since there are only $N-1$ exchange rates, the $N$ effective exchange rates defined above multiply to one if the same weights are used.

\(^{1/}\)Rhomberg (1976) has a thorough discussion of the indices of effective exchange rates in use at the time. See also World Financial Markets, August 1976, technical note, p. 14 and below in the text.

\(^{2/}\)Since July 1974 the Special Drawing Rights of the IMF satisfy this notion of 'world money'. See below in the text.
\[ \prod_{k=1}^{N} e^{ak} = 1, \]
so that, as claimed,
\[ \prod_{k=1}^{N} e^{ak} Q_k = Q. \]

In other words, in the same way that the numeraire has a bilateral rate of 1, world money has an effective rate of 1.

Therefore the purchasing power of currency \( k \) is obtained by multiplying the purchasing power of world money by the effective exchange rate for currency \( k \), or
\[ Q_k = Q_k e^{ak}. \]

This index of the purchasing power of currency \( k \) can be used to compute the purchasing power of any asset \( i \), the domestic currency price of which is \( A_k^i \), in terms of a particular international consumption basket, which would simply be
\[ A_k^i Q_k = A_k^i Q_k Q_k. \]

The purchasing power of bonds, of equity, of gold, or of raw materials would be defined in this way. An international "real wage" could similarly be defined as the purchasing power of labor in country \( k \) in terms of a given international consumption basket.

3. The notion of the purchasing power of a currency can be usefully applied without an explicit knowledge of the preferences of a given consumer-investor, depending on the purpose at hand. For example, a multinational
firm producing one particular good might be interested in the purchasing power of different currencies over that good, say \( j \), produced in \( N \) different countries, or over these \( N \) different products; this would be obtained by setting \( \beta_j^i = 1 \) for all \( i \).\(^1\)

Alternatively, the domestic weights \( \beta \) can be given implicitly by an available price index, so that the only explicit choice of weights refers to the expenditure shares across national outputs, given by \( \alpha \).

In that case, a decision has to be made about the relevant price index. This has been a long standing subject of controversy in connection with the purchasing power parity hypothesis.\(^2\) Keynes (1923) and Triffin (1937) among others favored broadly based indices such as the consumer price index or the value added deflator in the definition of an "equilibrium" exchange rate. The basic reason was that indices of relative prices with too large a share of traded goods - such as wholesale price indices - would move too closely with the exchange rate, so that the 'real' exchange rate would understate the departure from presumed base period purchasing power parity.\(^3\) Recently, however, the substantial movements in relative costs and prices for manufactures has made comparisons based on traded goods prices and unit labor costs quite popular in policy discussions.\(^4\)

\(^1\)The demand structure underlying the IMF Multilateral Exchange Rate Model is roughly one where there are \( M \) products produced in \( N \) countries so that \( M_i = M \) for all \( i \). See Armington (1969) and below in the text.

\(^2\)Officer (1976) and Katseli (1979) have useful surveys of the PPP literature.

\(^3\)Triffin (1957, p. 71, footnote 1) states that wholesale prices are 'totally irrelevant and misleading' for competitiveness calculations.

\(^4\)They have been reported monthly in World Financial Markets since November 1978 and in International Financial Statistics since January 1979.
In the framework above, a 'real' effective exchange rate of currency $k$ is nothing but the ratio of the purchasing power of currency $k$ from the perspective of an international investor and the purchasing of currency $k$ from the perspective of a national investor, defined as consuming a national basket, or $q_k = 1$. This can be written as

$$\tilde{s}_k^e = \frac{Q_k^d}{Q_k^l}$$

where $Q_k^d$ is the purchasing power of currency $k$ when the consumer is such that $q_k = 1$.

Using the definition of $Q_k$, we see that the 'real' effective exchange rate of currency $k$ is just the relative price of domestic and world output in the same units:

$$\tilde{s}_k^e = \sum_{i=1}^{N} (P_i \tilde{s}_i / P_k \tilde{s}_k)^{-q_i}$$

where $P_i = \sum_{i=1}^{M} P_{i,j} \beta_{i,j}$, $i = 1, \ldots, N$

when $\tilde{s}_k^e$ increases, there is real appreciation of the domestic currency and the terms of trade, defined as $1/\tilde{s}_k^e$, deteriorate.

It should be pointed out that the comparison underlying the construction of 'real' exchange rates cannot be grounded on optimizing behavior because the preferences of the domestic and international consumer-investor are different. Nevertheless, it is often of considerable heuristic value in quantifying medium and long run trends in international competitiveness relative to some presumed equilibrium relative price.1/

1/A recent example of the use of such indices for policy purposes is Dorabusch (1978). (See account of Business Week, April 3, 1979.) Historical perspectives are taken in Diaz (1979), Macedo (1979b) and Branson (1980).
In this context, an intermediate case may be mentioned. Consider a 'foreign' consumer-investor, defined as being located in country $k$ but only consuming goods of country $i$, so that $\alpha_i = 1$. The purchasing power of currency $k$ for such agent, denoted by $Q_k^i$, is simply the purchasing power of currency $k$ for the domestic consumer-investor of country $i$, $1/p_i$, times the bilateral exchange rate, or

$$Q_k^i = (S_i/S_k)p_i^{-1}$$

The real bilateral exchange rate between currency $i$ and currency $m$ can therefore be expressed as a ratio of the purchasing powers of the same currency for investors located in the two different countries, including the case where one of them is the domestic country. For example,

$$\tilde{S}_{im} = \frac{Q_k^m}{Q_k^i} = \frac{p_i S_m}{p_m S_i} \quad \text{for all } k.$$ 

Because the ratio of two nominal effective exchange rates is always the bilateral rate, the ratio of the purchasing powers of two currencies is also the bilateral rate. While it is obvious that world money cancels for the case of the international investor, this is also true for the foreign investor. For example for an investor located in country $i$ the ratio of the purchasing powers of currencies $k$ and $m$ is the price of currency $k$ in terms of currency $m$:

$$\frac{Q_k^i}{Q_m^i} = \frac{(p_i S_k/S_i)^{-1}}{(p_i S_m/S_i)^{-1}} = \frac{S_m}{S_k} = S_{km} \quad \text{for all } i.$$
4. In order to illustrate the application of the framework developed above, monthly indices of the purchasing power of major currencies from April 1973 to April 1978 are plotted in Figure 1. The consumption horizon of the hypothetical international investor was taken to be eight industrial countries, Canada (CA), France (FR), Germany (GE), Italy (IT), Japan (JA), Switzerland (SZ), the United Kingdom (UK) and the United States (US). The $\beta$ weights were assumed to be the same as the ones used in constructing the consumer price indices of the eight countries. Several alternative schemes for the $\alpha$ weights will be discussed below, but in Figure 1 weights given by the share in total income were used in the computation both of a monthly index of the purchasing power of world money with base = 100 in April 1973 and of a monthly index of the effective exchange rate of the eight currencies with the same base. Even though goods prices are a period average, end of period exchange rates were used in order to come as close as possible to prices at which actual transactions take place.

As already mentioned, the choice of the weights depends "on the particular policy objective".\textsuperscript{1/} The first choice, between bilateral and global weighting schemes is easy because the concept of an international investor requires that a rigorous comparison between the purchasing powers of different currencies be possible and this rules out bilateral weights.

Table 1 presents three alternative weighting schemes. The 'trade weights' labeled XM in the first column are measures of the average openness of the economy, using exports and imports. The 'MERM weights' in the second column are measures of marginal openness. The 'income weights' of the third

Table 1
Alternative Weighting Schemes (%)

<table>
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<tr>
<th></th>
<th>XM</th>
<th>MERM</th>
<th>Y</th>
</tr>
</thead>
<tbody>
<tr>
<td>CA</td>
<td>8.0</td>
<td>5.8</td>
<td>5.0</td>
</tr>
<tr>
<td>FR</td>
<td>12.2</td>
<td>17.4</td>
<td>9.6</td>
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<tr>
<td>GE</td>
<td>19.4</td>
<td>15.3</td>
<td>13.3</td>
</tr>
<tr>
<td>IT</td>
<td>8.2</td>
<td>7.8</td>
<td>5.0</td>
</tr>
<tr>
<td>JA</td>
<td>13.4</td>
<td>8.5</td>
<td>15.9</td>
</tr>
<tr>
<td>SZ</td>
<td>3.2</td>
<td>1.9</td>
<td>1.7</td>
</tr>
<tr>
<td>UK</td>
<td>11.1</td>
<td>6.0</td>
<td>6.7</td>
</tr>
<tr>
<td>US</td>
<td>24.5</td>
<td>37.3</td>
<td>42.7</td>
</tr>
</tbody>
</table>

Note: XM Total trade
      MERM IMF Multilateral Exchange Rate Model
      Y   Income
column are a prima facie measure of relative size.

The XM weights refer to the sum of average dollar exports and imports during the period 1973–77. The MERM weights are derived from the size of the effect of a ten percent depreciation of the domestic currency in the total balance in dollars, at the scale of world trade in 1977.¹ From the relative size of these total effects bilateral shares are derived, which, after adjustment for the initial trade imbalance is made, are used to obtain bilateral weights for the effective exchange rates reported in IFS. Here we take the main diagonal of the share matrix so that these weights give a ranking of the eight countries according to one measure of 'sensitivity' to external developments.²/

The income weights were derived from gross domestic product in 1975 domestic prices converted into dollars at the average yearly exchange rate and averaged from 1973 to 1977. As mentioned, they were used for the indices plotted in Figure 1.

Table 1 reveals interesting patterns. Thus, aside from the US, only France shows the marginal openness larger than the average openness. The US and Japan rank highest on income weights. In the UK, import weights are highest but the income share is higher than the MERM share. Canada, Germany, Italy and Switzerland show a ranking typical of smaller open economies, total trade first, MERM second and income last, even though Germany ranks a close third on the income scale.


²/ The sensitivity/openness issue was first addressed in Cooper (1968) and has become a popular topic in international political economy. See Keohane–Nye (1977) and Macedo–Peaslee (1971), expanded in Macedo (1977), for example.
We can use as illustration indices of the purchasing power of these eight currencies, where a weights are attached to the national consumer price indices. The first point is that there is hardly any difference in the aggregate between these weighting schemes. In fact, the values of the indices of the purchasing power of world money computed from total trade weights, income weights and MERM weights were in January 1970, 120.1; 119.0 and 118.9 respectively and in April 1978, 62.8; 63.9 and 63.8 respectively for a base value = 100 in April 1973.

This can also be seen in Table 2, where the mean annualized percentage change of the indices over a quarter together with the standard deviation, are reported for the period April 1973 - April 1978. Using income weights, the mean rate of change of the indices is always between the mean change using total trade weights and the mean change using MERM weights. Nevertheless, the income and MERM weights are generally closer together than they are to the total trade weights. The ranking of the variances is in fact the same for income and MERM weights, with the exception of France, which comes before Japan and Canada for the income weights and after for the MERM weights. The ranking of total trade weights also interchanges the relative ranking of Germany and Italy using the other weights and it brings Canada from the seventh to the fourth largest variance, coinciding there with the MERM ranking.

Among the many other weighting schemes that could be discussed¹, one is particularly relevant for the international perspective taken here. It has to do with the relative use of a particular currency in world payments. This measure is only relevant for the five major currencies, the US dollar,

¹See Macedo (1979, p. 11 ff.) for a discussion of four other weighting schemes, namely export and import shares, export shares in manufactures using the IMF World Trade Model and MERM "high elasticity" weights.
Table 2

Mean and Standard Deviation of Quarterly Changes in Purchasing Powers (in % p.a.) Apr 1973-Apr 1978

<table>
<thead>
<tr>
<th>Weights</th>
<th>Mean</th>
<th>s.d.</th>
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<tbody>
<tr>
<td></td>
<td>Mean</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Y</td>
<td>MERM</td>
</tr>
<tr>
<td></td>
<td>Y</td>
<td>MERM</td>
</tr>
<tr>
<td>CA</td>
<td>-11.97</td>
<td>-11.64</td>
</tr>
<tr>
<td>GE</td>
<td>-2.36</td>
<td>-2.12</td>
</tr>
<tr>
<td>IT</td>
<td>-17.01</td>
<td>-16.76</td>
</tr>
<tr>
<td>JA</td>
<td>-5.76</td>
<td>-5.44</td>
</tr>
<tr>
<td>SZ</td>
<td>1.56</td>
<td>1.83</td>
</tr>
</tbody>
</table>
the D. mark, the pound sterling, the French franc and the yen. Indeed, even though the SDR basket has been based on sixteen currencies, the return was computed on the basis of an average of these five currencies.\footnote{1}{Advocates of a greater commercial use of the SDR have long proposed the reduction of the number of currencies in the basket from sixteen to five\footnote{2}{and the step was officially taken by the Fund in September 1980. Table 3 contrasts these with the implicit SDR weights established in July 1974 and July 1978 as well as with the implicit income weights for the five countries using the shares in Table 1 third column.}

The reversal of the ranking of Japan, a larger economy and a smaller currency area, and of the UK, a smaller economy and a larger currency area, are noteworthy in the contrast of columns 1 and 4. Comparing the first three columns, the decline of the two reserve currencies of the Bretton-Woods system from 60% to 55% and the increase in the share of the D. mark are apparent but it is the stability of these shares that appears striking in light of the volatility of the foreign exchanges since 1974. We now turn to an interpretative account of the period, using these indices.

Looking back at Figure 1, the downward trend in the purchasing power of world money during the five year period is particularly noteworthy. The variations of the purchasing powers of the various currencies follow closely the ones of their nominal effective exchange rates, with built-in depreciation of about 7.25% p.a. over the eight years from 1970. Thus, the only currency for which purchasing power increased - by 3.7% - was the Swiss franc. The

\footnote{1}{See Polak (1979, p. 643).}
\footnote{2}{While the possibility of a private use of SDRs was raised in the Committee of Twenty in connection with intervention (see IMF (1974), p. 123), the 'simplified SDR' as a means of promoting its commercial use was proposed in World Financial Markets, August 1975, on the basis of the implicit weights of July 1974. These weights were used in Kouri-Macedo (1978, p. 129) to compute optimal portfolios.}
Table 3

SDR and income weights

<table>
<thead>
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<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
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<td>US</td>
<td>47</td>
<td>49</td>
<td>42</td>
<td>48</td>
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<td>UK</td>
<td>13</td>
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<tr>
<td>JA</td>
<td>11</td>
<td>11</td>
<td>13</td>
<td>18</td>
</tr>
<tr>
<td>FR</td>
<td>11</td>
<td>11</td>
<td>13</td>
<td>11</td>
</tr>
</tbody>
</table>

1  SDR 1974 definition
2  1978
3  1980
4  Income weights from Table 1
largest decline in purchasing power was the one of the Italian lira (57.8%), closely followed by the British pound (54.5%). The Deutsche mark had lost 15.1% of its purchasing power.\textsuperscript{1}

The drastic increase in the German discount rate soon after the beginning of generalized floating increased the effective D. mark rate substantially, as can be seen in Figure 1, where the purchasing power index for that currency was 114 in July. Similarly, the lifting of restrictions on the access of non residents to German capital markets appreciated the effective mark and led to an increase in purchasing power from 92 in September 1974 to 98 in February of 1975. The decline in domestic inflation since the first semester of 1976, on the other hand, propelled a continued appreciation of the effective mark which in March 1978 was 35% above its value five years earlier. This was just enough to keep the purchasing power index in the 84-87 range, while world money had declined from 74 to 64 since April 1976.

The slowing down of the decline in the purchasing power of the British pound in late 1976 at a value of around half of the base period can also be related to the increase of 8% in the effective pound rate after the IMF stabilization plan and the increase in oil exports.\textsuperscript{2} At the end of the sample period, on the other hand, the effective dollar rate was at its April 1973 low level, having appreciated slightly until late 1977. In terms of purchasing power, however, the main feature in Figure 1 is that world money has but smoothed slightly the dollar trend, whose weight is about 43%.

In Figure 2 we show the 'real' effective exchange rates of the same currencies. The greater variability is evident but there are discernible

\textsuperscript{1} The remaining values were CA = 54.9; FR = 61.3; JA = 73.7 and U.S. = 61.9

\textsuperscript{2} Between 1976, 11 and 1978, 4. We have been quoting the World Financial Markets nominal effective rates.
trends in the real world price of domestic consumption baskets, particularly for the U.S. and Japan. In fact, the yen shows a much higher appreciation than the purchasing power would have led to believe, given that the weight of the yen in world money is the second largest (16%). The real appreciation of the yen is of course much smaller when only traded goods prices are included.1/

Finally, Figure 3 displays the other measure discussed above of trends in the relative attractiveness of the U.S. dollar by comparing the increase of the U.S. consumer price index (line labelled "United States"), the purchasing power of the U.S. dollar for the international investor (line labelled "World") and the purchasing power of the U.S. dollar for investors located in Italy, the U.K., Germany and Switzerland. These four lines are simply the increase of the dollar value of the domestic price level, so that the decline in the purchasing power of the dollar is low in low inflation countries like Switzerland and high in high inflation countries like Italy.

1/ Thus in May 15, 1978 the real effective rate of the yen with 1976 bilateral manufactures trade shares as weights and March 1973 as base was 109.1 using wholesale prices of manufactures and 132.7 using consumer prices. Using export unit values leads to a real depreciation of 9%. See World Financial Markets, May 1978. The IMF values are 107.7 for relative wholesale prices in 1978 relative to 1973 and 123.6 for relative value added deflators during the same interval.
FIGURE 3
(April 1973 = 100)
II. Time invariant portfolio rules derived

The indices of purchasing power were derived in Section I as indices of value in the static optimization problem of a Bernoulli international consumer. They were used in suggesting recent trends in international financial markets because they can also be derived in the less restrictive framework of continuous time intertemporal consumption and investment optimization under uncertainty.\(^1\) Thus, in this section we retain the homotheticity of the utility function but we allow non unitary relative risk aversion. In order to preserve the separation between the consumption and investment decisions typical of the classic mean-variance framework of Markowitz (1958) and Tobin (1965), the stochastic processes generating proportional changes in prices and exchange rates are assumed to be Brownian motion, so that prices and exchange rates are stationary and lognormally distributed. While these assumptions are still quite restrictive, they are sufficient for the computation of optimal portfolios in the next section.

Since the purpose of this section is illustrative, we restrict the investor to a two country, two good world.

Consider the problem of maximizing the expected value of a utility function with constant relative risk aversion \(\gamma\) which is linear in the mean and variance of the real return on two assets:

\[
\text{Max } U = x_1(R_1 + dq_1) + x_2(R_2 + dq_2) \quad \frac{1}{2}(1 - \gamma)(dq_1 dq_2)^2
\]

subject to \(x_2 = 1 - x_1\)

where \(x_i\) is the proportion of wealth invested in the asset

\(^1\) See Merton (1969, 1971, 1973) and Merton-Samuelson (1974, p. 85 ff.)
denominated in currency $i$, $i=1, 2$.

$R_i$ is the known nominal return on the asset denominated in currency $i$;

and $dq_i$ is the proportional change in the purchasing power of currency $i$.

It is well known that the solution to this class of problems is an optimal portfolio obtained by adding a minimum variance portfolio, $x^m$, independent of risk aversion and of returns and a zero net worth speculative portfolio, $x^s$, scaled by risk aversion and chosen by comparing the return on each asset to the return of the minimum variance portfolio. The solution can be written as\(^1\):

$$
\begin{bmatrix}
  x_1 \\
  x_2
\end{bmatrix} = \begin{bmatrix}
  x_1^m \\
  x_2^m
\end{bmatrix} + \frac{1}{1-\gamma} \frac{1}{\omega_1^2 + \omega_2^2 - 2\omega_{12}} \begin{bmatrix}
  \omega_2^2 - \omega_{12} \\
  -\omega_{12} & \omega_1^2
\end{bmatrix} \begin{bmatrix}
  1-x_1^m \\
  -x_2^m
\end{bmatrix} \begin{bmatrix}
  r_1 \\
  r_2
\end{bmatrix}
$$

and

$$
\begin{bmatrix}
  x_1^m \\
  x_2^m
\end{bmatrix} = \frac{1}{\omega_1^2 + \omega_2^2 - 2\omega_{12}} \begin{bmatrix}
  \omega_2^2 - \omega_{12} \\
  \omega_1^2 - \omega_{12}
\end{bmatrix}
$$

where $\omega_i$ is the variance of changes in the purchasing power of currency $i$, $i=1, 2$.

$\omega_{12}$ is the covariance between purchasing power changes;

$r_i = R_i + E(dq_i)$ is the real return in the asset denominated in currency $i$.

\(^1\)See the Appendix for the derivation of the $N$ country case.
In this case the logs of the purchasing powers of the two currencies are expressed as

\[ q_1 = -\alpha_1 P_1 - \alpha_2 P_2 - (1-\alpha_1)s \]
\[ q_2 = -\alpha_1 P_1 - \alpha_2 P_2 + \alpha_1 s \]

where, denoting means respectively by \( \nu_1 \) and \( \nu \) and variances respectively by \( \xi_1 \) and \( \sigma \), proportional changes in prices and exchange rates are given by the following functions of Wiener processes \( dv_1 \) and \( dz \):

\[ dp_1 = \nu_1 dt + \xi_1 dv_1 \quad i=1, 2 ; \]
\[ ds = \nu dt + \sigma dz . \]

We can then express the proportional rate of change of the purchasing power of the two currencies as

\[ dq_1 = \{ -\alpha_1 \nu_1 - \alpha_2 \nu_2 - (1-\alpha_1)\nu + \frac{1}{2} \left[ \alpha_1^2 \xi_1 + \alpha_2^2 \xi_2 + 2\alpha_1 \alpha_2 \xi_1 \xi_2 \rho_{12} + (1-\alpha_1)^2 \sigma^2 \right] \}
\[ + 2(1-\alpha_1)(\alpha_1 \xi_1 \sigma_1 + \alpha_2 \xi_2 \sigma_2) \} dt - \alpha_1 \xi_1 dv_1 - \alpha_2 \xi_2 dv_2 - (1-\alpha_1)\sigma dz ; \]

\[ dq_2 = \{ -\alpha_2 \nu_2 - \alpha_2 \nu_2 + \alpha_1 \nu + \frac{1}{2} \left[ \alpha_1^2 \xi_1 + \alpha_2^2 \xi_2 + 2\alpha_1 \alpha_2 \xi_1 \xi_2 \rho_{12} + \alpha_1^2 \sigma^2 \right] \]
\[ - 2\alpha_1 (\alpha_1 \xi_1 \sigma_1 + \alpha_2 \xi_2 \sigma_2) \} dt - \alpha_1 \xi_1 dv_1 - \alpha_2 \xi_2 dv_2 + \alpha_1 \sigma dz ; \]

where \( \rho_{12} \) is the correlation coefficient between \( dv_1 \) and \( dv_2 \);

\( \rho_1 \) is the correlation coefficient between \( dv_1 \) and \( dz \).

The mean instantaneous expected change is given by the expressions in curly brackets and, by the multiplication rule of stochastic calculus, the expressions in square brackets give the instantaneous variance. The covariance is given by
\[ \omega_{12} = \alpha_1 \xi_1^2 + \alpha_2 \xi_2^2 + 2 \alpha_1 \alpha_2 \xi_1 \xi_2 \rho_{12} - \alpha_1 (1-\alpha_1) \sigma^2 - 2 \alpha_1 \xi_1 \sigma_1 \rho_1 \]

\[-2\alpha_1 \alpha_2 \xi_1 \xi_2 \sigma_2 + \alpha_1 \xi_1 \sigma_1 + \alpha_2 \xi_2 \sigma_2 .\]

It is thus clear that, for example,

\[ \omega_1^2 + \omega_2^2 - 2 \omega_{12} = \sigma^2 \]

so that we can rewrite the speculative portfolio as

\[
\begin{pmatrix}
\alpha_1^s \\
\alpha_2^s
\end{pmatrix} = \frac{1}{1-\gamma} \Sigma \begin{pmatrix} r_1 \\ r_2 \end{pmatrix}
\]

where \[ \Sigma = \frac{1}{\sigma^2} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \]

only depends on the variance of the exchange rate. Now, defining

\[ \Psi = \xi_i \sigma_i \]

the covariance between domestic prices and exchange rates, we rewrite the minimum variance portfolio as

\[
\begin{pmatrix}
x_1^m \\
x_2^m
\end{pmatrix} = \frac{1}{\sigma^2} \begin{bmatrix} \alpha_1 \sigma^2 - \alpha_1 \Psi - \alpha_2 \Psi \\
(1-\alpha_1) \sigma^2 + \alpha_1 \Psi + \alpha_2 \Psi \end{bmatrix} = (I - \Phi) \begin{pmatrix} \alpha_1 \\ \alpha_2 \end{pmatrix}
\]

where \[ \Phi = \begin{bmatrix} \xi_1 \rho_1 / \sigma & \xi_2 \rho_2 / \sigma \\ -\xi_1 \rho_1 / \sigma & -\xi_2 \rho_2 / \sigma \end{bmatrix} \]
measures the ratio of the covariance of prices and exchange rates and the variance of exchange rates. A high correlation coefficient could thus be offset by an exchange rate variance sufficiently higher than the variance of prices. If exchange rates and prices are uncorrelated or if price changes are perfectly anticipated φ becomes a zero matrix. Lower variance of exchange rates relative to prices would make the minimum variance portfolio differ more from expenditure shares. In fact the condition for the share in the minimum variance portfolio to be larger (smaller) than the expenditure share for country 1 (2) is given by

\[ \alpha_1 (1 - \frac{\zeta_1 \rho_1}{\zeta_2 \rho_2}) > 1 \text{ as } \rho_2 > 0. \]

This condition will hold when \( \rho_1 < 0 \) so that currency 1 is a good hedge because its price level is negatively correlated with the exchange rate. Given that \( \rho_1 < 0 \), the larger the covariance term in country 1 relative to country 2, the larger the difference \( x_1^m - \alpha_1 \), given \( \alpha_1 \). The converse is true when \( \rho_1 > 0 \).

The condition for a marginal increase in \( \alpha_1 \) to increase \( x_1^m \), on the other hand, is:

\[ \sigma > \zeta_1 \rho_1 - \zeta_2 \rho_2 \]

which will of course be true if \( \rho_1 < 0 \) and \( \rho_2 > 0 \), but also if the variance of exchange rate changes is larger than the algebraic difference sum of the covariances of exchange rates with inflation in both countries.

That the effect of an increase in the relative demand for country 1 goods increases the relative demand for country 1 currency is similar to the condition of stability in a flow view of the foreign exchange market,
whereby the demand for foreign currency is derived from the supply and demand for exports and imports. Given real returns, this condition would probably hold because, in practice, the variance of exchange rate changes is substantially larger than the variance of inflation.

The effect on total portfolio proportions has, however, to take into account that, because of Jensen's inequality, real return differentials depend not only on the variance of exchange rate changes but also on the covariance with prices. From the expressions for the changes in purchasing powers, in fact, we easily obtain an expression for the difference between real and nominal return differentials as:

\[ dq_1 - dq_2 = \left\{ -\pi + \frac{\sigma^2}{2} + \left[ \alpha_1 (-\sigma^2 + \xi_1 \rho_1 \sigma) + \alpha_2 \xi_2 \rho_2 \sigma \right] \right\} dt - \sigma dz \]

so that the term in square brackets is \( -x_1^m \sigma^2 \) and the speculative portfolio of currency 1 becomes

\[ x_1^s = \frac{1}{1-\gamma} \left[ \frac{\hat{R}}{\sigma^2} - x_1^m \right] \]

where \( \hat{R} = R_1 - R_2 - \pi + \frac{\sigma^2}{2} \)

and the total portfolio proportions become

\[
\begin{pmatrix}
  x_1 \\
  x_2
\end{pmatrix}
= \frac{1}{1-\gamma} \left[ -\gamma \begin{pmatrix}
  \alpha_1 \\
  \alpha_2
\end{pmatrix} + \gamma \begin{pmatrix}
  \psi_1 & \psi_2 \\
  -\psi_1 & -\psi_2
\end{pmatrix} \begin{pmatrix}
  \alpha_1 \\
  \alpha_2
\end{pmatrix} + \frac{\hat{R}}{\sigma^2} \begin{pmatrix}
  1 \\
  -1
\end{pmatrix} \right].
\]
The condition for the total effect to be positive is therefore that the above condition for the effect on the minimum variance portfolio be satisfied and that the relative degree of risk aversion be greater than one.

If the investor has a Bernouilli utility function the covariance term in the speculative portfolio exactly offsets the minimum variance portfolio and the total portfolio, independent of preferences, is given by

\[ x^B = \hat{\gamma}/\sigma^2. \]

If the investor is less risk averse than the Bernouilli investor the weight of speculative portfolio is correspondingly increased. Taking the case discussed in a letter of Gabriel Cramer to Daniel Bernouilli\(^1\)/ of \( \gamma = \frac{1}{2} \) we would have

\[ x^C = -x^m + 2\hat{\gamma}/\sigma^2. \]

In that case, the condition \( \frac{\partial x_1^C}{\partial \alpha_1} > 0 \) would require that the effect of an increase in the demand for goods of country 1 be associated with a lower share of currency 1 in the minimum variance portfolio, that is to say that

\[ \sigma < \xi_1^\rho_1 - \xi_2^\rho_2. \]

There is, however, some presumption that observed relative risk aversion, if taken to be constant, would probably be higher than the Bernouilli watershed, even for large organizations.\(^2\)/ In that case, of course, the condition for the effect on the minimum variance portfolio to be positive applies to the effect on the total portfolio as well.

\(^1\)/See Cramer (1728) and Bernouilli (1738) quoted in Samuelson (1977).

\(^2\)/See Samuelson (1977, p. 39 fn. 7).
III. Optimal portfolios for eight industrial countries computed

1. We now compute the optimal portfolio proportions that would obtain if in April 1978 an international investor with given expenditure shares and constant relative risk aversion would apply the time invariant portfolio rule derived in the previous section in the face of observed returns and ex post covariances of prices and exchange rates in the eight country world of Section I.

The assumption that the covariance structure of exchange rate changes and inflation is stationary is a strong one.\(^1\) Rather than assuming that the investor expects the change in purchasing power next period to be the same as the change in purchasing power this period, as implied by the Brownian motion assumption of Section II, forecasting rules could be postulated that would make the filtered covariance structure stationary.\(^2\)

Impressionistic evidence on the stationarity of the covariance structure from October 1973 to April 1978 is at best mixed. For example, the variance of exchange rate changes seems to have declined in the cases of the French franc, the D. mark and the yen. The lira and the Swiss franc show jumps rather than a downward trend. The Canadian dollar is quite stable with an upward jump in late 1976 and sterling shows a great deal of variation with a slight upward trend. Covariances between prices and exchange rates are typically dominated by the exchange rate variance. Filtering techniques are, however, an equally ad hoc alternative, particularly if different ARIMA

\(^1\) See the discussion of Branson (1980b, p. 192).

\(^2\) This was one of the procedures used in the simulations of Healy (1980a, pp. 209-220) who fits different integrated moving average processes to the exchange rate series and prefers the results to the ones obtained from sample averages.
processes are used for each series. While preliminary evidence for the more recent experience suggests that covariances might tend to stabilize, the assumption of stationarity has to be made mostly on grounds of convenience.

Given the time-invariant parameters so obtained, the optimal portfolio rule involves computing a minimum variance portfolio, \( x^m \), independent of mean real returns and a speculative portfolio, \( x^s \), dependent on mean real returns, where the quarterly change in the purchasing power of the currencies in terms of national consumer price indices is used to deflate nominal returns on money market instruments denominated in the eight different currencies.

From the previous section, we know that the "capital position" of the international investor is given by the portfolio proportions which minimize the variance of the return. This portfolio is the equivalent of the "risk-free" asset in domestic finance. However, since the minimum variance portfolio proportions need not be all positive, there is a certain amount of risk that the international investor has to bear, even when following the 'optimal rule'.

Using call money rates when readily available, the mean and standard deviation of nominal returns in percent per annum are reported in the first two columns of Table 4. Mean real returns, for the three different weighting schemes, obtained by adding the changes in purchasing powers from Table 2 above, are also reported in Table 4. It is apparent that the differences across weighting schemes are slight. Indeed the ranking of relative returns — indicated in Table 8 below — is the same independently of the weighting scheme.

Therefore, in spite of the fact that, as shown in the previous section, real return differentials depend on the minimum variance portfolio,
Table 4

Rates of Return Apr 1973-1978

<table>
<thead>
<tr>
<th></th>
<th>Nominal mean</th>
<th>Nominal s.d.</th>
<th>Mean Real Y</th>
<th>Mean Real XM</th>
<th>with weights MERM</th>
</tr>
</thead>
<tbody>
<tr>
<td>CA</td>
<td>7.55</td>
<td>1.12</td>
<td>-4.42</td>
<td>-4.09</td>
<td>-4.65</td>
</tr>
<tr>
<td>FR</td>
<td>9.56</td>
<td>2.10</td>
<td>.15</td>
<td>.39</td>
<td>-.21</td>
</tr>
<tr>
<td>GE</td>
<td>6.10</td>
<td>3.22</td>
<td>3.74</td>
<td>3.99</td>
<td>3.33</td>
</tr>
<tr>
<td>IT*</td>
<td>11.14</td>
<td>2.80</td>
<td>-5.87</td>
<td>-5.61</td>
<td>-6.18</td>
</tr>
<tr>
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<td>2.75</td>
<td>3.07</td>
<td>2.46</td>
</tr>
<tr>
<td>SZ*</td>
<td>5.52</td>
<td>1.24</td>
<td>7.08</td>
<td>7.34</td>
<td>6.69</td>
</tr>
<tr>
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<td>-4.89</td>
<td>-4.60</td>
<td>-5.22</td>
</tr>
<tr>
<td>US</td>
<td>7.13</td>
<td>2.36</td>
<td>-2.61</td>
<td>-2.28</td>
<td>-2.84</td>
</tr>
</tbody>
</table>

* Government bond yield. Others are call money rates.
empirically mean real returns are not too sensitive to differences in preferences, at least as captured by the types of weighting schemes postulated in Section I. As a consequence, since the empirical analysis of comparative effects of changes in preferences are beyond the scope of this section, the computations of the speculative portfolio are made in terms of real returns rather than in terms of nominal returns.

2. The minimum variance portfolio is given by the deviation between the expenditure shares \( x \) and a zero net worth weighted average of these shares, where the weights for the \( N-1 \) currencies are given by the covariance between changes in exchange rates and in prices in the \( N-1 \) countries. The share of the \( N \)th currency in this zero net worth component of the minimum variance portfolio is simply minus a weighted sum of the covariance between the \( N-1 \) exchange rates and domestic inflation relative to the variance covariance matrix of exchange rate changes. Using the notation of the previous section to refer to the 8 country world, we have

\[
x^m = (I - \Phi) x
\]

The \( \Phi \) matrix is thus obtained by multiplying the 7 by 8 covariance matrix between dollar exchange rate changes and inflation in the eight countries by the inverse of the variance covariance matrix of dollar exchange rate changes and using the fact that total minimum variance portfolio proportions sum to one. This matrix is reported in percentage terms in Table 5. It measures departures from anticipated inflation, which is the case when \( \Phi \) is a zero matrix because the variance of prices is zero. When the variance of the price level of country \( i \) is zero, then the \( i \)th column will be zero and an investor with \( x_i = 1 \) would have \( x_i = 1 \) and a diversification across the other currencies based on the covariance between other countries' prices
Table 5

The phi matrix

(\%)

<table>
<thead>
<tr>
<th></th>
<th>CA</th>
<th>FR</th>
<th>GE</th>
<th>IT</th>
<th>JA</th>
<th>SZ</th>
<th>UK</th>
<th>US</th>
</tr>
</thead>
<tbody>
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<td>CA</td>
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<td>-7.1</td>
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<td>-4.9</td>
<td>-24.0</td>
<td>-1.6</td>
</tr>
<tr>
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<td>-1.9</td>
<td>1.3</td>
<td>-8.4</td>
<td>-8.1</td>
<td>4.6</td>
<td>2.6</td>
<td>2.3</td>
</tr>
<tr>
<td>IT</td>
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<td>-1.2</td>
<td>5.5</td>
<td>15.0</td>
<td>2.4</td>
<td>-1.0</td>
<td>2.5</td>
<td>-5.6</td>
</tr>
<tr>
<td>JA</td>
<td>4.5</td>
<td>8.2</td>
<td>3.2</td>
<td>14.5</td>
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<td>20.3</td>
<td>4.5</td>
<td>4.8</td>
</tr>
<tr>
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<td>-0.6</td>
<td>2.7</td>
<td>-6.9</td>
<td>4.9</td>
<td>0.8</td>
<td>13.1</td>
<td>1.1</td>
</tr>
<tr>
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<td>0.2</td>
<td>1.1</td>
<td>-4.1</td>
<td>-1.6</td>
<td>-8.1</td>
<td>-6.6</td>
<td>5.0</td>
<td>-1.6</td>
</tr>
<tr>
<td>US</td>
<td>1.0</td>
<td>3.3</td>
<td>9.6</td>
<td>-33.6</td>
<td>25.0</td>
<td>-6.3</td>
<td>-4.2</td>
<td>11.8</td>
</tr>
</tbody>
</table>
and exchange rates. The less the "domestic" currency would be suitable as
a hedge against inflation for a "domestic" investor the larger the diagonal
element of $\Phi$ for that country. If the diagonal element is negative, the
currency is a superior hedge so that the "domestic" investor would borrow
in other currencies to increase the "domestic" component of its portfolio.

The effect of a marginal increase in expenditure on Japanese
goods, for example, is an increase of .79 in the share of yen in minimum
variance portfolio $(1 - .208)^{1/}$. The same "own" effect for Canada will be
1.1 and is in general given by one minus the diagonal of the $\Phi$ matrix.
The own effect being greater than or equal to one is an indication that
the hypothesis of a "preferred monetary habitat" need not rely on trans-
actions costs, loosely defined so as to include regulations requiring pay-
ments in domestic currency. Conversely the departure from the "preferred
habitat" in the cases of Japan, Italy, the US, the UK and, less so, Germany
and Switzerland is purely based on portfolio considerations. In the case of
France the effect is almost exactly one$^{2/}$.

Similarly, the effect of a marginal increase in the share of
Japanese goods would increase demand for Canadian dollars by .41, demand
for D. marks and pound sterling by .08 and decrease demand for US dollars
by .25. These are of course the same as the minimum variance portfolio pro-
portions for a "Japanese" investor. Other significant cross effects are

---

$^1/$ Including the effect from the speculative portfolio as well, we obtain
$-.79(\gamma/1-\gamma)$, or about .4 if $\gamma = -1$, the "Samuelson presumption" of the
previous section.

$^2/$ In Kouri-Macedo (1978), in a world of five countries using wholesale prices
from April 1973 to August 1977 the quarterly own effect for France and
Japan was substantially below one and the own effect for Germany, the UK
and the US substantially higher. Traded goods price changes were mostly
unanticipated.
\( \frac{\partial x_{US}^m}{\partial \alpha_{IT}} = .32 \), \( \frac{\partial x_{FR}^m}{\partial \alpha_{UK}} = .24 \) and \( \frac{\partial x_{JA}^m}{\partial \alpha_{SZ}} = -.2 \). Increases in the share of Italian goods increase the demand for dollars and increases in the share of British goods increase demand for French francs, whereas increases in the share of Swiss goods increase demand for yen.

While all columns of \( I-H \) give the minimum variance portfolio of the respective "domestic" investor, the 8th column is particularly interesting because it also gives the minimum variance portfolio for every investor if purchasing power parity obtains.\(^1\)

This portfolio would include long positions of 88% in US dollars and 11% in Canadian dollars and short positions in all the "strong" currencies whose exchange rates show a positive correlation with US inflation.\(^2\). This is of course in sharp contrast with the minimum variance portfolios actually obtained with the three weighting schemes, reported in Table 6. The combined North American portfolio ranges from 37% (KM) to 46.5 (MERM) rather than the 99% implied by purchasing power parity.

\(^{1/}\) If, on top of this, inflation is known in the Nth country, the minimum variance portfolio disappears as analyzed in Kouri (1975) reproduced in (1977).

\(^{2/}\) The coefficients of correlation are .23 for Japan, .11 for Germany and .09 for Switzerland. The negative correlation with Italy of .23, almost twice as large as with Canada, .12, is however more than offset by the fact that the Italian variance is seven times larger than the Canadian. That Canada's exchange rate with the US dollar has by far the lowest variance is a well-known fact which has traditionally been a source of difficulties in the testing of flexible exchange rate theories. There again, it gives Canada a much larger share in the portfolio of all national investors, as is evident from the first row of \( \phi \) in Table 5. The positive relation with the Japanese yen comes from the fact that the yen has the second lowest variance (about double of the Canadian dollar's) and a positive covariance with the US price level, as pointed out in the text. The variance of the DM-dollar rate, which is the highest, is offset by the positive correlation with US prices.
Table 6

Minimum variance portfolios for alternative weighting schemes (%)

<table>
<thead>
<tr>
<th></th>
<th>XM</th>
<th>Y</th>
<th>MERM</th>
</tr>
</thead>
<tbody>
<tr>
<td>CA</td>
<td>18.6</td>
<td>18.1</td>
<td>15.4</td>
</tr>
<tr>
<td>FR</td>
<td>16.5</td>
<td>12.5</td>
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<tr>
<td>GE</td>
<td>19.9</td>
<td>13.7</td>
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</tr>
<tr>
<td>IT</td>
<td>7.2</td>
<td>5.7</td>
<td>8.0</td>
</tr>
<tr>
<td>JA</td>
<td>5.1</td>
<td>7.8</td>
<td>1.0</td>
</tr>
<tr>
<td>SZ</td>
<td>1.0</td>
<td>-0.4</td>
<td>0.6</td>
</tr>
<tr>
<td>UK</td>
<td>13.2</td>
<td>9.1</td>
<td>7.9</td>
</tr>
<tr>
<td>US</td>
<td>18.4</td>
<td>33.5</td>
<td>31.1</td>
</tr>
</tbody>
</table>
Another way to obtain the relative importance of the departure from the no inflation case is to compare the proportions of the minimum variance portfolio with the expenditure shares, their difference being given by $\delta \alpha$. Taking the absolute percent difference between $x^m$ and $\alpha$ for the income weights and comparing it with the corresponding value for the trade and MERM weights, we can confirm that the weighting scheme does not make too much difference. For income weights the largest differences between $x^m$ and $\alpha$ occurs for Canada (13%), the U.S. (-11%), and Japan (-8%). This seems to be due to the combination of the low variance of the exchange rate and the large covariance (in absolute value) with the Nth country's price level.\(^1\)

---

1/ Another comparison of interest is between the ranking in the variance in the purchasing power (Table 2 above) and in the minimum variance portfolio. For income weights the ranking is identical except for Japan and Germany, whose relative position is interchanged. In forms of variance alone U.S. is first, Canada second, Japan third and Germany seventh (last is Switzerland). Including the covariance between purchasing powers - to arrive at the minimum variance portfolio - makes Germany third and Japan seventh. With MERM weights Japan starts fourth, in terms of variance and there are minor changes in the rankings of the other currencies. With total trade weights, Japan's relative positions change in the same way as with income weights, but Germany ranks highest in the minimum variance portfolio, followed by Canada (up from 5th).
3. Turning to the speculative portfolio, the augmented inverse of the variance covariance matrix of exchange rate changes, denoted in the previous section as the $\Sigma$ matrix and used to weight the vector of real returns is reported in Table 7, after division by 400 to make it compatible with annual percentage rates of return. The diagonal elements of the $\Sigma$ matrix measure the own effect of real rate of return changes on the speculative portfolio and the off diagonal elements indicate the degree of substitutability or complementarity between currencies.

The large own effect of the U.S. dollar, and to a lesser degree of the Canadian dollar, is apparent. As shown in Table 8 last column the Canadian dollar rate exhibits a much smaller variance than any of the other 7 dollar rates and has therefore a larger own effect. The large value for the U.S. dollar comes from the sum of the cross effects of the dollar and the remaining currencies, namely the Canadian dollar.

In fact the two North American currencies are strong substitutes for each other. An increase in the real return differential of one leads to a decline of $11.5%/1-\gamma$, the share of the other in the speculative portfolio. The substitutability between the D. mark and the Swiss franc is weaker (1.6%) and in fact smaller in absolute value than the complementarity between the U.S. dollar and the D. mark.$^{1/}$

---

$^{1/}$This complementarity, pointed out in Kouri-Macedo (1978), is also found in Healy (1980a) when the sample mean is used. When a weighted average of past exchange rate changes or ARIMA techniques are used, however, the complementarity is between the D. mark and Canadian dollar. This ambiguity does not justify empirical applications of a two country framework where assets are forced to be substitutes. Such a framework has however been used to discuss the "dollar overhang" in Dornbusch (1980).
Table 8  
The Sigma Matrix  
(\%)  

<table>
<thead>
<tr>
<th></th>
<th>CA</th>
<th>FR</th>
<th>GE</th>
<th>IT</th>
<th>JA</th>
<th>SZ</th>
<th>UK</th>
<th>US</th>
</tr>
</thead>
<tbody>
<tr>
<td>CA</td>
<td>9.76</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>FR</td>
<td>1.00</td>
<td>1.65</td>
<td>2.98</td>
<td>1.08</td>
<td>1.48</td>
<td>0.25</td>
<td>0.84</td>
<td>0.08</td>
</tr>
<tr>
<td>GE</td>
<td>-1.10</td>
<td>-1.38</td>
<td>3.23</td>
<td>1.08</td>
<td>1.48</td>
<td>0.25</td>
<td>0.84</td>
<td>0.08</td>
</tr>
<tr>
<td>IT</td>
<td>1.00</td>
<td>1.08</td>
<td>1.48</td>
<td>1.00</td>
<td>1.48</td>
<td>0.25</td>
<td>0.84</td>
<td>0.08</td>
</tr>
<tr>
<td>JA</td>
<td>-0.98</td>
<td>-0.36</td>
<td>-1.64</td>
<td>0.12</td>
<td>-0.40</td>
<td>0.06</td>
<td>-0.31</td>
<td>0.06</td>
</tr>
<tr>
<td>UK</td>
<td>0.06</td>
<td>-0.26</td>
<td>0.06</td>
<td>-0.31</td>
<td>-0.40</td>
<td>0.06</td>
<td>-0.31</td>
<td>0.06</td>
</tr>
<tr>
<td>US</td>
<td>-11.46</td>
<td>-2.28</td>
<td>1.78</td>
<td>-1.58</td>
<td>-2.34</td>
<td>0.88</td>
<td>-0.40</td>
<td>15.39</td>
</tr>
</tbody>
</table>


Note also that the magnitude of own and cross effects as given by the $\Sigma$ matrix in Table 7 is larger than the difference in real rates of return in Table 4, so that the speculative portfolio for these three weighting schemes is the same at two decimal points. In Table 8, the portfolio proportions of speculative currency holdings in decreasing order are reported together with the ranking of real returns from Table 4 above and the variance of exchange rate changes (in percent per annum).

The dominant share of the U.S. dollar (28%) is exactly offset by a short position in Canadian dollars. The high positive covariance between the two rates implied by the strong cross effects leads the investor to borrow substantially in the currency with a slightly lower rate of return. The 15% long position in Swiss francs, and the 7% long position in yen are no surprise given the ranking of real returns. The short position in D. mark, a direct consequence of the complementarity with the U.S. dollar and the strong substitutability with the Swiss franc, is nevertheless startling.

The percentages indicated in Table 8 are the speculative portfolio for the Bernouilli investor. If the relative degree of risk aversion is higher the less the speculative portfolio will contribute to the total portfolio proportions. Indeed the minimum variance portfolio of Table 6 above becomes the total portfolio when risk aversion becomes infinite. In practice values of $\gamma$ of about -10 are enough to make the speculative portfolio negligible. When $\gamma=-1$ the percentages shown in Table 8 are cut into half and they are doubled for the Cramer investor.

Total portfolio proportions for the three different weighting schemes and the three degrees of risk aversion are reported in Table 9. For the Bernouilli investor the total share of the U.S. ranges from 46%
<table>
<thead>
<tr>
<th>Rank from Table 4</th>
<th>x^S(%)*</th>
<th>(\sigma^2(% \text{ p.a.}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>US</td>
<td>28</td>
</tr>
<tr>
<td>1</td>
<td>SZ</td>
<td>15</td>
</tr>
<tr>
<td>3</td>
<td>JA</td>
<td>7</td>
</tr>
<tr>
<td>2</td>
<td>GE</td>
<td>-2</td>
</tr>
<tr>
<td>4</td>
<td>FR</td>
<td>-3</td>
</tr>
<tr>
<td>8</td>
<td>IT</td>
<td>-8</td>
</tr>
<tr>
<td>7</td>
<td>UK</td>
<td>-9</td>
</tr>
<tr>
<td>6</td>
<td>CA</td>
<td>-28</td>
</tr>
</tbody>
</table>

Note: *Rounded to offset the effects of different weighting schemes on the decimals.
to 62% and the share of the D. mark from 12% to 18%. The high share of the French franc when the MERM weights are used is also noteworthy. The startling feature of Table 9 is however the sensitivity of the proportions to assumptions about $\gamma$. This is particularly true for the U.S. and Canada. The share of the U.S. dollar, for example, almost doubles from the high to the low risk aversion cases using MERM and income weights and more than doubles when trade weights are used. This suggests that the considerations that led to presume more risk aversion than the Bernouilli case in the previous section have quite an important bearing on the final proportions. Increasing risk aversion would ultimately bring the total portfolio proportions very close to the minimum variance portfolio and therefore to an optimum portfolio that would be entirely independent of returns. Optimum portfolio proportions would then become akin to the weighting schemes underlying the SDR standards described in Table 3. Indeed using $\gamma=-1$ and XM weights we find shares close to the 1980 SDR definition. The implicit share of the U.S. dollar is 42%, the share of the D. mark is 25%, the share of the French franc 20%, of sterling 11% and, surprisingly, a low 2% for the yen. Figure 4 shows the difference between the optimal dollar share and the expenditure share as a function of risk aversion. With $\gamma$ at about -2 the difference goes to zero.
Table 9

Total portfolios (%)

<table>
<thead>
<tr>
<th>Weights</th>
<th>Total trade (XM)</th>
<th>Income (Y)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\gamma=1$</td>
<td>$\gamma=0$</td>
</tr>
<tr>
<td>CA</td>
<td>4.6</td>
<td>-9.4</td>
</tr>
<tr>
<td>FR</td>
<td>15.0</td>
<td>13.5</td>
</tr>
<tr>
<td>GE</td>
<td>18.9</td>
<td>17.9</td>
</tr>
<tr>
<td>IT</td>
<td>3.2</td>
<td>-8.8</td>
</tr>
<tr>
<td>JA</td>
<td>1.6</td>
<td>12.1</td>
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<tr>
<td>SZ</td>
<td>8.5</td>
<td>16.0</td>
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<tr>
<td>UK</td>
<td>8.7</td>
<td>4.2</td>
</tr>
<tr>
<td>US</td>
<td>32.4</td>
<td>46.4</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Weights</th>
<th>MERM</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\gamma=-1$</td>
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<tr>
<td>CA</td>
<td>1.4</td>
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<tr>
<td>FR</td>
<td>19.0</td>
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<td>GE</td>
<td>14.5</td>
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<td>IT</td>
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<td>JA</td>
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<tr>
<td>SZ</td>
<td>8.1</td>
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<tr>
<td>UK</td>
<td>3.4</td>
</tr>
<tr>
<td>US</td>
<td>45.1</td>
</tr>
</tbody>
</table>

Source: Tables 6 and 8
Figure 4

Difference between the dollar share in the optimal portfolio and in consumption depending on relative risk aversion.
Conclusion

Even though an explicit test of the framework presented in Section II is beyond the scope of this paper— in part because data on actual currency proportions in central banks is highly aggregated across countries— it was claimed that the use of the indices of purchasing power and of the optimal portfolio proportions could yield a great deal of insight on the problems and proposed reforms of the present international monetary system.

Old puzzles like the strong substitution between the U.S. and the Canadian dollars were interpreted in this framework without resort to expectations of a 'normal' relative price of the two currencies. Previous findings like the complementarity between the U.S. dollar and the D. mark and the large importance of the U.S. dollar in a world without the economies of scale of the vehicle currency were confirmed in a more general framework. Other findings, like the limited attractiveness of the yen for relatively risk averse investors, came as surprises.

Significant departures from purchasing power parity, and the related presence of significant unanticipated inflation in high inflation countries were other findings of interest. All these conclusions should be further examined with indices using traded goods prices and alternative assumptions about the stationarity of covariances. These are only some of the uses that could be cited for the framework developed here. Others would include the computation of the real forward premiums taking the cross effects given by the Σ matrix into account, the investigation of a weighting scheme that would make a given portfolio optimal, the analysis of the effects of nominal return differentials and the exploration of market equilibrium.

To sum up, the framework developed in this paper shows the importance of the microeconomic foundations of international financial intermediation.
and provides a guide for currency diversification policy on the part of large organizations, in particular central banks, a policy that single exchange rate models have to neglect entirely.
Appendix

The optimal portfolio rule in the N country case

The maximization problem can be written as

$$\text{Max } U = E(x'r) - \frac{1}{2}(b)V(x'r)$$

subject to $x'e = 1$

where $x = (x_1 \ldots x_N)$ is the vector of portfolio proportions;

$r = (r_1 \ldots r_N)$ is the vector of real returns;

and $e$ is a $N$ column vector of ones and $b = 1-\gamma$.

Since nominal returns $R_i$ are known, the variance of mean return can be written as

$$V(x'r) = E(x'r r'x) = x'\Omega x$$

where $\Omega = [\omega_{ij}] = E(dq_i dq_j)^2$ is the $N$ by $N$ variance covariance matrix of changes in purchasing powers;

and $dq_i = r_i - R_i$.

Since $dq_i$ and $\omega_{ij}$ are given we find the optimal portfolio by differentiating the maximand with respect to the decision variables $x_i$, imposing the constraint and accepting short sales, so that $x_i$ can be negative. We form the Lagrangean

$$L = x'r - \frac{1}{2}b x'\Omega x + \lambda(x'e - 1).$$
Differentiating \( L \) with respect to the vector of instruments we find the \( N + 1 \) first order conditions as

\[
\frac{\partial L}{\partial x} = r - b\Omega x + \lambda e = 0 ;
\]

\[
\frac{\partial L}{\partial \lambda} = x'e - 1 = 0 .
\]

The variance covariance matrix being non singular we can solve for \( x \)

\[
x = \frac{1}{b} \Omega^{-1} r + \frac{\lambda}{b} \Omega^{-1} e
\]

using the constraint we obtain

\[
\lambda = (b - e^'\Omega^{-1} r) / e^'\Omega^{-1} e ,
\]

so that the first order conditions are rewritten as

\[
r - b\Omega x + \frac{1}{e^'\Omega^{-1} e} (b - e^'\Omega^{-1} r)e = 0
\]

or

\[
x = \frac{\Omega^{-1} e}{e^'r^{-1}e} + \frac{1}{b} \Omega^{-1} (r - e^'\Omega^{-1} r) / e^'\Omega^{-1} e .
\]

Factoring \( r \) we obtain the portfolio rule

\[
x = \frac{\Omega^{-1} e}{e^'\Omega^{-1} e} + \frac{1}{b} \Omega^{-1} \left[ I - e^'\Omega^{-1} / e^'\Omega^{-1} e \right] r .
\]
In this derivation it is obvious that the speculative portfolio is chosen by comparing the real return on each asset to the return of the minimum variance portfolio.\(^1\) Using the notation in the text, we can write the rule as

\[ x = (I - \gamma)\alpha + \frac{1}{1-\gamma} \Sigma r. \]

\(^1\)This is further elaborated in Macedo (1979), p. 23.
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